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Concerning a proof of  $\aleph_{\alpha+1} \leq 2^{\aleph_\alpha}$  without axiom of choice: A correction

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CONCERNING A PROOF OF  $\aleph_{\alpha+1} \leq 2^{\aleph_\alpha}$  WITHOUT AXIOM OF  
CHOICE : A CORRECTION

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The last three statements on p.112 (CMUC 6,1) are to be corrected:

Theorem. Any regular cardinal number  $\aleph_{\alpha+1}$  in  $\Sigma$  such that  $\aleph_{\alpha+1} \neq 2^{\aleph_\alpha}$  is an inaccessible cardinal number in  $\Delta$ .

Corollary 1. If the system of axioms  $\Sigma_0 + (\exists \alpha)$  [ $\aleph_{\alpha+1} \neq 2^{\aleph_\alpha}$  &  $\aleph_{\alpha+1}$  regular] is consistent, the system  $\Sigma^* +$  "there exists an inaccessible cardinal number" is consistent too.

Corollary 2. If the existence of an inaccessible cardinal number contradicts with the axioms of set theory, then " $\aleph_{\alpha+1}$  regular  $\rightarrow \aleph_{\alpha+1} \leq 2^{\aleph_\alpha}$ " is provable without using of the axiom of choice.

Similar result is proved in the paper of Specker (Zur Axiomatik der Mengenlehre, Zeits. für Math. Logik 3(1957) p. 203, 2.32.