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CODENSITY AND BINDING CATEGORIES

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Abstract: There is found a two-object category M such that, supposing the set axiom (M), the equivalence of M with a full subcategory of a complete and well-powered category A make any concrete category to be equivalent with a full subcategory of A .

Key words: Complete category, codense subcategory, full and faithful functor, binding category.

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The main result of this paper is quoted (in a weakened form) in the abstract. A certain result of this type follows from the Isbell's paper [8]. From the following description of the proof of our result it will be seen what kind of Isbell's result is meant. Before stating it, we remind that no set of one-object categories can replace a two-object category M (see 3.3) and that under non (M) there is no small category with the testing property of M (see [13], 4.7). Our proof goes out from the deep theorem of Hedrlín and Kučera asserting that supposing (M) any concrete category is fully embeddable into a binding category (see [1]). We recall that this theorem is a consequence of the Kučera's result that any concrete category can be fully embedded into the category

$S(P^-)$ and the fact that under (M) $S(P^-)$ can be fully embedded into a binding category proved by Hedrlín and Pultr in [3]. Further, we recall that (M) denotes that there is a cardinal p such that any ultrafilter closed under intersections of p elements is trivial and that it is equivalent with the fact that the full subcategory of the category Ens of sets determined by a set p (of cardinality p) is codense in Ens (see [5]).

Further, in § 1 we shall prove a theorem on a "preservation" of codensity by left adjoints, which makes it possible to show that the category of all algebras with two unary operations, which is binding by [2], has (as any equational class of unary algebras with equations involving only one variable) a two-object dense and codense subcategory. Isbell has proved in [8] that this category (as any equational class of unary algebras) has a small dense and codense (adequate in his terminology) subcategory. The mentioned theorem of § 1 yields further contribution to the Isbell's discussion on small codense subcategories of equational classes of algebras; namely that the existence of a small codense subcategory in an equational class of algebras with a set of operations implies (M). It guarantees the existence of small codense or dense subcategories in another concrete categories, too. Particularly, under (M) it yields a small codense subcategory of the category of topological spaces and of $S(P^-)$. It gives another proof of the mentioned theorem of Hedrlín and Pultr concerning the full embeddability of $S(P^-)$ into a category of algebras.

Finally, it suffices to use the result of [13] asserting that a full and faithful functor from a dense and codense full subcategory of C into a complete and well-powered category A can be extended to the whole C (remaining full and faithful). Again, Isbell has proved such a theorem in 8 for the case of A being complete, cocomplete, closed under (large) intersections of extremal monomorphism and (large) cointersections of extremal epimorphisms. As well, § 2 contains a discussion of these two techniques of extensions of full and faithful functors.

Concerning concepts and results of the category theory see [10] and [4]. By limits, colimits, completeness and cocompleteness we shall understand small ones.

§ 1. Density. It is better to define a left U-generating family of objects instead of a left U-generating subcategory (see [13], 1.3).

1.1. Proposition: Let C be a complete category, X a cogenerating set of objects of C and $U: C \longrightarrow A$ a functor which preserves limits and reflects isomorphisms. Then X right U-generates C.

Proof. Let $f: Uc \longrightarrow Uc'$ be a morphism of A such that for any $x \in X$ and any $h: c' \longrightarrow x$ there exists a morphism h' such that $U(h\ ) = U(h)f$. Let D be an inclusion functor of a subcategory of C consisting of all $h: c' \longrightarrow x$ and corresponding $h': c \longrightarrow x$ for any $x \in X$). Let d be a limit of D with projections $p_1: d \longrightarrow c$ and $p_2: d \longrightarrow c'$. Since $\{h: c' \longrightarrow x / x \in X\}$ is a mono-source (a concept of a mono-source (epi-sink) is taken from [4]; in [13] this thing was called a jointly mono (epi) family), p_1 have to be mono.

There is a morphism $g: Uc \longrightarrow Ud$ such that $U(p_1)g = 1_{Uc}$ and $U(p_2)g = f$ because $1_{Uc}: Uc \longrightarrow Uc$ and $f: Uc \longrightarrow Uc'$ determine a cone to the functor UD and U preserves limits. Hence $U(p_1)$ is an isomorphism and so is p_1 for U reflects isomorphisms. Put $f' = p_2 p_1^{-1}$. It holds $U(f') = U(p_2)U(p_1)^{-1} = U(p_2)g = f$.

This proposition is a reformulation of Theorem 35.1 of [4]. Analogously, 35.9 can be modified to

1.2. <u>Proposition</u>: Let C be a complete and well-powered category, X an extremally cogenerating set of objects of C and U be a functor which preserves mono-sources. Then X right U-generates C.

Notice that we do not need products in the proof of 1.1 (and likewise in 1.2).

1.3. <u>Theorem</u>: Let $U: C \longrightarrow A$ be a faithful functor having a left adjoint F, $K_0: B \longrightarrow A$ a dense functor and $K: D \longrightarrow C$ a functor such that $FK_0 = KJ$ for a functor $J: B \longrightarrow D$. Then K is dense if and only if the family K(D) left U-generates C.

<u>Proof</u>: Denote by $\varphi = \varphi_{a,c}: C(Fa,c) \longrightarrow A(a,Uc)$ the adjunction isomorphism. Let K(D) left U-generate C and $\tau : C(K-,c) \longrightarrow C(K-,c')$ be a natural transformation. We get a natural transformation

$$A(K_0-,Uc) \xrightarrow{\varphi^{-1}} C(FK_0-,c) = C(KJ-,c) \xrightarrow{\tau} C(KJ-,c') = $$
$$= C(FK_0-,c') \xrightarrow{\varphi} A(K_0-,Uc') .$$

Since K_0 is dense, there is a unique morphism $f: Uc \longrightarrow Uc'$ of A such that $A(K_0-,f) = \varphi \tau \varphi^{-1}$. Take morphisms $h: Kd \longrightarrow c$ and $g: K_0 b \longrightarrow UKd$. We have $fU(h)g =$ $= \varphi \tau \varphi^{-1}(U(h)g) = \varphi \tau (h\, \varphi^{-1}(g)) = \varphi(\tau(h)\, \varphi^{-1}(g)) =$ $= U(\tau(h))g$. Hence $fU(h) = U(\tau(h))$ for K_0 is dense. Since $K(D)$ U-generates C , there is an $f': c \longrightarrow c'$ with $U(f') = f$. It holds $C(K-,f') = \tau$ because $U(f'h) = U(\tau(h))$ and U is faithful. If $\tau = C(K-,t)$ for a morphism t , then $A(K_0-,U(f')) = A(K_0-,U(t))$ and $f' = t$ for f is unique and U faithful. Therefore K is dense.

Let K be dense. Then $\{U(h) \;/\; h: FK_0 b \longrightarrow c \;,\; b \in B\}$ is an epi-sink for any $c \in C$ because $\{\varphi(h) \;/\; h: FK_0 b \longrightarrow c,\; b \in B\}$ is and $\varphi(h) = U(h)\eta_{K_0 b}$, where η is the unit of the adjunction. By 1.8 of [13] $K(D)$ left U-generates C .

Remark: It suffices to assume that functors $A(K_0 b,U-)$ are representable for any $b \in B$.

Another result of this kind is Proposition 2.4 of [13]. A consequence of Theorem 1.3 is the well-known Isbell s result that a free algebra on n generators determines a dense subcategory of an equational class of algebras with operations at most n-ary. It also recovers that complete graphs (without loops) $\underline{1}$ and $\underline{2}$ having one and two vertices determine a dense subcategory of the category G of graphs. Here $\underline{1}$ is a free graph on a one-element set (a dense subcategory in Ens) and $\underline{2}$ left U-generates G (U is a usual forgetful functor). Similarly, $(1,\emptyset)$ and $(n,\{id_n\})$ form a dense subcategory in the category $S(Ens(n,-))$ (concerning the definition of categories $S(F)$ see [14]). Dually, we can treat categories $S(Ens(-,n))$. The important case of $n = 2$ is given in the first of the following examples.

1.4. <u>Examples</u>: Suppose that (M) hold.

a) Let $S(P^-)$ be a category objects of which are pairs $(a, \mathcal{U})$, where a is a set and $\mathcal{U}$ a system of subsets of a, and morphisms $f: (a, \mathcal{U}) \longrightarrow (b, \mathcal{B})$ correspond with maps $f: a \longrightarrow b$ such that $f^{-1}x \in \mathcal{U}$ for any $x \in \mathcal{B}$. The forgetful functor $U: S(P^-) \longrightarrow Ens$ has a right adjoint R, where $Ra = (a, \emptyset)$. The object $(2, \{\{1\}\})$, where $2 = \{0,1\}$, right U-generates $S(P^-)$. Therefore the dual of 1.3 implies that the full subcategory of $S(P^-)$ determined by $(p, \emptyset)$ and $(2, \{\{1\}\})$ is codense in $S(P^-)$. Here p is a set of cardinality p from (M).

b) Similarly, $(p, \{\emptyset, p\})$ and $(2, \{\emptyset, \{1\}, 2\})$ determine a codense subcategory of the category of topological spaces.

c) Let u be a closure operation on a three-point set $3 = \{0,1,2\}$ such that $u\emptyset = \emptyset$, $u\{0\} = \{0,1\}$ and $ux = 3$ otherwise. Then $(3,u)$ right U-generates the category of closure spaces (in the sense of the Čech's book (Topological spaces, Prague 1966)) and it determines, together with $(p, \{\emptyset, p\})$ a codense subcategory of this category.

Since any category with a small dense subcategory is isomorphic to a full category of algebras and the dual of a full category of algebras is a full category of algebras under (M) (see [6]), all categories listed in the preceding examples are fully embeddable into a category of algebras under (M), which is proved by Hedrlín and Pultr in [3].

Another application of Theorem 1.3, which is crucial for our purpose, is on categories of unary algebras. As it is quoted in the introduction, Isbell has proved in [8] that, under (M), any equational class of unary algebras has a small codense subcategory. Namely, he has showed that the full subcategory of all algebras of at most p^q elements is codense, where q is a cardinality of a free algebra on one generator. We shall show that any equational class C of unary algebras with equations involving only one variable has a one-object codense subcategory, namely, a cofree algebra on p generators. The existence of such an algebra, i.e. the existence of a right adjoint to a forgetful functor can be established in the following way. Any such C can be viewed as a category of functors $S \longrightarrow Ens$, where S is a monoid of all unary operations of C considered as a one-object category. A usual forgetful functor $U: Ens^S = C \longrightarrow Ens = Ens^1$ equals to Ens^K, where $K: 1 \longrightarrow S$ is a unique functor of a one-morphism category 1 into S. Then a left (right) Kan extension along K yields a left (right) adjoint to U. This right adjoint assigns to each set x a unary algebra $(x^S, \{r_s\}_{s\in S})$, where x^S is a product of S copies of x with projections p_s and r_s is a unique map $x^S \longrightarrow x^S$ such that $p_t r_s = p_{ts}$ for any $t \in S$. Now, from the dual of 1.3 and from 1.1 we obtain the following result.

1.5. <u>Corollary</u>: Let (M) hold, C be an equational class of unary algebras with equations involving only one variable and F or R a left or right adjoint resp. to the forgetful functor $U: C \longrightarrow Ens$. Then the full subcategory M of C determined by $F(1)$ and $R(p)$ is dense and codense in C.

1.6. <u>Corollary</u>: Let n be a cardinal and C an equational class of algebras with at most n-ary operations having a small codense subcategory B. Then (M) holds (a full subcategory on objects $U(B) \cup \{n\}$ is codense in Ens). Particularly, no equational class of algebras with finitary operations has a codense subcategory consisting of finite algebras.

Proof. Since F is faithful and has a right adjoint U, following the dual of 1.3 it suffices to show that $\{n\}$ right F-generates Ens. Let $f: Fa \longrightarrow Fb$ be a homomorphism such that for any map $h: b \longrightarrow n$ there is a map $h': a \longrightarrow n$ with $F(h)f = F(h')$. In order to show that $f = F(f')$ we have to prove that $U(f)(x) \in b$ for any $x \in a$. Since C has at most n-ary operations, there is $y \in UFn$ and $g: n \longrightarrow b$ such that $UF(g)(y) = U(f)(x)$. Let $g = g_2 g_1$ be an epi-mono factorization of g, where $g_1: n \longrightarrow k$, and $g_3: k \longrightarrow n$ be the right inverse to g_1. It holds that $g_3 = hg_2$ for some $h: b \longrightarrow n$ and thus $g_1 = g_1 g_3 g_1 = g_1 h g_2 g_1 = g_1 hg$. We get $UF(g_1)(y) = UF(g_1 hg)(y) = U(F(g_1 h)f)(x) = UF(g_1 h')(x) \in k$. Hence $U(f)(x) = UF(g_2 g_1)(y) \in b$.

The example of the category of compact Hausdorff spaces, which has a small codense subcategory (see [5]), shows that the assumption that C has only a set of operations is substantial.

§ 2. Extensions of full and faithful functors. Let M, C and A be categories and $K: M \longrightarrow C$ and $T: M \longrightarrow A$ full and faithful functors. We shall set the construction of a functor $L : C \longrightarrow A$ from [11] into its proper context. Namely, we shall show that L_* is a left Kan extension of T along K computed, instead of in A, in an appropriate full reflective subcategory A_T of A containing $T(M)$. Let A_T be a full subcategory of A determined by all $a \in A$ such that for any pair f, g of parallel morphisms with the domain in $T(M)$ and the codomain a there is a morphism h with the codomain in $T(M)$ such that $hf \neq hg$.

2.1. Lemma. Let A be extremally co-well powered and have coequalizers and cointersections of extremal epimorphisms. Then A_T is a reflective subcategory of A .

Proof. Given $a \in A$, we construct a system $\{a_\alpha\}$ of objects of A indexed by ordinal numbers and morphisms $f_{\beta,\alpha}$: $: a_\beta \longrightarrow a_\alpha$ for each $\beta < \alpha$ such that $f_{\beta,\alpha} f_{\gamma,\beta} = f_{\gamma,\alpha}$, in which $a_0 = a$ and $f_{\alpha,\alpha+1} : a_\alpha \longrightarrow a_{\alpha+1}$ is a cointersection of all coequalizers of parallel pairs of morphisms f, g with the domain in $T(M)$ such that $hf = hg$ for any h with the codomain in $T(M)$. Further, each a_α indexed by a limit number is a cointersection of all $f_{0,\beta}$, where $\beta < \alpha$, with $f_{\beta,\alpha}$ as components of a colimit cone. This system is a right multistrict analysis in the sense of Isbell [7] and following 2.3 of [7] any $f_{\beta,\alpha}$ is an extremal epimorphism. Since A is extremally co-well-powered, this construction stops at an ordinal γ and $f_{0,\gamma}$ is a desired reflection from a into A_T . Namely, having a morphism t from a into A_T we construct a unique k with $t = kf_{0,\gamma}$ by an induction because $tf = tg$ for any morphisms f , g from the construction.

In fact, this proof is made by a technique used in the proof of Theorem 2.4 in [7]. Now, whenever $\mathrm{Lan}_K T$ exists it is preserved by the reflector $F: A \longrightarrow A_T$, $F\ \mathrm{Lan}_K T =$ $= \mathrm{Lan}_K (FT)$. In the crucial case of K being dense $F . \mathrm{Lan}_K T =$ $= L_*$ because $\mathrm{Lan}_K T$ is right M-full by [13] 1.9 and 1.7. In what follows, we shall need Theorem 1.11 from [13] asserting that L_* is full and faithful whenever K is dense and codense. As it is quoted in the introduction, [8] contains a result concerning extensions of full embeddings from a dense and codense subcategory. This result is based on Theorem 3.2 from [6]. On the other hand, we can state a result of the kind of this Theorem 3.2 if we suppose in our situation that M is small, $C = Ens^{M^{op}}$ and $K: M \longrightarrow Ens^{M^{op}}$ is the Yoneda embedding. Then K is dense and L_* exists provided that A is cocomplete and extremally co-well powered. If $F: M^{op} \longrightarrow$ $\longrightarrow Ens$ is a functor such that L_* is full and faithful on a full subcategory determined by $K(M) \cup \{F\}$, then $Fm \cong$ $\cong Ens^{M^{op}}(M(-,m),F) \cong A(Tm, L_*(F))$ because of the Yoneda Lemma and full- and faithfulness of L_*. Hence $F \cong A(T-, L_*(F))$, i.e. F is representable in A. If F is reflective (see [5]), then $K(M)$ is also codense in the full subcategory with objects $K(M) \cup \{F\}$ (see [5] 1.8) and by [13] Th. 1.11 and the preceding computation F is representable in A, which is the announced result of a kind of 3.2 of [6]. Further, if F is a subfunctor of a product of representable functors, then $K(M)$ cogenerates the category obtained by the joining of F and by [11] Prop. 3 F is representable in A if and only if $A(T-, L_*(F))$ is its representation.

In [15], [12] and [13] there are developed some techniques of extensions of full and faithful functors from a left and right U-generating subcategory of a category endowed with a faithful functor U into Ens. It is possible to replace the category of sets by a category X. It permits us to dualize some considerations from [12] and [13] (taking $X = Ens^{op}$). Since these results are rather technical and we shall not need them, we do not state them. But it deserves to mention the following special case of Theorem 3.5 from [13].

2.2. Remark: Let C be a category, $U: C \longrightarrow Ens$ a faithful functor having a right adjoint R and M a full subcategory of C. Denote by $\varphi : (U-,-) \cong (-,R-)$ the adjunction isomorphism and by $\varepsilon : UR \longrightarrow 1$ the counit of the adjunction. Let $m \in M$, $c \in C - M$ and $g: m \longrightarrow c$ be a morphism of C. Suppose that there is a set x such that card $x \geq$ $\geq \min \{\text{card } Um, 2\}$ and $Rx \in M$. Then a) and b) from Theorem 3.5 of [13] are satisfied for $n = Rx$ and $h_0 = \varphi(k)$, where $k: Uc \longrightarrow x$ induces a monic on the image of $U(g)$. Indeed:

a) Let $y \in Un - U(h_0 g)(Um)$. Choose $s' = 1_n$ and $s: Rx \longrightarrow$ $\longrightarrow Rx$ such that $\varphi^{-1}(s)$ equals to ε_x on $URx - \{y\}$ and $\varphi^{-1}(s)(y) \neq \varepsilon_x(y)$. It holds $\varepsilon_x U(s)(y) = \varphi^{-1}(s)(y)$ and thus $U(s)(y) \neq y$. Further, $\varphi^{-1}(sh_0 g) = \varphi^{-1}(s)U(h_0 g) =$ $= \varepsilon_x U(h_0 g) = \varphi^{-1}(h_0 g)$ and therefore $sh_0 g = h_0 g$.

b) Let $m' \in M$ and $h: c \longrightarrow m'$. Since $k = \varepsilon_x U(h_0)$, $U(h_0)$ induces a monic on the image of $U(g)$ nd thus there is a map r such that $rU(h_0 g) = U(hg)$. Evidently, $\{k: Um' \longrightarrow x\}$ is a mono-source and therefore $\{U\varphi(k)\}$ is a mono-source. Since $\varphi^{-1}(\varphi(k)hg) = kU(hg) = krU(h_0 g) = \varphi^{-1}(\varphi(kr)h_0 g)$, we have $\varphi(k)hg = \varphi(kr)h_0 g$ and so $\{U(t) \ / \ t: m' \longrightarrow n,\ thg = t'h_0 g$ for a $t': n \longrightarrow n\}$ is a mono-source.

§ 3. Binding categories. Let C be a category of algebras with two unary operations. Let M be a full subcategory of C described in 1.5.

3.1. <u>Theorem</u>. Let (M) hold and A be a cocomplete and extremally co-well powered category, or complete and extremally well-powered resp., such that M is equivalent to a full subcategory of A. Then any concrete category is equivalent to a full subcategory of A.

<u>Proof</u> follows from 1.5, 1.11 of [13] and from the results of Hedrlín, Kučera and Pultr quoted in the introduction.

We can weaken completeness properties of A taking a more complicated small category M.

3.2. <u>Theorem</u>. Let (M) hold and b be a regular infinite cardinal number. Then there is a small category M_b such that any concrete category is equivalent to a full subcategory of an arbitrary extremally co-well powered category having b-filtered colimits, coequalizers and cointersections of extremal epimorphisms, or extremally well-powered monomorphisms resp., which has a full subcategory equivalent with M_b.

<u>Proof</u>: Let C' be a full subcategory of C consisting of $R(p)$ and of all algebras from C without one-element subalgebras and N_b be a full subcategory of C' determined by $R(p)$ and by all epimorphic images of $F(b)$. C' is binding, which follows, for instance, from the proof of Theorem 3 of [2]. Since any algebra of C has a homomorphism into $R(p)$, $R(p)$ contains one-element subalgebras and thus it has no homomorphism into $C' - \{R(p)\}$. Following [13], 4.1 the comma category $(N_b \downarrow c)$ is b-filtered for any $c \in C$ because a coproduct of less than b copies of $F(b)$ is equal to $F(b)$ and hence coproducts of objects of $N_b - \{R(p)\}$ are epimorphic images of $F(B)$. By 2.1 it follows from 1.11 of [13] that

a category having N_b and $(N_b)^{op}$ as components is a desired small category M_b .

The situation without the set axiom (M) is dealt with in [15], [12] and [13]. If we want to diminish the number of objects of testing categories in this case from three to two, we can obtain the following contribution (using 3.5 of [13] and 2.2).

If a full subcategory of C on objects F(1) and $R(\aleph_0)$ can be pseudorealized into a cocomplete category A endowed with a colimit compressing faithful functor into $\mathcal{E}ns$, then any category of algebras is equivalent to a full subcategory of A .

However, the author did not succeed in proving that a pseudorealization can be replaced by a full embedding. Concerning the possibility of diminishing from two to one, we have the following result.

3.3. Example: For any set S of one-object categories there is a complete, cocomplete, well-powered and co-well powered category A which contains any category from S as a full subcategory but there is a small category not equivalent to a full subcategory of A .

Let N be a disjoint union of categories from S . By Proposition 4.1 of [9] N is a generating and cogenerating full subcategory of a category A with requested properties. We shall show that A does not contain a full subcategory with two objects a , b such that there are at least two morphisms from a to a and from b to b , there is a morphism from a to b and there is no morphism from b to a . Since N generates and cogenerates A , there are morphisms $c_1 \longrightarrow a$, $a \longrightarrow c_2$, $c_3 \longrightarrow b$ and $b \longrightarrow c_4$, where $c_i \in N$ for $i = 1,\dots,4$. Since there are no morphisms between distinct objects of N , $c_1 = c_2 = c_3 = c_4$. However, then we have a morphism from b to a , which is a contradiction.

It remains a question there, whether all one-object categories suffice.

R e f e r e n c e s :

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