Commentationes Mathematicae Universitatis Carolinae

Ladislav Bican

A note on factor-splitting Abelian groups of rank two

Commentationes Mathematicae Universitatis Carolinae, Vol. 22 (1981), No. 3, 569--578

Persistent URL: http://dml.cz/dmlcz/106098

COMMENTATIONES MATHEMATICAE UNIVERSITATIS CAROLINAE
22,3 (1981)

A NOTE ON FACTOR-SPLITTING ABELIAN GROUPS OF RANK TWO

Ladislav BICAN

Abstract: The purpose of this note is to prove that a torsionfree abelian group G of rank two is factor-splitting if and only if the set π of all primes decomposes into $\pi = \pi_1 \cup \pi_2$ in such a way that $G \otimes Z_{\pi_1}$ is homogeneous and $G \otimes Z_{\pi_2}$ is either a Butler group or it is generated by the (infinite) set of elements of (all) maximal types. As a consequence we obtain a characterization of Butler groups of rank two as finite extensions of groups generated by elements of maximal types provided that the type set is not ordered.

Key words: Factor-splitting group, Butler group, homogeneous group, completely decomposable group, p-rank.

Classification: 20K15

By the word "group" we shall always mean an additively written abelian group. The symbol π will denote the set of all primes. If $\pi' \subseteq \pi$ then $Z_{\pi'}$ will denote the group of rationals with denominators prime to every $p \in \pi'$. If $m \in Z$, $(m,p) = 1$ for every $p \in \pi'$ then we shall write $(m, \pi') = 1$. Any maximal linearly independent set of elements of a torsionfree group G is called a basis. If G is a torsionfree grou then the set of all elements g of G having infinite p-height is a subgroup of G which will be denoted by $G[p^{\infty}]$. It is well-known (see [12]) that if G is a torsionfree group of finite rank and F its free subgroup of the same rank then the number $r_p(G)$ of summands $C(p^{\infty})$ in G/F does not depend on the particular choice of F and this number is called the p-rank of G. Recall [13] that a torsionfree group G is said to be factor-splitting if each homomorphic image of G splits, and [1],[6] that G is called a Butler group (purely finitely generated group) if it contains elements $g_1, g_2, \dots, g_m$ such that $G = \sum_{i=1}^{m} \langle g_i \rangle_*^G$. The type set of a torsionfree group G is denoted by $\hat{\tau}(G)$. Other notations and terminology is essentially that as in [8].

We start with some known results on factor-splitting and Butler groups. If p is a prime then we shall say that a basis $\{u,v\}$ of a torsionfree group G of rank two satisfies (FSp) if either $h_p(u) = h_p(v)$ or $G \otimes Z_p = (\langle u \rangle_* \otimes Z_p) \oplus$ $\oplus (\langle v \rangle_* \otimes Z_p)$.

1. Lemma ([2; Theorem 1]): A torsionfree group G of rank two is factor-splitting if and only if every basis of G satisfies (FSp) for almost all primes p.

2. Lemma ([2; Theorem 2]): Any homogeneous torsionfree group of rank two is factor-splitting.

3. Lemma ([4; Lemma 8] or [3; Lemma 5]): Let $\pi = \bigcup_{i=1}^{m} \pi_i$ and let G be a torsionfree group. If $G \otimes Z_{\pi_i}$, $i = 1,2,\dots,m$, is factor-splitting then G is factor-splitting.

4. Lemma ([5; Theorem 4]): Every Butler group is factor-splitting.

5. Lemma ([7; Theorem 5]): Every Butler group with ordered type set is completely decomposable.

6. **Lemma**: Let $\{g,h\}$ be a basis of a torsionfree group G of rank two. If $G \otimes Z_p = (\langle g\rangle_* \otimes Z_p) \oplus (\langle h\rangle_* \otimes Z_p)$ for each prime $p \in \pi' \subseteq \pi$ then $G \otimes Z_{\pi'} = (\langle g\rangle_* \otimes Z_{\pi'}) \oplus$ $\oplus (\langle h\rangle_* \otimes Z_{\pi'})$.

Proof: If $p \in \pi'$ and $0 \neq g \in G$ are arbitrary then by the hypothesis there is $0 \neq \beta_p \in Z$ with $\beta_p g = x_p + y_p$, $(\beta_p, p) = 1$, $x_p \in \langle g\rangle_*$, $y_p \in \langle h\rangle_*$. If $q,\dots,r$ are all primes from π' dividing β_p, then similarly $\beta_q g = x_q + y_q$, $(\beta_q, q) = 1$, $x_q \in \langle g\rangle_*$, $y_q \in \langle h\rangle_*, \dots, \beta_r g = x_r + y_r$, $(\beta_r, r) = 1$, $x_r \in \langle g\rangle_*$, $y_r \in \langle h\rangle_*$. Denoting $d = (\beta_p, \beta_q, \dots$ $\dots, \beta_r)$, we obviously get $(d, \pi') = 1$, $dg = x + y$, $x \in \langle g\rangle_*$, $y \in \langle h\rangle_*$ and the assertion follows easily.

Now we are ready to prove the main result.

7. Theorem: A torsionfree group G of rank two is factor-splitting if and only if there is a decomposition $\pi' =$ $= \pi_1 \cup \pi_2$ such that $G \otimes Z_{\pi_1}$ is homogeneous and either

(1) $G \otimes Z_{\pi_2}$ is a Butler group

or

(2) $G \otimes Z_{\pi_2} = \sum_{i=1}^{\infty} \langle g_i\rangle_* \otimes Z_{\pi_2}$ where $\{\hat{\tau}(g_i) = \hat{\tau}_i \mid i =$ $= 1,2,\dots\}$ is the set of all maximal elements of $\hat{\tau}(G)$, $\hat{\tau}_i \cap \hat{\tau}_j = \hat{\tau}$ for all $i,j = 1,2,\dots$, $i \neq j$, and from $\lambda g_k =$ $= \mu g_i + \nu g_j$ it follows $h_p(\lambda g_k) = \min\{h_p(\mu g_i),$ $h_p(\nu g_j)\}$ for almost all primes p with $h_p(g_i) \neq h_p(g_j)$.

Proof: Sufficiency. If (1) holds then G is factor-splitting by Lemmas 2, 4 and 3.

Assume (2). With respect to Lemmas 2 and 3 we can restrict ourselves to the case $\pi_1 = \emptyset$ (i.e. $Z_{\pi_2} = Z$). First we shall show that $G \otimes Z_p = (\langle g_i\rangle_* \otimes Z_p) \oplus (\langle g_j\rangle_* \otimes Z_p)$

for almost all primes p with $h_p(g_i) \neq h_p(g_j)$. Without loss of generality we can restrict ourselves to the case $k = h_p(g_1) < h_p(g_2) = l < \infty$ (the case $l = \infty$ being trivial). In view of $\hat{\tau}_i \cap \hat{\tau}_2 = \hat{\tau}_1 \cap \hat{\tau}_2 = \hat{\tau}$ it is $h_p(g_i) = h_p(g_1)$ for almost all primes considered. Moreover, for almost all such primes the equality $\alpha_i g_i = \beta_i g_1 + \gamma_i g_2$, $(\alpha_i, \beta_i, \gamma_i) = 1$, $i = 3,4,\ldots$, yields $h_p(\alpha_i g_i) = \min\{h_p(\beta_i g_1), h_p(\gamma_i g_2)\}$. Then, obviously, $s_i = h_p(\alpha_i) \leq h_p(\beta_i)$ and $s_i \leq l - k$, for otherwise one easily obtains $h_p(\gamma_i) > 0$ which contradicts the hypothesis $(\alpha_i, \beta_i, \gamma_i) = 1$. Now for each such prime p there are elements $x, y, x_i \in G$ with $p^k x = g_1$, $p^l y = g_2$, $p^k x_i = g_i$, $i = 3,4,\ldots$, and it suffices to show that each element $g \in G$ with $p^r g = \lambda x + \mu y$ lies in $\langle x,y \rangle$. By hypothesis, $\nu g = \lambda_1 x + \lambda_2 y + \sum_{i=3}^{n} \lambda_i x_i$, $(\nu, p) = 1$. Setting $\alpha = \alpha_3 \alpha_4 \ldots \alpha_n = \alpha_i \bar{\alpha}_i$ we get $p^l \nu \alpha g = p^{l-k} \alpha \lambda_1 g_1 + \alpha \lambda_2 g_2 + \sum_{i=3}^{n} p^{l-k} \lambda_i \bar{\alpha}_i \cdot (\beta_i g_1 + \gamma_i g_2) = (p^{l-k} \alpha \lambda_1 + p^{l-k} \sum_{i=3}^{n} \lambda_i \bar{\alpha}_i \beta_i) g_1 + (\alpha \lambda_2 + p^{l-k} \sum_{i=3}^{n} \lambda_i \bar{\alpha}_i \gamma_i) g_2 = p^l((\alpha \lambda_1 + \sum_{i=3}^{n} \lambda_i \bar{\alpha}_i \beta_i) x + (\alpha \lambda_2 + p^{l-k} \sum_{i=3}^{n} \lambda_i \bar{\alpha}_i \gamma_i) y)$ and so $\nu \alpha g = (\alpha \lambda_1 + \sum_{i=3}^{n} \lambda_i \bar{\alpha}_i \beta_i) x + (\alpha \lambda_2 + p^{l-k} \sum_{i=3}^{n} \lambda_i \bar{\alpha}_i \gamma_i) y = \beta x + \gamma y$. Now $s = \sum_{i=3}^{n} s_i = h_p(\alpha)$, $p^s \alpha' = \alpha$, $(\alpha', p) = 1$, and it is easy to see that $p^s \mid \beta$, $p^s \mid \gamma$, $\beta = p^s \beta'$, $\gamma = p^s \gamma'$. Thus $\alpha' \nu g = \beta' x + \gamma' y$, $(\alpha' \nu, p) = 1$, which together with $p^r g = \lambda x + \mu y$ yields $g \in \langle x,y \rangle$.

Now let $\{u,v\}$ be an arbitrary basis of G. Since $\hat{\tau}(G) = \{\hat{\tau}, \hat{\tau}_1, \hat{\tau}_2, \ldots\}$, there are essentially three possibilities. If $\hat{\tau}(u) = \hat{\tau}(v) = \hat{\tau}$ then $\{u,v\}$ obviously satisfies (FSp) for almost all primes p. If $\hat{\tau}(u) = \hat{\tau}$ and $\hat{\tau}(v) = \hat{\tau}_i$

for some i, then $\langle v\rangle_* = \langle g_i\rangle_*$ (otherwise $\langle g_i, v\rangle_* = G$ and each non-zero element of G is of the type $\geq \hat{\tau}_i$). Thus $\rho v = \sigma g_i$ and $\eta u = \lambda g_1 + \mu g_i$ for some non-zero integers $\rho, \sigma, \eta, \lambda, \mu$. For each prime p relatively prime to these integers we have $g_1 \otimes 1 = \lambda g_1 \otimes (1/\lambda) =$ $= (\eta\mu - \mu g_i) \otimes (1/\lambda) = u \otimes (1/\eta\lambda) - g_i \otimes (1/\mu\lambda)$ in $G \otimes Z_p$. Hence, by the preceding part, $G \otimes Z_p =$ $= (\langle u\rangle_* \otimes Z_p) \oplus (\langle v\rangle_* \otimes Z_p)$ for almost all primes with $h_p(g_1) \neq h_p(g_i)$ and $\{u,v\}$ satisfies (FSp) for almost all primes. Finally, if $\langle u\rangle_* = \langle g_i\rangle_*$, $\langle v\rangle_* = \langle g_j\rangle_*$ for some $i \neq j$, then $G \otimes Z_p = (\langle u\rangle_* \otimes Z_p) \oplus (\langle v\rangle_* \otimes Z_p)$ for almost all primes p with $h_p(g_i) \neq h_p(g_j)$ and $\{u,v\}$ satisfies (FSp) for almost all primes p. So G is factor-splitting by Lemma 1.

Necessity. Put $\pi'_1 = \{p \in \pi \mid r(G[p^\infty]) < r_p(G)\}$ and $\pi'_2 = \pi \setminus \pi'_1$. Suppose, first, that the set of all maximal elements of $\hat{\tau}(G)$ is finite, say $\{\hat{\tau}_1, \hat{\tau}_2, \dots, \hat{\tau}_n\}$, and let $g_1, g_2, \dots, g_n$ be the elements of G of types $\hat{\tau}_1, \hat{\tau}_2, \dots$ $\dots, \hat{\tau}_n$, respectively. If $\pi'_{ij} = \{p \in \pi'_2 \mid h_p(g_i) < h_p(g_j)\}$, $i,j = 1,2,\dots,n$, $i \neq j$, then Lemma 1 yields $G \otimes Z_p =$ $= (\langle g_i\rangle_* \otimes Z_p) \oplus (\langle g_j\rangle_* \otimes Z_p)$ for a cofinite subset π_{ij} of π'_{ij}. Putting $\pi_2 = \bigcup_{i,j} \pi_{ij}$ and $\pi_1 = \pi'_1 \cup (\pi'_2 \setminus \pi_2)$ we easily see by Lemma 6 and [5; Theorem 2] that $G \otimes Z_{\pi'_1}$ is a Butler group. Further, for $p \in \pi'_1$ the group $G \otimes Z_p$ is obviously indecomposable and so Lemma 1 yields the homogeneity of $G \otimes Z_{\pi'_1}$. The set $\pi'_2 \setminus \pi_2$ decomposes into $(\pi'_2 \setminus (\bigcup_{i,j} \pi'_{ij})) \cup ((\bigcup_{i,j} \pi'_{ij}) \setminus (\bigcup_{i,j} \pi_{ij}))$ where the last subset is finite. For each $p \in \pi'_2 \setminus (\bigcup_{i,j} \pi'_{ij})$ it is $h_p(g_i) =$ $= h_p(g_j)$ for all $i,j = 1,2,\dots,n$. Now, if $g \in G$ is such that

$h_p(g) > h_p(g_i)$ for an infinite set of primes p from $\pi'_2 \setminus (\bigcup_{i,j} \pi'_{ij})$, then, obviously, $h_p(g) = h_p(g_j)$ for almost all primes $p \in \pi_{ji}$ and so $\hat{\tau}(g) \parallel \hat{\tau}_i$ for each i = = 1,2,...,n (the sets π_{ji} are infinite owing to the incomparability of $\hat{\tau}_i$, $\hat{\tau}_j$). However, $\hat{\tau}(g)$ is maximal in $\hat{\tau}(G)$, G being of rank two, which contradicts the choice of $\hat{\tau}_1, \hat{\tau}_2, \dots, \hat{\tau}_n$. From this contradiction it immediately follows that $G \otimes Z_{\pi_1}$ is a homogeneous group.

Now we proceed to the infinite case. Let $\{\hat{\tau}_1, \hat{\tau}_2, \dots\}$ be the set of all maximal elements of $\hat{\tau}(G)$. If $c_1, c_2 \in G$ are elements of types $\hat{\tau}_1$, $\hat{\tau}_2$, respectively, then, by Lemma 1, $G \otimes Z_p = (\langle c_1 \rangle_* \otimes Z_p) \oplus (\langle c_2 \rangle_* \otimes Z_p)$ for almost all primes p with $h_p(c_1) \neq h_p(c_2)$ and so we can choose suitable multiplies g_1, g_2 of c_1, c_2 such that

(3) $G \otimes Z_p = (\langle g_1 \rangle_* \otimes Z_p) \oplus (\langle g_2 \rangle_* \otimes Z_p)$

for all primes with $h_p(g_1) \neq h_p(g_2)$. Since $\{g_1, g_2\}$ is a basis of G we can choose elements $c_3, c_4, \dots$ in G of types $\hat{\tau}_3, \hat{\tau}_4, \dots$, respectively, which are linear combinations of g_1, g_2. Put $\tau = \tau(g_1) \cap \tau(g_2)$ and assume we have constructed the elements $g_1, g_2, \dots, g_n$ such that

(4) $\hat{\tau}(g_i) = \hat{\tau}_i$, i = 1,2,...,n,

(5) $\tau(g_i) \cap \tau(g_j) = \tau$, i,j = 1,2,...,n, $i \neq j$,

(6) $G \otimes Z_p = (\langle g_1 \rangle_* \otimes Z_p) \oplus (\langle g_i \rangle_* \otimes Z_p)$ for all i = = 2,...,n and all primes p with $h_p(g_1) < h_p(g_i)$.

If for each i,j = 1,2,...,n, $i \neq j$, we denote $\pi_{ij} = \{p \in \pi \mid h_p(g_i) < h_p(g_j)\}$ and $\bar{\pi}_2 = \pi_{12} \cup \pi_{21}$, then by Lemma 6 we have

(7) $G \otimes Z_{\pi_2} = (\langle g_1\rangle_* \otimes Z_{\pi_2}) \oplus (\langle g_2\rangle_* \otimes Z_{\pi_2})$

and

(8) $G \otimes Z_{\pi_{1i}} = (\langle g_1\rangle_* \otimes Z_{\pi_{1i}}) \oplus (\langle g_i\rangle_* \otimes Z_{\pi_{1i}})$

for all $i = 3,\dots,n$.

Further, for $i,j = 1,2,\dots,n$, $i \neq j$, and $p \in \pi_{1i} \cap \pi_{1j}$ we have by (5) $\min\{h_p(g_i), h_p(g_j)\} = h_p(g_1) < \min\{h_p(g_i), h_p(g_j)\}$, a contradiction showing $\pi_{1i} \cap \pi_{1j} = \emptyset$. Similarly we shall show

$$(9) \qquad \begin{aligned} &\pi_{1i} \cap \pi_{ij} = \emptyset, \quad \pi_{21} \cap \pi_{1i} = \emptyset \\ &\pi_{ij} = \pi_{1j}, \quad \pi_{i1} = \pi_{21} \end{aligned}$$

for all $i,j,=2,3,\dots,n$, $i \neq j$.

Since c_{n+1} is a linear combination of g_1, g_2, it is $h_p(c_{n+1}) \geq \tau(p)$ for all primes p and, by (7) and (8), $h_p(c_{n+1}) = \tau(p)$ for almost all primes $p \in \pi_{21} \cup \pi_{12} \cup \dots \cup \pi_{1n}$. Thus there is $d_{n+1} \in \langle c_{n+1}\rangle_*$ with $h_p(d_{n+1}) = \tau(p)$ for all primes $p \in \pi_{21} \cup \pi_{12} \cup \dots \cup \pi_{1n}$. Further, from the incomparability of $\hat{\tau}_1, \hat{\tau}_2, \dots, \hat{\tau}_n, \hat{\tau}_{n+1}$ it follows that $h_p(d_{n+1}) > \tau(p)$ for infinitely many primes $p \notin \pi_{21} \cup \pi_{12} \cup \dots \cup \pi_{1n}$. By Lemma 1, $G \otimes Z_p = (\langle g_1\rangle_* \otimes Z_p) \oplus (\langle d_{n+1}\rangle_* \otimes Z_p)$ for almost all primes p with $h_p(g_1) < h_p(d_{n+1})$ and hence for suitable element $g_{n+1} \in \langle d_{n+1}\rangle_*$ the equality (6) holds for all primes $p \in \pi_{1,n+1}$. Moreover, the relations (4) and (5) obviously hold for all $i,j = 1,2,\dots,n+1$, $i \neq j$. Thus, by induction, we have constructed the elements $g_1, g_2, \dots$ such that the formulas (4),(5),(6) (and consequently (7),(8),(9)) hold for all $i,j = 1,2,\dots, i \neq j$. Now, by Lemma 1, $G \otimes Z_p = (\langle g_i\rangle_* \otimes Z_p) \oplus (\langle g_j\rangle_* \otimes Z_p)$ for almost all primes p with $h_p(g_i) \neq h_p(g_j)$ and so for $\lambda g_k = \mu g_i +$

$+ \nu g_j$ the equality $h_p(\lambda g_k) = \min\{h_p(\mu g_i), h_p(\nu g_j)\}$ holds for almost all such primes.

Put $\pi_2 = \pi_{21} \cup \bigcup_{i=2}^{\infty} \pi_{1i}$ and $\pi_1 = \pi \setminus \pi_2$. Then $G \otimes Z_{\pi_1}$ is homogeneous, for otherwise we easily obtain an element from G, the type of which is incomparable with all the types $\hat{\tau}_1, \hat{\tau}_2, \dots$. Concerning the equality $G \otimes Z_{\pi_2} = \sum_{i=1}^{\infty} \langle g_i \rangle_* \otimes Z_{\pi_2}$ we can without loss of generality suppose that $\pi_1 = \emptyset$. If $0 \neq g \in G$ is an arbitrary element then, by (7), $\beta_2 g = x_1^{(2)} + x_2$, $x_1^{(2)} \in \langle g_1 \rangle_*$, $x_2 \in \langle g_2 \rangle_*$, and $(\beta_2, \pi_2) = 1$. If β_2 has components in $\pi_{13}, \pi_{14}, \dots, \pi_{1n}$ only, then (8) yields $\beta_i g = x_1^{(i)} + x_i$ with $x_1^{(i)} \in \langle g_1 \rangle_*$, $x_i \in \langle g_i \rangle_*$ and $(\beta_i, \pi_{1i}) = 1$ for all $i = 3, \dots, n$. Now $(\beta_2, \beta_3, \dots, \beta_n) = 1$ yields $\sum_{i=2}^{n} \beta_i \gamma_i = 1$ for suitable integers $\gamma_2, \gamma_3, \dots, \gamma_n$, so that $g = \sum_{i=2}^{n} \gamma_i x_1^{(i)} + \sum_{i=2}^{n} \gamma_i x_i \in \sum_{i=1}^{\infty} \langle g_i \rangle_*$ and Theorem 7 is proved.

8. <u>Corollary</u>: A torsionfree group G of rank two is a Butler group if and only if G is either completely decomposable with ordered type set or if the subgroup $H = \sum_{i=1}^{n} \langle g_i \rangle_*$, where $\{\hat{\tau}(g_1), \hat{\tau}(g_2), \dots, \hat{\tau}(g_n)\}$ is the set of all maximal elements of $\hat{\tau}(G)$, is of finite index in G.

<u>Proof</u>: Only the necessity must be proved. If $\hat{\tau}(G)$ is ordered then G is completely decomposable by Lemma 5. So assume that $\{\hat{\tau}_1, \hat{\tau}_2, \dots, \hat{\tau}_n\}$ is the set of all maximal elements of $\hat{\tau}(G)$, $n \geq 2$. By Lemma 4 G is factor-splitting, so that by Theorem 7 (and its proof) we have a decomposition $\pi = \pi_1 \cup \pi_2$ such that $G \otimes Z_{\pi_1}$ is homogeneous and $G \otimes Z_{\pi_2} = \sum_{i=1}^{n} \langle g_i \rangle_* \otimes Z_{\pi_2}$ where $\hat{\tau}(g_i) = \hat{\tau}_i$, $i = 1, 2, \dots, n$. The group $G \otimes Z_{\pi_1}$ is obviously a Butler group and

so it is completely decomposable by Lemma 5. Then the subgroup $(\langle g_1\rangle_* \otimes Z_{\pi_1}) \oplus (\langle g_2\rangle_* \otimes Z_{\pi_1})$ is of finite index in $G \otimes Z_{\pi_1}$ by [8; Theorem 48.1] and now it is easy to see that H is of finite index in G.

9. Remarks: (a) If I, J are two p-reduced torsionfree groups of rank one with incomparable types, then it is not too hard to show that the subgroup $H = \langle pI, pJ, u - v\rangle$ of $G = I \oplus J$, where $u \in I$, $v \in J$, $h_p(u) = h_p(v) = 0$, is indecomposable. Hence a Butler group of rank two with exactly two maximal types need not be completely decomposable, but it contains a completely decomposable subgroup of finite index.

(b) The situation in the class of factor-splitting groups of rank at least 3 is more complicated. One of the difficulties arises from the fact that not all homogeneous groups of rank at least 3 are factor-splitting (see [4; Example 2]).

References

[1] D.M. ARNOLD: Pure subgroups of finite rank completely decomposable groups (preprint, 31 p.).

[2] L. BICAN: Factor-splitting abelian groups of rank two, Comment. Math. Univ. Carolinae 11(1970), 1-8.

[3] L. BICAN: Factor-splitting abelian groups of finite rank, Comment. Math. Univ. Carolinae 17(1976), 473-480.

[4] L. BICAN: Factor-splitting abelian groups of arbitrary rank, Comment. Math. Univ. Carolinae 19(1978), 653-672.

[5] L. BICAN: Purely finitely generated abelian groups, Comment. Math. Univ. Carolinae 21(1980), 209-218.

[6] L. BICAN: Splitting in abelian groups, Czech. Math. J. 28 (1978), 356-364.

[7] M.C.R. BUTLER: A class of torsionfree abelian groups of finite rank, Proc. London Math. Soc. 15(1965), 680-698.

[8] L. FUCHS: Abelian groups, Budapest 1958.

[9] L. FUCHS: Infinite abelian groups I, Academic Press, 1970.

[10] L. FUCHS: Infinite abelian groups II, Academic Press, 1973.

[11] L. FUCHS: Notes on torsionfree abelian groups (preprint, 18 p.).

[12] L. PROCHÁZKA: O p-range abelevych grup bez kručenija konečnogo ranga, Czech. Math. J. 12(1962), 3-43.

[13] L. PROCHÁZKA: Zametka o faktornorasčepljajemych abelevych gruppach, Čas. pěst. mat. 87(1962), 404-414.

Matematicko-fyzikální fakulta, Universita Karlova, Sokolovská 83, 18600 Praha 8, Československo

(Oblatum 6.4. 1981)