Josef Kolomý

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(Preliminary communication)

J. KOLOMÝ, Praha

The method of successive approximations has been developed in recent years by many mathematicians (for instance see [1] - [6]). But in practical computations it is often shown that the method of successive approximations converges very slowly. Hence it is important to accelerate this method. For such accelerating methods see for example [7] -[9] .

In the present note we introduce a simple new method of solving the equation

(1) $$A x = f ,$$

where A is a linear bounded operator in real or complex Hilbert space H , $f \in H$. The method is based on the following theorem.

<u>Theorem</u>. Let A be a linear bounded operator in H such that A^{-1} exists and is bounded in H. Furthermore let the inequality $0 < \vartheta < 1/\|A\|^2$ be fulfilled. Then the sequence $\{x_n\}$ defined by equalities:

(2) $$x_{n+1} = \vartheta A^* f + \beta_n (I - \vartheta A^* A) x_n ,$$
$$\beta_n = Re \langle f, A x_n \rangle / \| A x_n \|^2, \quad (n = 0, 1, 2..)$$

converges in the norm (of H) to the unique solution x^* of (1), $\| x^* - x_n \| = O(q^n)$, where $q = \| I - \vartheta A^* A \| < 1$, A^* is adjoint with A and $x_0 \neq 0$ is an arbitrary element from H. Further-more $\|x_{n+1} - x^*\| < \|x_n - x^*\|$ for every n ($n = 0, 1, 2..$).

We shall omit the proof of this theorem, because we intend to publish it with proofs of [9] and other theorems in another paper.

The parameters β_n ($n = 0, 1, 2, \dots$) are determined in (2) from the conditions, that $\|x^* - \beta_n x_n\|^2 = Min$ (for $n = 0, 1, 2, \dots$). It is clear that this method is quicker than the method of successive approximations and all so it is simpler than the methods of the type of steepest descent [10].

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