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CONCERNING A PROOF OF $\aleph_{\alpha+1} \leq 2^{\aleph_\alpha}$ WITHOUT AXIOM OF CHOICE : A CORRECTION

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The last three statements on p.112 (CMUC 6,1) are to be corriged:

Theorem. Any regular cardinal number $\aleph_{\alpha+1}$ in Σ such that $\aleph_{\alpha+1} \nleq 2^{\aleph_\alpha}$ is an inaccessible cardinal number in Δ .

Corollary 1. If the system of axioms $\Sigma_0 + (\exists \alpha)$ $[\aleph_{\alpha+1} \nleq 2^{\aleph_\alpha} \;\&\; \aleph_{\alpha+1}$ regular$]$ is consistent, the system $\Sigma^* +$ "there exists an inaccessible cardinal number" is consisten· ·oo.

Corollary 2. If the existence of an inaccessible cardinal number contradicts ·ith the axioms of set theory, then " $\aleph_{\alpha+1}$ regular $\rightarrow \aleph_{\alpha+1} \leq 2^{\aleph_\alpha}$ " is provable without using of the axiom of choice.

Similar result is proved in the paper of Specker (Zur Axiomatik der Mengenlehre, Zeits. für Math. Logik 3(1957) p. 203, 2.32.