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Two results concerning string graphs

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ANNOUNCEMENTS OF NEW RESULTS

(of authors having an address in Czechoslovakia)

TWO RESULTS CONCERNING STRING GRAPHS

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String graphs are defined as intersection graphs of curves in the plane. Two problems on string graphs have long been open: whether the string graphs can be characterized by a finite set of forbidden induced minors and whether recognizing string graphs is NP-hard. Here the answers to these questions are stated.

It is easy to see that every induced minor (i.e. graph obtained by vertex deletions and edge contractions) of a string graph is a string one as well. This fact leads to introducing the class of critical (with respect to the induced-minor order) nonstring graphs, and to the question of its finiteness. We claim:

Theorem 1. The class of critical nonstring graphs is infinite.

It follows from the works of Robertson-Seymour that every minor closed class of graphs is polynomially recognizable. It is also known that recognizing and induced-minor closed class of graphs may be NP-hard or even undecidable. However, no natural induced-minor closed class recognizing that would be NP-hard was known. We claim:

Theorem 2. Recognizing string graphs is NP-hard.

Note that every string graph has a string representation in which any two strings share a finite number of common intersecting points. It seemed that recognizing realizability of graphs by intersection graphs might be easier, if one puts constraints on the number of common intersecting points that two strings may share. A bit surprisingly, the method of our proof of Theorem 2 yields the following:

Theorem 3. Determining whether a given string graph has a string representation in which any two strings share at most one common intersecting point is NP-complete.

The proofs in detail are supposed to appear elsewhere.

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NOWHERE CONTINUOUS SOLUTIONS TO ELLIPTIC SYSTEMS

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For each given F_σ set M in $\mathbf{R}^3$ there is a linear elliptic system with measurable bounded coefficients in $\mathbf{R}^3$ which has a weak solution u with the