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ON THE SIGNORINI PROBLEM WITH FRICTION IN LINEAR THERMOELASTICITY: THE QUASI-COUPLED 2D-CASE

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Summary. The Signorini problem with friction in quasi-coupled linear thermo-elasticity (the 2D-case) is discussed. The problem is the model problem in the geodynamics. Using piecewise linear finite elements on the triangulation of the given domain, numerical procedures are proposed. The finite element analysis for the Signorini problem with friction on the contact boundary Γ_α of a polygonal domain $G \subset R^2$ is given. The rate of convergence is proved if the exact solution is sufficiently regular.

Keywords: contact problems, variational inequalities, numerical analysis, mechanics, geophysics.

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1. INTRODUCTION

The problem studied is a simulation of a dynamic plate tectonic model, mathematically describing the collision zones in the sense of the new global tectonics. The aim of the present paper is to extend the results of [23] to the case of plate collision with friction on the contact boundary between the colliding plates and blocks. Similar to [21], [23] we shall assume that the collision model can be investigated from the point of view of 2D – quasi-coupled thermoelasticity.

In the following we shall deal with the quasi-steady-state problem consisting of the equilibrium equation

$$(1.1) \qquad (c_{ijkl}\, e_{kl}(u) + \beta_{ij}(T - T_p))_{,j} + f_i = 0 \quad \text{in} \quad G$$

and of the heat conduction equation

$$(1.2) \qquad (\varkappa_{ij} T_{,j})_{,i} + W = \varrho c_e v_j T_{,j} \quad \text{in} \quad G\,,$$

where G is the region occupied by the obducting or subducting plate with boundary ∂G. The boundary consists of three parts, $\partial G = \bar{\Gamma}_\tau \cup \bar{\Gamma}_u \cup \bar{\Gamma}_\alpha$. Thus we consider the following three types of boundary conditions: On the Earth's surface

(1.3a–c) $$\tau_{ij}n_j = P_{0i}\,, \quad T = T_0 \quad \text{or} \quad \varkappa_{ij}T_{,j}n_i = q_0 \quad \text{on} \quad \Gamma_\tau\,,$$

where T_0, q_0 are the temperature and the heat flow and P_0 is the loading on the Earth's surface;

– on the boundary Γ_u the displacement vector u and the temperature T are prescribed, i.e.

(1.4a, b) $$u = u_0\,, \quad T = T_1 \quad \text{or} \quad \varkappa_{ij}T_{,j}n_i = 0 \quad \text{on} \quad \Gamma_u\,;$$

– the boundary Γ_α represents the contact boundary between the colliding plates, thus the Signorini conditions and the Coulombian law of friction are given:

(1.5a) $$u_n \leqq 0\,, \quad \tau_n(u) \leqq 0\,, \quad u_n\tau_n = 0 \quad \text{on} \quad \Gamma_\alpha \quad \text{(the Signorini conditions)}$$

(1.5b) $$|\tau_t(u)| \leqq \mathscr{F}|\tau_n(u)|\,, \quad |u_t|\,(|\tau_t(u)| - \mathscr{F}|\tau_n(u)| = 0$$

$$(\mathscr{F}\tau_n(u))\,(x) < 0 \Rightarrow \exists\lambda \geqq 0\,, \quad u_t(x) = -(\lambda\tau_t)\,(x)$$

(the Coulombian law of friction),

(1.5c) $$T \leqq T_2\,, \quad q \leqq 0\,, \quad (T - T_2)\,q = 0 \quad \text{on} \quad \Gamma_\alpha\,,$$

where u_n, u_t are normal and tangential components of displacement and $\tau_n(u) = \tau_{ij}(u)\,n_in_j$, $\tau_t = \tau - \tau_n n$ are the normal and tangential components of the stress vector.

In linear elasticity contact problems with friction in the sense of the Coulomb law (see [3]) were solved for the first time by Nečas et al. [15] who solved the case of a strip in R^2, and Jarušek [12]–[14] who solved the case of a strip in R^3 and the general case of the contact of three-dimensional elastic bodies with a sufficiently smooth boundary. Numerical analysis of Haslinger [7] and Haslinger, Hlaváček [8] gives the ideas how to solve these types of problems numerically. Due to the difficulties of the problem, Duvaut [2] introduced a modified friction law for the Signorini problem replacing the normal stress on the contact boundary by its mollifier.

The aim of the paper is to present the mathematical analysis of the model problem of the contemporary geodynamics as well as the theory of thermoelasticity, and to prove the existence and convergence theorems.

2. VARIATIONAL SOLUTION OF THE SIGNORINI PROBLEM WITH FRICTION

Let $G \subset R^2$ be the plane region with a Lipschitz boundary ∂G, occupied by either an obducting or a subducting plate at the moment $t = t_p$. The boundary ∂G consists of the parts $\Gamma_\tau, \Gamma_u, \Gamma_\alpha$, $\partial G = \bar{\Gamma}_\tau \cup \bar{\Gamma}_u \cup \bar{\Gamma}_\alpha$. Let $x = (x_1, x_2)$ be Cartesian coordinates. Let $n = (n_1, n_2)$, $t = (t_1, t_2) = (-n_2, n_1)$ denote the unit outward normal and tangential vectors to the boundary ∂G. Let us look for the temperature $T \in H^1(G)$

and the displacement vector $u = (u_1, u_2) \in W^1 = [H^1(G)]^2$, where $H^k(G) = W^{k,2}(G)$, $k \in R^1$ denotes the Sobolev space in the usual sense. Let $e_{ij}(u)$ be the strain tensor and let Duhamel - Neumann's law be considered, i.e.

$$(2.1\text{a, b}) \qquad \tau_{ij} = c_{ijkl}\, e_{kl}(u) - \beta_{ij}(T - T_p)\,, \quad e_{ij} = \tfrac{1}{2}(u_{i,j} + u_{j,i})$$

where $\tau_{ij} = \tau_{ij}(x)$ is the stress tensor, $T_p = T_p(x)$ is the input temperature at which the materials are in the initial strain and stress state, $\beta_{ij}(x) \in C^1(\bar{G})$ is the coefficient of the thermal expansion. The elastic coefficients $c_{ijkl}(x) \in C^1(\bar{G})$ satisfy the usual symmetry conditions and the conditions of Lipschitz continuity and ellipticity

$$(2.2\text{a, b}) \qquad c_{ijkl} = c_{jikl} = c_{klij}\,,$$
$$0 < a_0 \leqq (c_{ijkl}(x)\, \xi_{kl}\xi_{ij})\, |\xi|^{-2} \leqq A_0 < \infty \quad \forall x \in G\,, \quad \xi = (\xi_{ij}) \in R^4\,,$$

a_0, A_0 are constants independent of $x \in G$ and $\xi \in R^4$.

Let $\varkappa_{ij} = \varkappa_{ij}(x) \in C^1(\bar{G})$ be the thermal conductivity fulfilling

$$(2.2\text{c, d}) \qquad \varkappa_{ij} = \varkappa_{ji}\,, \quad \varkappa_{ij}(x)\, \zeta_i\zeta_j \geqq c|\zeta|^2\,, \quad \forall x \in G\,, \quad \forall \zeta \in R^2\,, \quad c = \text{const.} > 0\,.$$

Let $W = W(x) \in L^2(G)$ be the heat sources in the lithospheric plate, $\varrho(x) \in C(\bar{G})$ and $c_e(x) \in C(\bar{G})$ the density and the specific heat, $f \in [L^2(G)]^2$ the vector of the body forces.

The stress tensor satisfies the equilibrium conditions

$$(2.3) \qquad \tau_{ij,j} + f_i = 0\,.$$

On ∂G we define the stress vector τ, its normal and tangential components by

$$\tau_i = \tau_{ij}(x)\, n_j\,, \quad \tau_n = \tau_i n_i = \tau_{ij} n_j n_i\,, \quad \tau_t = \tau_i t_i = \tau_{ij} n_j t_i\,,$$

and the normal and tangential displacement components by $u_n = u_i n_i$ and $u_t = u_i t_i$.

Further, denote by $(\cdot, \cdot)$ the scalar product in $[L^2(G)]^2$, by $\langle \cdot, \cdot \rangle$ the scalar product in $[L^2(\Gamma_\alpha)]^2$ and by $[\cdot, \cdot]$ the scalar product in $[L^2(\partial G)]^2$. Denote by $H^{-1/2}(\Gamma_\alpha)$ the dual space of $\mathring{H}^{1/2}(\Gamma_\alpha) = \{v \mid v \in H^{1/2}(\partial G), v|_{\mathrm{cl}(\partial G \setminus \Gamma_\alpha)} = 0$, with the norm $H^{1/2}(\partial G)\}$.

We shall look for such T, u, T replaced by $T + z$, u replaced by $u + w$, where z is a sufficiently smooth scalar function in $\bar{G} = G \cup \partial G$ satisfying (1.3b), (1.4b) and $z = 0$ on Γ_α, and w is a sufficiently smooth vector function in $\bar{G} = G \cup \partial G$ satisfying (1.4a) and $w = 0$ on Γ_α. Then due to (1.1)–(1.5) and this transformation we have the following problem:

Problem (P_f): find a scalar function T and a vector function u satisfying

$$(2.4\text{a, b}) \qquad -(\varkappa_{ij}(x)\, T_{,j})_{,i} + \varrho c_e v_j T_{,j} = Q\,, \quad (c_{ijkl}(x)\, e_{kl}(u))_{,j} + F_i = 0\,,$$
$$i = 1, 2 \quad \text{in} \quad G\,,$$

where $F_i = f_i - (\beta_{ij}(T - T_p))_{,j} + (c_{ijkl} e_{kl}(w))_{,j} \in L^2(\bar{G})$,

$$Q = W + (\varkappa_{ij} z_{,j})_{,i} - \varrho c_e v_j z_{,j} \in L^2(\bar{G})\,,$$

(2.5a, b) $$T = 0\,,\quad \tau_{ij} n_j = P_i \quad \text{on} \quad \Gamma_\tau\,,$$

where $P_i = P_{0i} - c_{ijkl}\, e_{kl}(w)\, n_j$,

(2.6a, b) $$T = 0\,,\quad u_i = 0\,,\quad i = 1, 2 \quad \text{on} \quad \Gamma_u\,,$$

(2.7a–c) $$T \leqq T_2\,,\quad q \leqq 0\,,\quad (T - T_2)\, q = 0 \quad \text{on} \quad \Gamma_\alpha\,,$$

$$u_n \leqq 0\,,\quad \tau_n \leqq 0\,,\quad u_n \tau_n = 0 \quad \text{on} \quad \Gamma_\alpha\,,$$

and

$$|\tau_t(u)| \leqq \mathscr{F} |\tau_n(u)|\,,\quad |u_t| \left(|\tau_t(u)| - \mathscr{F} |\tau_n(u)|\right) = 0\,,$$

$$(\mathscr{F} \tau_n(u))\,(x) < 0 \Rightarrow \exists \lambda \geqq 0\,,\quad u_t(x) = -(\lambda \tau_t)\,(x)\,,$$

where $\mathscr{F}$ is the coefficient of friction, λ is a non-negative function on Γ_α, q is the heat flow, and $F \in [L^2(\bar{G})]^2$, $P \in [H^{-1/2}(\Gamma_\tau)]^2$, $u_0 \in [H^{1/2}(\Gamma_u)]^2$, $q_0 \in L^2(\Gamma_\tau)$, $T_p \in H^{-1/2}(G)$, $T_1 \in H^{-1/2}(\Gamma_u)$, $T_2 \in H^{-1/2}(\Gamma_\alpha)$, $Q \in L^2(\bar{G})$, $T_0 \in H^{-1/2}(\Gamma_\tau)$.

Let us suppose that $\Gamma_\alpha \in C^{2,1}(\Gamma_\alpha)$, $\mathscr{F} \in C^{0,1}$ have a compact support and let dist $(\operatorname{supp} \mathscr{F}, \partial G \setminus \Gamma_\alpha) > 0$. Let us denote by

$${}^1V = \{w \mid w \in H^1(G),\ w = 0 \text{ on } \Gamma_u \cup \Gamma_\tau \text{ in the sense of traces}\}\,,$$

$$V = \{v \mid v \in W^1,\ v = 0 \text{ on } \Gamma_u \text{ in the sense of traces}\}$$

the spaces of virtual temperatures and displacements, respectively, and by

$${}^1K = \{w \mid w \in {}^1W,\ w \leqq T_2 \text{ on } \Gamma_\alpha \text{ in the sense of traces}\}\,,$$

$$K = \{v \mid v \in V,\ v_n \leqq 0 \text{ on } \Gamma_\alpha \text{ in the sense of traces}\}$$

the sets of admissible virtual temperatures and admissible virtual displacements, respectively. Further, denote

$\mathscr{H} = \{g_n \in H^{-1/2}(\Gamma_\alpha),\ g_n \leqq 0$ in the dual sense to the ordering on $\{w \in H^{1/2}(\partial G),\ w|_{\mathrm{cl}(\partial G \setminus \Gamma_\alpha)} = 0$, provided with the norm of $H^{1/2}(\partial G)\}$ given by the restriction of the canonical ordering on $L^2(\Gamma_\alpha)\}$.

As our quasi-coupled problem is indeed not coupled, therefore both the problems in thermics and elasticity can be solved separately and the coupling terms $(\beta_{ij}(T - T_p))_{,j}$ have the meaning of body forces. Our further investigations will be based on the results of [23], [12]–[15].

We shall introduce an auxiliary problem, in which $\tau_n(u)$ in (2.7) is replaced by g_n. Let $g_n \in \mathscr{H}$ be arbitrary. For T, $w \in H^1(G)$, u, $v \in W^1$ we put

$$b(T, w) = b_1(T, w) + b_2(T, w) = \int_G (\varkappa_{ij}(x)\, T_{,j} w_{,i} + \varrho c_e v_j T_{,j} w)\, dx\,,$$

(2.8) $$s(w) = \int_G Q w\, dx + \int_{\Gamma_\tau} q_0 w\, ds \quad \text{or} \quad s(w) = \int_G Q w\, dx\,,$$

$$B(u, v) = \int_G c_{ijkl}\, e_{ij}(u)\, e_{kl}(v)\, \mathrm{d}x\,, \quad S(v) = \int_G F_i v_i\, \mathrm{d}x + \int_{\Gamma_\tau} P_i v_i\, \mathrm{d}s\,,$$

$$L(v) = L_0(v) + j_{g_n}(v)\,, \quad L_0(v) = \tfrac{1}{2}B(v, v) - S(v)\,,$$

$$j_{g_n}(v) = \langle \mathscr{F}|g_n|, |v_t|\rangle = \int_{\Gamma_\alpha} \mathscr{F}|g_n|\,|v_t|\, \mathrm{d}s\,.$$

Definition 2.1. *By a variational solution of the auxiliary problem* $(P_f)_v$ *we mean a pair of functions* (T, u), $T \in {}^1K$, $u \in K$, *such that*

(2.9a, b) $$b(T, w - T) \geqq s(w - T)\,,$$

$$B(u, v - u) + \langle \mathscr{F}|g_n|, |v_t| - |u_t|\rangle \geqq S(v - u)\,, \quad \forall w \in {}^1K\,, \quad \forall v \in K$$

where $|g_n|$ *means* $-g_n$ *for* $g_n \in \mathscr{H}$. *For* $g_n = 0$ *we have the case without friction (see* [23]*).*

For $u \in W^1$ we define $\tau_n(u)$ by

(2.10) $$\langle \tau_n(u), v_n\rangle = B(u, v) - (F, v) \quad \forall v \in W^1\,, \quad v_t|_{\Gamma_\alpha} = 0\,, \quad v_n|_{\mathrm{cl}(\delta G \setminus \Gamma_\alpha)} = 0\,.$$

Definition 2.2. *A solution* (T, u), $T \in {}^1K$, $u \in K$, *of* (2.9) *such that* $\mathscr{F}\tau_n(u) = \mathscr{F}g_n$ *with* $\tau_n(u)$ *from* (2.10) *is called a weak solution of the Signorini problem with friction in linear quasi-coupled thermoelasticity.*

Remark. As the problem is quasi-coupled the problem (2.9b) is equivalent to the following variational formulation: $u \in K$ such that

(2.9b′) $$L(u) \leqq L(v) \quad \forall v \in K\,.$$

As the set K is a nonempty, closed and convex subset of V, the functional $L_0(v)$ is due to Theorem 2.1 of [23], convex and differentiable, the functional $j_{g_n}(v) = \langle g|v_t|\rangle$, $g = \mathscr{F}|g_n|$ is convex and non-differentiable, hence due to [1], (2.9b) and (2.9b′) are equivalent.

It can be shown that any classical solution of the problem discussed is a weak solution, and conversely, if the weak solution of the discussed problem is smooth enough, then it represents a classical solution of the problem discussed.

The next theorem gives the existence and unicity of the solution of the problem $(\mathrm{P_f})_\mathrm{v}$ and an estimate of it. The theorem states that there is a unique solution and that the problem is well-posed. By the well-posedness we shall mean the existence, uniqueness, and continuous dependence of the solution on the given data (T_0, T_1, Q, u_0, F, P_0). From the physical point of view, it reflects the fact that the solution only changes a little for little changes in the displacements and surface body forces and temperatures.

Theorem 2.1. *Let* (2.2a–d) *hold. Then for every* $Q \in L^2(G)$, $q_0 \in L^2(\Gamma_\tau)$, $g \in L^\infty(\Gamma_\alpha)$, $P \in [L^2(\Gamma_\tau)]^2$, $F \in [L^2(G)]^2$ *there exists a unique solution of Problem* $(\mathrm{P_f})_\mathrm{v}$. *Further-*

more, there exist constants c_0, c_1 independent of g_n such that

$$\|T\|_1 \leqq c_0\|q_0\|_{L^2(\Gamma_\tau)} + \|Q\|_{L^2(G)} + \|T_1\|_1 + \|T_0\|_1 ,$$
$$\|u\|_{W^1} \leqq c_1\|P\|_{[L^2(\Gamma_\tau)]^2} + \|F\|_{[L^2(G)]^2} + \|u_0\|_{W^1} .$$

Proof. The bilinear form $b\colon {}^1V x\, {}^1V \to R^1$ is ${}^1V$ – elliptic and bounded on ${}^1V$ and $Q \in ({}^1V)'$. To complete the first part of the proof for the thermics, the analogy of the Lax-Milgram theorem for variational inequalities can be used. For the second part of the proof for elasticity, the set K is closed and convex in ${}^1W$, hence it is weakly closed. The functional L_0 is strictly convex and weakly lower semicontinuous (see [23]). The functional $\langle g, |v_t|\rangle$ is convex and continuous, and thus it is weakly lower semicontinuous (see [4]). According to the Korn inequality $B(u, u) \geqq c_1\|u\|_{W^1}$ which is satisfied on V and $j_{g_n}(v) = \langle g, |v_t|\rangle \geqq 0$ (for an arbitrary $g \in L^\infty(\Gamma_\alpha)$, we have $\langle g, |v_t|\rangle \geqq 0$ as $g \geqq 0$ a.e. on Γ_α), we find $L(u) \to \infty$ as $\|u\|_{W1} \to \infty$. Further, substituting $v = u_0, u_0 \in W^1, u_0|_{\Gamma_u} = 0$, into (2. b) we get $B(u, u - u_0) \geqq S(u_0 - u)$. Applying the Korn inequality, we conclude that

$$c_1\|u\|^2_{W^1} \leqq B(u, u) \leqq B(u, u - u_0) + S(u - u_0) \leqq$$
$$\leqq \|u\|_{W^1}\,\|u_0\|_{W^1} + \|F\|_{[L^2(G)]^2}\,\|u\|_{W^1} + \|P\|_{]L^2(\Gamma_\tau)]^2}\,\|u\|_{W^1} ,$$

which completes the proof. Q.E.D.

To prove the existence theorem of the contact problem with bounded friction, estimates of the admissible coefficient of friction are necessary (Theorem 2.2). The problem was completely solved by Jarušek [12], [13] for the case of linear elasticity. As we see from his results and results of Nečas et al. [15], the estimate of the maximal admissible magnitude of the coefficient of friction $\mathscr{F}$ depends on the elastic coefficients, precisely on constants a_0 and A_0 of (2.2b) near the contact surface in the nonhomogeneous anisotropic case, and on the Lamé constants λ and μ in the nonhomogeneous isotropic case, and is independent of the acting body and surface forces. As in this paper we deal with the quasi-coupled problem, which is not coupled, the results of Jarušek [12], [13] can be fully accepted also in this thermoelastic case.

Lemma 2.1. *A mapping $\Phi\colon \mathscr{F}g_n \mapsto \mathscr{F}\,\tau_n(u)$ is continuous on $H^{-1/2}(\Gamma_\alpha)$ and $\tau_n(u) \in \mathscr{H}$ for every $g_n \in \mathscr{H}$.*

For the proof see [13].

The next lemma gives the regularity of Φ.

Lemma 2.2. *A mapping Φ acts from $H^{-1/2+\gamma}(\Gamma_\alpha)$ to $H^{-1/2+\gamma}(\Gamma_\alpha)$, $\gamma \in (0, 1/2)$ such that*

$$\|\Phi(g_n)\|_{-1/2+\gamma} \leqq c(\|\mathscr{F}\|_\infty)\,\|g_n\|_{-1/2+\gamma} + c_1(F, P, u_0)\,, \tag{2.11}$$

where $\|\cdot\|_{-1/2+\gamma}$ is the norm in $H^{-1/2+\gamma}(\Gamma_\alpha)$, $\|\cdot\|_\infty$ is the norm in $L^\infty(\Gamma_\alpha)$.

The proof is based on the use of certain renormation technique by means of shifts in arguments in (2.9b), the Fourier transformation and the method of local coordinates. For the proof, see [13]. Finally, we have the following result.

Theorem 2.2. *Let G be the domain with the boundary $\partial G = \Gamma_\tau \cup \Gamma_a \cup \Gamma_u$ defined above. Let $\varkappa_{ij}(x)$, $c_{ijkl}(x)$ satisfy (2.2). Let $T_0 \in H^{-1/2}(\Gamma_\tau)$, $T_1 \in H^{-1/2}(\Gamma_u)$, $T_2 \in H^{-1/2}(\Gamma_a)$, $u_0 \in W^1$ such that $u_0|_{cl(\partial G \setminus \Gamma_u)} = 0$. Let $F \in [L^2(\bar{G})]^2$, $Q \in L^2(\bar{G})$, $q_0 \in L^2(\Gamma_\tau)$, $P \in [H^{-1/2}(\Gamma_\tau)]^2$ such that $[P, w] = 0$ for every $w \in [H^{1/2}(\partial G)]^2$ such that $w|_{cl(\partial G \setminus \Gamma_\tau)} = 0$. Let $\mathscr{F} \in C^{0,1}(\Gamma_a)$ have a compact support in Γ_a. Let*

$$(2.12a, b) \quad \|\mathscr{F}\|_\infty < \left(\frac{a_0}{2A_0}\right)^{1/2}, \quad or \quad \|\mathscr{F}\|_\infty < \left(\frac{\mu}{\lambda + 2\mu}\right)^{1/4} \sim (c_s/c_p)^{1/2}$$

for the nonhomogeneous anisotropic case or for the homogeneous isotropic case, respectively, where $c_s^2 = \mu\varrho^{-1}$, $c_p^2 = (\lambda + 2\mu)\,\varrho^{-1}$, λ, μ are the Lamé coefficients, ϱ the density, c_s, c_p the velocities of S and P waves, respectively, then there exists at least one solution of the Signorini problem with friction for the quasi-coupled case in thermoelasticity.

Proof. The proof of the thermic part of this theorem is equivalent to that of Theorem 2.1. Further, the coupling term $(\beta_{ij}(T - T_p))_{,j} \in [L^2(G)]^2$ acts as a body force (see [23]). As the estimate of $\mathscr{F}$ does not directly depend on the body and surface forces, it does not depend on the term $(\beta_{ij}(T - T_p))_{,j}$, either. Thus the same technique of Jarušek of [13] can be used. Then using (2.11), the above mentioned properties of Φ on $H^{-1/2}(\Gamma_a)$, the Sobolev imbedding theorem and the reflexivity of the spaces involved, the weak continuity of Φ on $\mathscr{H} \cap H^{-1/2+\gamma}(\Gamma_a)$ is proved. Further, a convex subset $M \subset \mathscr{H} \cap H^{-1/2+\gamma}(\Gamma_a)$ which is mapped by Φ into itself is found, provided $c(\|\mathscr{F}\|_\infty) < 1$, for which the estimate (2.12) must be fulfilled. Applying Tichonov's fixed point theorem (see e.g. [13]) we prove the existence of a fixed point of Φ, for which the appropriate u is a solution of the contact problem with friction considered. The proof is given in detail in [13] for linear elasticity.

Remark. Theorem 2.2 and especially the estimates (2.12a, b) have practical significance in geophysics, above all in the analysis of the geodynamic processes taking place in the regions of collision of lithospheric plates and blocks. The meaning of these estimates is following: the expression $\mathscr{F}|g_n| = \mathscr{F}|\tau_n|$ represents the friction forces acting on the contact boundary Γ_a, which must be overridden in order to shift the lithospheric blocks. These estimates and the estimates based on the numerical analysis give us some information about the distribution of the stress field in the collision zone and allow us to interpret some observed anomalous geophysical fields discussed by the author for the Carpathians (see [19], [22]).

3. NUMERICAL SOLUTION

For the numerical solution the finite element method, based on the Galerkin technique, will be used. Let us assume the case with the given friction $g = \mathscr{F}|g_n| \geqq 0$, $g \in L^\infty(\Gamma_a)$ a.e. on Γ_a, where g is the given frictional force. Then we define

(3.1) $$j_{g_n}(v) = \langle g, |v_t| \rangle = \int_{\Gamma_\alpha} g|v_t| \, ds .$$

Let the given bounded domain $G \subset R^2$ with the polygonal boundary ∂G be „triangulated", as in the case without friction ([23]). Let $\{\mathcal{T}_h\}$ be a system of regular triangulations defined as in [23] with the end points $\bar{\Gamma}_u \cap \bar{\Gamma}_\tau$, $\bar{\Gamma}_u \cap \bar{\Gamma}_\alpha$, $\bar{\Gamma}_\alpha \cap \bar{\Gamma}_\tau$ coinciding with the vertices of the triangles T_h.

Let ${}^1V_h$, V_h be the spaces of linear finite elements

$$ {}^1V_h = \{w \mid w \in C(\bar{G}),\ w|_{T_h} \in P_1,\ w = 0 \text{ on } \Gamma_u \cup \Gamma_\tau,\ \forall T_h \in \mathcal{T}_h\} ,$$
$$ V_h = \{v \mid v \in [C(\bar{G})]^2,\ v|_{T_h} \in [P_1]^2,\ v = 0 \text{ on } \Gamma_u,\ \forall T_h \in \mathcal{T}_h\} ,$$

where P_1 is the space of linear polynomials. Further, define

$$ {}^1K_h = \{w \mid w \in {}^1V_h,\ w \leqq T_2 \text{ on } \Gamma_\alpha\} = {}^1V_h \cap {}^1K , \quad \text{hence} \quad {}^1K_h \subset {}^1K \quad \text{for} \quad \forall h ,$$
$$ K_h = \{v \mid v \in V_h,\ v_n \leqq 0 \text{ on } \Gamma_\alpha\} = V_h \cap K , \quad \text{hence} \quad K_h \subset K \quad \text{for} \quad \forall h .$$

Definition 3.1. *We say that a pair of functions* (T_h, u_h), $T_h \in {}^1K_h$, $u_h \in K_h$ *is a finite element approximation of the problem* $(\mathrm{P_f})_\mathrm{v}$, *if*

(3.2a, b) $$b(T, w - T_h) \geqq s(w - T_h) ,$$
$$B(u, v - u_h) + \langle g, |v_t| - |(u_h)_t| \rangle \geqq S(v - u_h) \quad \forall w \in {}^1K_h , \quad v \in K_h .$$

Remark. As the problem is quasi-coupled, a finite element approximation (3.2b) of the primary problem (2.9b) is equivalent to the following formulation:

find $u_h \in K_h$ such that

(3.2b′) $$L(u_h) \leqq L(v) \quad \forall v \in K_h .$$

Theorem 3.1. *There exists a unique solution of the finite element approximation* (3.2) *for* $\forall h$, $h \in (0, 1)$.

Proof. Since K_h is a nonempty, closed and convex subset of W^1, it is weakly closed. Thus the proof of both parts of the theorem is similar to that of Theorem 2.2, in which the functional $j_{g_n}(v) = \langle g, |v_t| \rangle$ is convex. Q.E.D.

Lemma 3.1. *For* $T \in {}^1K$, $u \in K$, $T_h \in {}^1K_h$, $u_h \in K_h$ *we have*

(3.3a, b) $$\|T - T_h\|_1 \leqq c\{b(T_h - T, w_h - T) + b(T, w - T_h) +$$
$$+ b(T, w_h - T) - (Q, w - T_h) - (Q, w_h - T)\}^{1/2} \quad \forall w \in {}^1K , \quad w_h \in {}^1K_h ,$$
$$c = \text{const.} > 0 ,$$
$$\|u - u_h\|_{W^1} \leqq c_0\{B(u_h - u, v_h - u) + B(u, v - u_h) +$$
$$+ B(u, v_h - u) - (F, v - u_h) - (F, v_h - u) + j_{g_n}(v_h) - j_{g_n}(u) +$$
$$+ j_{g_n}(v) - j_{g_n}(u_h)\}^{1/2} , \quad \forall v \in K , \quad v_h \in K_h , \quad c_0 = \text{const.} > 0 .$$

The proof is similar to that in [5], [23].

Theorem 3.2. *Let Γ_α be polygonal. Let* $T_2 \in H^2(\Gamma_\alpha) \cap H^1(\Gamma_\alpha)$, $T \in {}^1K \cap H^2(G)$, $T|_{\Gamma_\alpha} \in H^2(\Gamma_\alpha)$, $u \in K \cap W^2$, $\tau(u) \in [L^\infty(\Gamma_\alpha)]^2$, $u \in [W^{1,\infty}(\Gamma_\alpha)]^2$, $g \in L^\infty(\Gamma_\alpha)$. *Let* ${}^1K_h \subset {}^1K$, $K_h \subset K$. *Let the changes* $u_n < 0 \to u_n = 0$ *and* $u_t = 0 \to u_t \neq 0$ *occur at finitely many points of* Γ_α *only. Then*

$$\|T - T_h\|_1 = 0(h), \quad \|u - u_h\|_{W^1} = 0(h).$$

Proof. To prove this theorem, the technique of the proofs of Theorems 2.6 and 2.10 of [23] will be used. We have ${}^1K_h \subset {}^1K$, $K_h \subset K$, $\forall h \in (0, 1)$. Using Lemma 3.1, Green's theorem and the assumption that T and u are sufficiently regular we obtain

$$\text{(3.4a)} \qquad \|T - T_h\|_1^2 \leqq c\Big\{1/2\varepsilon\|T - T_h\|_1^2 + 1/2\varepsilon^{-1}\|T - w_h\|_1^2 + $$

$$+ c_3\|T_h - T_1\|_1 \|w_h - T\|_{L^2(G)} + \int_{\Gamma_\alpha} T_{,n}(w - T_h)\,\mathrm{d}s + c_2\|w_h - T\|_{L^2(\Gamma_\alpha)}\Big\}^{1/2},$$

$$\varepsilon > 0 \quad \text{arbitrary},$$

for the thermal part, and

$$\text{(3.4b)} \qquad \|u - u_h\|_{W^1} \leqq c_0\Big\{1/2\varepsilon\|u_h - u\|_{W^1}^2 + 1/2\varepsilon^{-1}\|v_h - u\|_{W^1}^2 + $$

$$+ \int_{\partial G} \tau_{ij}(u)\, n_j(v_h - u)_i\,\mathrm{d}s\Big\}^{1/2}, \quad \varepsilon > 0 \quad \text{arbitrary},$$

for the elastic part.

To estimate the right hand side of (3.4a) we put $w_h = T_{LI}$, where $T_{LI} \in {}^1V_h$ is the Lagrange interpolation of T on the triangulation $\mathscr{T}_h$. As $(T_{LI})_n \leqq T_2$ on Γ_α, we have $T_{LI} \in {}^1K$ and since $T_{LI} \in {}^1V_h$, also $T_{LI} \in {}^1K_h$. Thus

$$\text{(3.5)} \qquad \|T_{LI} - T\|_1 \leqq c_r h\|T\|_2, \quad \|(T_{LI})_n - T_n\|_{L^2(\Gamma_\alpha)} \leqq c_s h^2\, \Sigma\|T_h\|_{H^2(\Gamma_\alpha)},$$

$$\|T_{LI} - T\|_{L^2(G)} \leqq c_p h^2\|T\|_2.$$

Due to [23],

$$\int_{\Gamma_\alpha} T_{,n}(w - T_h)\,\mathrm{d}s = 0(h^4).$$

Thus

$$\|T - T_h\|_1 \leqq c\{1/2\varepsilon c_{r1}\, h\|T\|_2 + 1/2\varepsilon^{-1} c_{r2}\, h\|T\|_2 + $$

$$+ c_3 c_r\, h\|T\|_2 + 0(h^4) + c_2 c_s h^2 \Sigma\|T_h\|_{H^2(\Gamma_\alpha)} = 0(h),$$

which proves the first part of the theorem.

To estimate the right hand side of (3.4b) we set $v_h = u_{LI}$, where $u_{LI} \in V_h$ is the Lagrange interpolation of u on the triangulation $\mathscr{T}_h$. As $(u_{LI})_n \leqq 0$ on Γ_α, $u_{LI} \in K$. Since $u_{LI} \in V_h$, we have $u_{LI} \in K_h$. Thus

$$\|u_{LI} - u\|_{W^1} \leqq c_r\, h\|u\|_{W^2}, \quad \|(u_{LI})_n - u_n\|_{[L^2(\Gamma_\alpha)]^2} \leqq c_s h^2 \Sigma\|u_n\|_{[H^2(\Gamma_\alpha)]^2}.$$

In (3.3b) the terms $B(u, v - u_h) - (F, v - u_h)$ and $B(u, v_h - u) - (F, v_h - u)$ are estimated by using Green's theorem and later by using a suitable choice of $v_h \in K_h$, $v \in K$. Thus applying Green's theorem we obtain

$$B(u, v_h - u) - (F, v_h - u) = \int_{\Gamma_\alpha} \tau_n(u)\,(v_h - u)_n\,\mathrm{d}s + \int_{\Gamma_\alpha} \tau_t(u)\,(v_h - u)_t\,\mathrm{d}s \leqq 0\,.$$

To estimate the integrals

$$J_1(\Gamma_\alpha) = \int_{\Gamma_\alpha} \tau_n(u)\,(v_h - u)_n\,\mathrm{d}s \quad \text{and} \quad J_2(\Gamma_\alpha) = \int_{\Gamma_\alpha} \tau_t(u)\,(v_h - u)_t\,\mathrm{d}s$$

we assume that $\Gamma_\alpha = \bigcup_i \Gamma_{\alpha_i}$, where $\bigcup_i \bar{\Gamma}_{\alpha_i}$ approximates piecewise linearly the boundary Γ_α. To estimate the first integral we distinguish several cases:

– the case $u_n(x) < 0$, $x \in \bar{\Gamma}_{\alpha_i}$. Then due to $u_n\tau_n = 0$, the integral $J_1(\bar{\Gamma}_{\alpha_i}) = 0$.

– the case $u_n(x) = 0$ and $u_n(x) < 0 \to u_n = 0$, $x \in \bar{\Gamma}_{\alpha_i}$. Let us put $v_h = u_{LI}$.

Due to the properties of u_{LI} discussed above, either $(u_{LI})_n = 0$ on $\bar{\Gamma}_{\alpha_i}$ and then $J_1(\bar{\Gamma}_{\alpha_i}) = 0$ or $(u_{LI})_n < 0$ and then

$$|J_1(\bar{\Gamma}_{\alpha_i})| \leqq \|(u_{LI})_n - u_n\|_{L^\infty(\bar{\Gamma}_{\alpha_i})} \int_{\bar{\Gamma}_{\alpha_i}} |\tau_n|\,\mathrm{d}s \leqq c_{s_i} h^2\,.$$

The last inequalities hold because for $u \in [W^{1,\infty}(\Gamma_\alpha)]^2$ and $\tau \in [L^\infty(\Gamma_\alpha)]^2$, $u_n \in W^{1,\infty}(\Gamma_{\alpha_i})$, $\tau_n \in L^\infty(\bar{\Gamma}_{\alpha_i})$, and because $(u_{LI})_n$ is the Lagrange interpolation of u_n. Hence $|J_1(\Gamma_\alpha)| \leqq c_s h^2$.

Now we estimate $|\int_{\Gamma_\alpha} \tau_t(u)\,(v_h - u)_t\,\mathrm{d}s + j_{g_n}(v_h) - j_{g_n}(u)|$. We have several cases:

– the case $u_t(x) > 0$ for $x \in \Gamma_\alpha$. Due to (3.2b) we have $J_1(\Gamma_\alpha) + J_2(\Gamma_\alpha) + j_{g_n}(v) - j_{g_n}(u) \geqq 0$. Further, $g|u_t| + \tau_t u_t = 0$ a. e. on Γ_α. Hence $g = -\tau_t$ a.e. on $\bar{\Gamma}_{\alpha_i}$. Let us put $v_h = u_{LI}$. Thus $(u_{LI})_t = (u_t)_{LI}$ and $(u_{LI})_t\,(s_i)$, $s_i \in \Gamma_{\alpha_i}$, s_i are the points of the triangulation on Γ_α. Then $(u_t)_{LI} > 0$ on $\bar{\Gamma}_{\alpha_i}$ and

$$\int_{\Gamma_{\alpha_i}} \{-g[(u_t)_{LI} - u_t] - g[(u_t)_{LI} - u_t]\}\,\mathrm{d}s = 0\,,$$

– the case $u_t(x) = 0$ for $x \in \Gamma_\alpha$. As $(u_t)_{LI} = 0$ on $\bar{\Gamma}_{\alpha_i}$, we have

$$\int_{\Gamma_{\alpha_i}} \{\tau_t(u)\,[(u_t)_{LI} - u_t] + g[|(u_t)_{LI}| + |u_t|]\}\,\mathrm{d}s = 0\,.$$

– the case $u_t(x) < 0$ for $x \in \Gamma_\alpha$. Since $u_t < 0$ on Γ_α, we have $|u_t| = -u_t$ and $(u_t)_{LI} < 0$. Thus $g = \tau_t$ a.e. on $\bar{\Gamma}_{\alpha_i}$. Let us set $v_h = u_{LI}$. Then

$$\int_{\bar{\Gamma}_{\alpha_i}} \{g[(u_t)_{LI} - u_t] + g[-(u_t)_{LI} + u_t]\}\,\mathrm{d}s = 0\,,$$

– the case when $u_t(x) = 0$ changes to $u_t(x) \neq 0$ for $x \in \Gamma_\alpha$. Let $v_h = u_{LI}$. Since $u \in [W^{1,\infty}(\Gamma_\alpha)]^2$, $\tau \in [L^\infty(\Gamma_\alpha)]^2$, $g \in L^\infty(\Gamma_\alpha)$, we have $u_t \in W^{1,\infty}(\bar{\Gamma}_{\alpha_i})$ and $\tau_t \in L^\infty(\bar{\Gamma}_{\alpha_i})$.

Then

$$\left|\int_{\Gamma_{\alpha_i}} \{\tau_t(u)\,[(u_t)_{LI} - u_t] + g[|(u_t)_{LI}| - |u_t|]\}\,\mathrm{d}s\right| \leqq \|(u_t)_{LI} - u_t\|_{L\infty(\bar{\Gamma}_{\alpha_i})} \,.$$

$$\cdot \int_{\bar{\Gamma}_{\alpha_i}} (|\tau_t(u)| + g)\,\mathrm{d}s \leqq c_{r_i} h^2 \,.$$

Thus

$$\left|\int_{\Gamma_\alpha} \tau_t(u)\,[(u_t)_{LI} - u_t]\,\mathrm{d}s + j_{g_n}(u_{LI}) - j_{g_n}(u)\right| \leqq c_r h^2 \,.$$

Finally, we obtain

$$\|u - u_h\|_{W^1} \leqq \bar{c}_o\{\varepsilon h\|u\|^2_{W^1} + \varepsilon^{-1}h^2\|u\|^2_{W^1} + c_s h^2 + c_r h^2\}^{1/2} = O(h)\,,$$

which completes the proof.

ALGORITHM

Since the problem is quasi-coupled the algorithm is divided into two parts. The first for the thermal part is based on the technique of quadratic programming. The second for the elastic contact problem with friction is based on the numerical approximation of a saddle point [1], [16]. Sufficient conditions for the existence of a saddle point can be found in [4].

Definition 3.2. *A point $(u_{sh}, \lambda_h) \in K_h \times \Lambda \subset K \times \Lambda$ is said to be an approximate saddle point of a functional $\mathscr{L}$ on $K_h \times \Lambda$ if a saddle point $(u_s, \lambda) \in K \times \Lambda$ exists and if*

$$\mathscr{L}(u_{sh}, \mu) \leqq \mathscr{L}(u_{sh}, \lambda_h) \leqq \mathscr{L}(v, \lambda_h) \quad \forall(v, \mu) \in K_h \times \Lambda\,.$$

The problem (3.2b) is equivalent to the following problem:

$$\text{find} \quad u_h \in K_h \quad \text{such that}$$

$$L(u_h) = \min_{v \in K_h} \sup_{\mu \in \Lambda} \mathscr{L}(v, \mu)$$

where $\Lambda = \{\mu|\ \mu \in L^2(\Gamma_\alpha), |\mu| \leqq 1 \text{ a.e. on } \Gamma_\alpha\}$, $\mathscr{L}$ is the Lagrangian, $K_h \subset K \subset V$. Then $\int_{\Gamma_\alpha} g|v_t|\,\mathrm{d}s = \sup_{\mu \in \Lambda} \int_{\Gamma_\alpha} \mu g v_t\,\mathrm{d}s$, where $g v_t$ is a Lipschitz operator mapping $V \to L^2(\Gamma_\alpha)$.

Thus we have the following problem:

$$\text{find} \quad u_h \in K_h \quad \text{such that}$$

$$L(u_h) = \min_{v \in K_h} \sup_{\mu \in \Lambda} \mathscr{L}(v, \mu) = \min_{v \in K_h} \sup_{\mu \in \Lambda} \left\{L_0(v) + \int_{\Gamma_\alpha} \mu g v_t\,\mathrm{d}s\right\}.$$

The existence of a saddle point follows from Propositions 1.2 and 2.2 of [4] and the Korn inequality. By Proposition 2.2 we also obtain uniqueness of its approximation $(u_{sh}, \lambda_h) \in K_h \times \Lambda$.

Let $v \in K$. Then there exists $v_h \in K_h$ such that $\|v_h - v\|_{h \to 0} \to 0$. Thus a sequence $\{v_h\}$, $v_h \in K_h$ can be chosen such that $v_h \to u_s$ in V. Due to Propositions 1.2 and 2.2 of [4] and this assumption,

$$\mathscr{L}(u_{sh}, \lambda_h) = \min_{v \in K_h} \sup_{\mu \in \Lambda} \mathscr{L}(v, \mu) = \min_{v \in K_h} L(v) = L(u_{sh}) \leqq L(v_h) \xrightarrow[h \to 0]{} L(u_s)$$

as the functional L is continuous. Since Λ is closed and $\lim_{\|v\| \to \infty} L(v) \to \infty$ subsequences $\{u_{shi}\}, \{\lambda_{hi}\}$ of $\{u_{sh}\}, \{\lambda_h\}$ can be chosen such that $u_{shi} \rightharpoonup u_0$ in V, $\lambda_{hi} \rightharpoonup \lambda_0$ in Λ. Since $v_h \in K_h$, $v_h \rightharpoonup v$ in V we have $v \in K$ and thus $u_0 \in K$. Since Λ is closed, it is weakly closed and thus $\lambda_0 \in \Lambda$.

As $L(u_{sh}) \leqq L(v_h) \xrightarrow[h \to 0]{} L(u_s)$ and as L is weakly lower semicontinuous, we have

$$L(u_0) \leqq \liminf_{h_i \to 0} L(u_{shi}) \leqq L(u_s) .$$

Since u_s is a solution of the problem (2.9b′), we have $u_s = u_0$ and the sequence $\{u_{shi}\}$ can be chosen quite arbitrarily. Thus $\{u_{sh}\} \to u_s$. As

$$L(v_h) \geqq L(u_{sh}) = L_0(u_{sh}) + j_{g_n}(u_{sh}) \geqq L_0(u_{sh}) + B(u_s, u_{sh} - u_s) -$$
$$- S(u_{sh} - u_s) + j_{g_n}(u_{sh}) + 1/2c_0\|u_{sh} - u_s\|^2 , \quad \forall v_h \in K_h ,$$

we have

$$\|u_s - u_{sh}\| \leqq c\{L_0(v_h) + j_{g_n}(v_h) - L_0(u_{sh}) +$$
$$+ B(u_s, u_s - u_{sh}) - S(u_s - u_{sh}) - j_{g_n}(u_{sh})\}^{1/2} , \quad \forall v_h \in K_h .$$

Let us choose $v_h \in K_h$ such that $v_h \to u_s$. Then $L_0(v_h) \to L_0(u_s)$, $j(v_h) \to j(u_s)$ for $h \to 0_+$. Thus

$$\lim_{h \to 0_+} \sup \|u_s - u_{sh}\| \leqq c\{j_{g_n}(u_s) - \liminf_{h \to 0_+} j_{g_n}(u_{sh})\}^{1/2} \leqq 0$$

as the functional j_{g_n} is weakly lower semicontinuous. Thus $\|u_s - u_{sh}\| \to 0$ for $h \to 0_+$.

As (u_{sh}, λ_h) is saddle point of $\mathscr{L}$ on $K_h \times \Lambda$, we have

$$L_0(u_{sh}) + \int_{\Gamma_\alpha} \mu g(u_{sh})_t \, ds \leqq L_0(u_{sh}) + \int_{\Gamma_\alpha} \lambda_h g(u_{sh})_t \, ds , \quad \forall \mu \in \Lambda .$$

Hence

$$\int_{\Gamma_\alpha} (\mu - \lambda_h) \, g(u_{sh})_t \, ds \leqq 0 \quad \forall \mu \in \Lambda \quad \text{and}$$

$$\int_{\Gamma_\alpha} ((\lambda_h + \varrho g(u_{sh})_t) - \lambda_h)(\mu - \lambda_h) \, ds \leqq 0 \quad \forall \varrho > 0 , \quad \forall \mu \in \Lambda .$$

Thus

(3.6) $\quad \lambda_h = P(\lambda_h + \varrho g(u_h)_t) , \quad \varrho > 0 , \quad P$ is a projection of $L^2(\Gamma_\alpha)$ onto Λ .

Due to Proposition 1.7 of [4],

$$B(u_{sh}, v - u_{sh}) + \int_{\Gamma_\alpha} \lambda_h g(v_t - (u_{sh})_t)\, \mathrm{d}s \geqq S(v - u_{sh}) \quad \forall v \in K_h$$

or

$$L_0(u_{sh}) + \int_{\Gamma_\alpha} \lambda_h g(u_{sh})_t\, \mathrm{d}s \leqq L_0(v) + \int_{\Gamma_\alpha} \lambda g v_t\, \mathrm{d}s \quad \forall v \in K_h\,. \tag{3.7}$$

The inequalities (3.7) and (3.6) give an idea for numerical solution. Such a type of problems are solved by Uzawa's or Arrow-Hurwicz's algorithms (see e.g. in [1], [4], [6], or [16]).

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Souhrn

SIGNORINIHO ÚLOHA SE TŘENÍM V LINEÁRNÍ TERMOELASTICITĚ: QUASI-SDRUŽENÝ 2D-PŘÍPAD

JIŘÍ NEDOMA

V článku je diskutována Signoriniho úloha se třením v quasi-sdružené lineární termopružnosti (2D-případ). Diskutovaná úloha je modelovou úlohou v geodynamice litosférických desek. Je dokázána existence a jednoznačnost řešení modelové úlohy. Je dokázána konvergence konečně prvkové aproximace k přesnému řešení. Je proveden odhad koeficientu tření v závislosti na fyzikálních parametrech prostředí.

Резюме

ЗАДАЧА СИНЬОРИНИ С ТРЕНИЕМ ДЛЯ ЛИНЕЙНОЙ ТЕРМОУПРУГОСТИ: КВАЗИ-СОПРЯЖЕННЫЙ 2D СЛУЧАЙ

JIŘÍ NEDOMA

В статье обсуждается задача Синьорини с трением для квази-сопряженной линейной термоупругости. Обсуждаемая задача является за моделью для геодинамики литосферических плит. Доказывается существование решения модельной задачи и его однозначность. Далее приводится доказательство сходимости апроксимации с конечным числом элементов к точному решению. Дается оценка коэффициента трения в зависимости от физических параметров среды.

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