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ANNOUNCEMENT OF NEW RESULTS

BASE AND ESSENTIAL BASE IN PARABOLIC POTENTIAL THEORY

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Let F be the fundamental solution of the heat equation in $\mathbb{R}^{n+1}$. For $z \in \mathbb{R}^{n+1}$ and $c \in \mathbb{R}$ denote by $B(z,c) = \{w \in \mathbb{R}^{n+1};\ F(z-w) \geqq (4\pi c)^{-n/2}\} \cup \{z\}$ (the heat ball with the "center" z and radius c), $A(z,c) = \overline{B(z,c) \setminus B(z,c/2)}$. Let b(E) stand for the base of a set $E \subset \mathbb{R}^{n+1}$, i.e. the set of all points at which E is not parabolically thin. For a compact set $K \subset \mathbb{R}^{n+1}$, the thermal capacity of K is defined by $\gamma(K) = \sup\{\nu(K); \operatorname{spt}\nu \subset K, F*\nu \leqq 1\}$ and the continuous thermal capacity by $\alpha(K) = \sup\{\nu(K); \operatorname{spt}\nu \subset K,\ F*\nu \leqq 1,\ F*\nu$ continuous$\}$ (here ν runs over nonnegative Radon measures in $\mathbb{R}^{n+1}$ and $F*\nu$ denotes the thermal potential defined by the convolution of F and ν). The inner continuous thermal capacity $\alpha_*(E)$ and the outer thermal capacity $\gamma^*(E)$ of a set $E \subset \mathbb{R}^{n+1}$ are defined in a usual way.

Theorem 1: For an arbitrary set $E \subset \mathbb{R}^{n+1}$, the following conditions are equivalent:

(i) $z \in b(E)$;

(ii) $\int_0^1 \gamma^*(E \cap B(z,c))/c^{n/2+1}\, dc = \infty$;

(iii) $\sum_{k=1}^{\infty} 2^{kn/2}\, \gamma^*(E \cap B(z,2^{-k})) = \infty$;

(iv) $\sum_{k=1}^{\infty} 2^{kn/2}\, \gamma^*(E \cap A(z,2^{-k})) = \infty$.

In the proof, the criterion of the regularity established in [1] is used in an essential way.

Let $E \subset \mathbb{R}^{n+1}$ be an arbitrary set. The smallest finely closed set $L \subset \mathbb{R}^{n+1}$ such that $E \setminus L$ is semi-polar is called the essential base of the set E and denoted by $\beta(E)$.

Theorem 2: For an arbitrary Borel set $E \subset \mathbb{R}^{n+1}$, the following conditions are equivalent:

(i) $z \in \beta(E)$

(ii) $\int_0^1 \alpha_*(E \cap B(z,c))/c^{n/2+1}\, dc = \infty$;

(iii) $\sum_{k=1}^{\infty} 2^{kn/2} \alpha_*(E \cap B(z,2^{-k})) = \infty$;

(iv) $\sum_{k=1}^{\infty} 2^{kn/2} \alpha_*(E \cap A(z,2^{-k})) = \infty$.

Results from [2] are important for the proof of Theorem 2. For a bounded open set $U \subset \mathbb{R}^{n+1}$, the points of the Choquet boundary $Ch_{K(U)}\bar{U}$ of $\bar{U}$ with respect to the space K(U) of all functions continuous on $\bar{U}$ and caloric on U can be characterized in terms of the continuous capacity. Namely, for $z \in \partial U$, the condition $z \in Ch_{K(U)}\bar{U}$ is equivalent to (ii) - (iv) from Theorem 2 where E is replaced by $\mathbb{R}^{n+1} \setminus U$ and α_* by α. Geometric conditions guaranteeing that $z \in b(E)$, or $z \in \beta(E)$, can be deduced from Theorems 1 and 2.

References:

[1] EVANS, L.C., GARIEPY, R.F.: Wiener's criterion for the heat equation, Arch.Rational Mech.Anal. 78(1982), 293-314.

[2] HANSEN,W.: Semi-polar sets and quasi-balayage, Math.Ann. 257 (1981), 495-517.