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Исправление к: "Смешанные краевые задачи уравнения теплопроводности для бесконечной полосы"

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ИСПРАВЛЕНИЕ К: "СМЕШАННЫЕ КРАЕВЫЕ ЗАДАЧИ УРАВНЕНИЯ ТЕПЛОПРОВОДНОСТИ ДЛЯ БЕСКОНЕЧНОЙ ПОЛОСЫ"

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Куйбышев

В нашей статье того-же названия опубликованной в CMUC 9,1 (1968), при получении уравнения (5) (стр.5) по нашему недосмотру допущена неточность, должно быть:

$\bar{\bar{u}}'_{+}(p,l,s) = \bar{\bar{u}}'_{+}(p,0,s)$, поэтому решение задачи сводится к функциональному уравнению, не совпадающему с функциональным уравнением для полуплоскости, а к уравнению аналогичному ему:

$$(5)\quad \bar{\bar{u}}_{-}(p,0,s)\gamma + \bar{\bar{u}}'_{+}(p,0,s)\, th\frac{\gamma l}{2} = -\gamma \bar{\bar{f}}_2(p,s), \quad -m<\beta<m,$$

которое после факторизации принимает вид

$$(6)\quad \bar{\bar{u}}'_{+}(p,0,s)\frac{\Gamma(\frac{1}{2}+i\frac{\gamma l}{2\pi})}{i\sqrt{p+ik}\,\Gamma(i\frac{\gamma l}{2\pi})} + \bar{\bar{u}}_{-}(p,0,s)\frac{\sqrt{p-ik}\,\Gamma(1-i\frac{\gamma l}{2\pi})}{\Gamma(\frac{1}{2}-i\frac{\gamma l}{2\pi})} = -\frac{\bar{\bar{f}}_2(p,s)\sqrt{p-ik}\,\Gamma(1-i\frac{\gamma l}{2\pi})}{\Gamma(\frac{1}{2}-i\frac{\gamma l}{2\pi})}.$$

Решая уравнение (6), находим:

$$\underline{\underline{\bar{u}_{(\sigma)}(x,y,t)}} = -\frac{\sqrt{a}}{e\sqrt{\pi}}\int_0^{t+0} dt_0 \int_0^{\infty} \frac{W_2(x_0,t_0)}{\sqrt{t-t_0}} \exp\left[-\frac{(x-x_0)^2}{4a(t-t_0)}\right]\cdot$$

$$\cdot\left\{1+2\sum_{m=1}^{\infty} \exp\left[-\frac{4a\pi^2 m^2}{l^2}(t-t_0)\right]\cdot\cos\frac{2\pi m y}{l}\right\}dx_0,$$

где

$$W_2(x,t) = -\frac{1}{2\pi\sqrt{2\pi}}\int_{\sigma_1-i\infty}^{\sigma_1+i\infty} e^{st}ds \int_{\sigma_2-i\infty}^{\sigma_2+i\infty} e^{-ipx}\left\{\frac{\Gamma(i\frac{\gamma l}{2\pi})\bar{\bar{f}}_2(p,s)\sqrt{p^2+k^2}\,\Gamma(1-i\frac{\gamma l}{2\pi})}{\Gamma(\frac{1}{2}+i\frac{\gamma l}{2\pi})\cdot\Gamma(\frac{1}{2}-i\frac{\gamma l}{2\pi})}+\right.$$

$$+\frac{\Gamma(i\frac{\gamma l}{2\pi})\sqrt{p+ik}}{\Gamma(\frac{1}{2}+i\frac{\gamma l}{2\pi})}\Big[\Big[\sum_{n=0}^{\infty}\frac{\bar{\bar{f}}_2\big[i\sqrt{k^2+\frac{4\pi^2}{l^2}(n+1)^2};s\big]\sqrt{i}\sqrt{\sqrt{k^2+\frac{4\pi^2}{l^2}(n+1)^2}-k}\,(-1)^n 2^{n+1}}{n!\,(2n+1)!!\sqrt{\pi}\,(i\sqrt{k^2+\frac{4\pi^2}{l^2}(n+1)^2}-p)}+$$

$$+\frac{1}{2\pi}\int_0^{\infty}\frac{\bar{\bar{f}}_2(i\rho+ik,s)\sqrt{i\rho}}{(i\rho+ik-p)}\Big[\frac{\Gamma(1+\frac{l}{2\pi}\sqrt{\rho^2+2\rho k})}{\Gamma(\frac{1}{2}+\frac{l}{2\pi}\sqrt{\rho^2+2\rho k})}+\frac{\Gamma(1-\frac{l}{2\pi}\sqrt{\rho^2+2\rho k})}{\Gamma(\frac{1}{2}-\frac{l}{2\pi}\sqrt{\rho^2+2\rho k})}\Big]d\rho\Big]\Big]\Big\}dp;$$

$$\underline{\underline{u_{(a)}(x,y,t)}}=-\frac{2\sqrt{a}}{l\sqrt{\pi}}\int_0^{t+0}dt_0\int_0^{\infty}\frac{W_1(x_0,t_0)}{\sqrt{t-t_0}}\exp\Big[-\frac{(x-x_0)^2}{4a(t-t_0)}\Big]\sum_{m=0}^{\infty}\exp\Big[-\frac{a\pi^2(2m+1)^2}{l^2}\cdot$$

$$\cdot(t-t_0)\Big]\cos\frac{(2m+1)\pi y}{l}\,dx_0\ ,$$

где

$$W_1(x,t)=\frac{1}{2\pi\sqrt{2\pi}}\int_{\sigma_1-i\infty}^{\sigma_1+i\infty}e^{st}ds\int_{\sigma_2-i\infty}^{\sigma_2+i\infty}e^{-ipx}\Big\{\frac{\bar{\bar{f}}_1(p,s)\ k^2+p^2\,\Gamma(\frac{1}{2}-i\frac{\gamma l}{2\pi})\Gamma(\frac{1}{2}+i\frac{\gamma l}{2\pi})}{\Gamma(i\frac{\gamma l}{2\pi})\Gamma(1-i\frac{\gamma l}{2\pi})}-$$

$$-\frac{\sqrt{p+ik}\,\Gamma(\frac{1}{2}+i\frac{\gamma l}{2\pi})}{2\pi\,\Gamma(i\frac{\gamma l}{2\pi})}\int_0^{\infty}\frac{\bar{\bar{f}}_1(i\rho+ik,s)\sqrt{i\rho}}{(p-ik-i\rho)}\Big[\frac{\Gamma(\frac{1}{2}+\frac{l}{2\pi}\sqrt{\rho^2+2\rho k})}{\Gamma(1+\frac{l}{2\pi}\sqrt{\rho^2+2\rho k})}+$$

$$+\frac{\Gamma(\frac{1}{2}-\frac{l}{2\pi}\sqrt{\rho^2+2\rho k})}{\Gamma(1-\frac{l}{2\pi}\sqrt{\rho^2+2\rho k})}\Big]d\rho-\frac{\sqrt{p+ik}\,\Gamma(\frac{1}{2}+i\frac{\gamma l}{2\pi})}{\Gamma(i\frac{\gamma l}{2\pi})}\cdot$$

$$\cdot\sum_{n=0}^{\infty}\frac{(-1)^n 2^{n+1}\bar{\bar{f}}_1(i\sqrt{k^2+\frac{\pi^2}{l^2}(2n+1)^2};s)\sqrt{i}\sqrt{\sqrt{k^2+\frac{\pi^2}{l^2}(2n+1)^2}-k}}{n!\,(2n+1)!!\sqrt{\pi}\,(p-i\sqrt{k^2+\frac{\pi^2}{l^2}(2n+1)^2})}\Big\}dp\ ;$$

$$\underline{\underline{u_{(b)}(x,y,t)}}=\frac{2\sqrt{2\pi}}{l^2}\int_0^{t+0}dt_0\int_0^{\infty}\frac{W_2(x_0,t_0)}{\sqrt{t-t_0}}\exp\Big[-\frac{(x+x_0)^2}{4a(t-t_0)}\Big]\sum_{m=0}^{\infty}\exp\cdot$$

$$\cdot\Big[-\frac{a\pi^2(2m+1)^2(t-t_0)}{l^2}\Big](2m+1)\sin\frac{(2m+1)\pi y}{l}\,dx_0\ ,$$

$$W_3(x,t)=\frac{1}{2\pi\sqrt{2\pi}}\int_{\sigma_1 i-\infty}^{\sigma_1 i+\infty} e^{st}ds\int_{\sigma_2-i\infty}^{\sigma_2+i\infty} e^{-inx}\{\bar{\bar{\Psi}}_1'(n,s)\cdot$$

$$\cdot\frac{\Gamma(\frac{1}{2}-i\frac{\gamma\ell}{2\pi})\Gamma(\frac{1}{2}+i\frac{\gamma\ell}{2\pi})}{\Gamma(i\frac{\gamma\ell}{2\pi})\Gamma(1-i\frac{\gamma\ell}{2\pi})\sqrt{n^2+k^2}}+\frac{\Gamma(\frac{1}{2}+i\frac{\gamma\ell}{2\pi})}{\Gamma(i\frac{\gamma\ell}{2\pi})\sqrt{n+ik}}\cdot$$

$$\cdot[[\sum_{n=0}^{\infty}\frac{\bar{\bar{\Psi}}_1'(i\sqrt{k^2+\frac{\pi^2}{\ell^2}(2n+1)^2};s)(-1)^n 2^{n+1}}{\sqrt{\pi}\,n!(2n+1)!!\sqrt{i}\sqrt{\sqrt{\frac{\pi^2}{\ell^2}(2n+1)^2+k^2}-k}\,(i\sqrt{\frac{\pi^2}{\ell^2}(2n+1)^2+k^2}-n)}+$$

$$+\frac{1}{2\pi}\int_0^{\infty}\frac{\bar{\bar{\Psi}}_1'(i\rho+ik,s)}{(i\rho+ik-n)}[\frac{\Gamma(\frac{1}{2}+\frac{\ell}{2\pi}\sqrt{\rho^2+2\rho k})}{\Gamma(1+\frac{\ell}{2\pi}\sqrt{\rho^2+2\rho k})}+\frac{\Gamma(\frac{1}{2}-\frac{\ell}{2\pi}\sqrt{\rho^2+2\rho k})}{\Gamma(1-\frac{\ell}{2\pi}\sqrt{\rho^2+2\rho k})}]d\rho]]\}dn;$$

$$\underline{\underline{u_{(2)}}}(x,y,t)=\frac{4\sqrt{a\pi}}{\ell^2}\int_0^{t+0}dt_0\int_0^{\infty}\frac{W_4(x_0,t_0)}{\sqrt{t-t_0}}\exp[-\frac{(x+x_0)^2}{4a(t-t_0)}]\sum_{m=1}^{\infty}m\cdot$$

$$\cdot\exp[-\frac{4a\pi^2m^2(t-t_0)}{\ell^2}]\sin\frac{2m\pi y}{\ell}\,dx_0\quad,$$

где

$$W_4(x,t)=\frac{1}{2\pi\sqrt{2\pi}}\int_{\sigma_1 i-\infty}^{\sigma_1 i+\infty} e^{st}ds\int_{\sigma_2-i\infty}^{\sigma_2+i\infty} e^{-inx}\frac{\Gamma(i\frac{\gamma\ell}{2\pi})}{\sqrt{n+ik}\,\Gamma(\frac{1}{2}+i\frac{\gamma\ell}{2\pi})}\cdot$$

$$\cdot\{-\bar{\bar{\Psi}}_2'(n,s)\frac{\Gamma(1-i\frac{\gamma\ell}{2\pi})}{\sqrt{n-ik}\,\Gamma(\frac{1}{2}-i\frac{\gamma\ell}{2\pi})}-$$

$$-\sum_{n=0}^{\infty}\frac{\bar{\bar{\Psi}}_2'(i\sqrt{k^2+\frac{4\pi^2}{\ell^2}(n+1)^2};s)(-1)^n 2^{n+1}}{\sqrt{\pi}\,n!(2n+1)!!\sqrt{i}\sqrt{\sqrt{k^2+\frac{4\pi^2}{\ell^2}(n+1)^2}-k}\,(i\sqrt{k^2+\frac{4\pi^2}{\ell^2}(n+1)^2}-n)}-$$

$$- \frac{1}{2\pi} \int_0^\infty \frac{\bar{\bar{\psi}}_2' (i\rho + ik, s)}{\sqrt{i\rho}\,(i\rho + ik - p)} \left[\frac{\Gamma\left(1 + \frac{\ell}{2\pi}\sqrt{\rho^2 + 2\rho k}\right)}{\Gamma\left(\frac{1}{2} + \frac{\ell}{2\pi}\sqrt{\rho^2 + 2\rho k}\right)} + \right.$$

$$\left. + \frac{\Gamma\left(1 - \frac{\ell}{2\pi}\sqrt{\rho^2 + 2\rho k}\right)}{\Gamma\left(\frac{1}{2} - \frac{\ell}{2\pi}\sqrt{\rho^2 + 2\rho k}\right)} \right] d\rho \,\Big\}\, dp \,.$$

Соответственно изменится решение рассмотренного в статье примера.