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# VARIETIES WITH TOLERANCE AND CONGRUENCE EXTENSION PROPERTY

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**Abstract.** A *tolerance* on an algebra $\mathfrak{A}$ si a reflexive and symmetric binary relation on $\mathfrak{A}$ which has the Substitution property with respect to all operations of $\mathfrak{A}$. An algebra $\mathfrak{A}$ satisfies the *Tolerance Extension Property* if each tolerance on any subalgebra $\mathfrak{B}$ of $\mathfrak{A}$ is a restriction of some tolerance on $\mathfrak{A}$. The paper contains characterizations of such varieties of algebras in polynomial conditions and characterizations of varieties with Congruence Extension Property in a special case. All results are illustrated by examples.

**Key words:** congruence extension property, tolerance relation, variety of algebras, polynomial conditions.

The problems of *Congruence Extension Property,* briefly (CEP) and of *Principal Congruence Extension Property,* (PCEP), were investigated by A. Day, G. Grätzer and H. Lakser in [6], [5], [7], [8]. A. Day proved in [5] that in a variety of algebras, conditions (CEP) and (PCEP) are equivalent. We shall proceed to generalize these concepts for tolerances and characterize varieties satisfying such properties.

Let $\mathfrak{A} = (A, F)$ be an algebra and $T$ be a binary relation on the set $A$. The relation $T$ is called a *tolerance on* $\mathfrak{A}$ if it is reflexive and symmetric and it has the *Substitution Property with respect to F,* i.e. $T$ is a subalgebra of the direct product $\mathfrak{A} \times \mathfrak{A}$. Thus every congruence on $\mathfrak{A}$ is a tolerance on $A$ but not vice versa (see [3], [10]). Denote by $LT(\mathfrak{A})$ the set of all tolerances on $\mathfrak{A}$. Clearly $LT(\mathfrak{A})$ is a complete lattice with respect to the set inclusion and the meet in $LT(\mathfrak{A})$ coincides with the set-intersection, [2] (moreover, it is an algebraic lattice, see [3]). Denote by $\bigvee_A$ the join in $LT(\mathfrak{A})$. If $a, b \in A$, denote by $T_A(a, b)$ the least tolerance on $\mathfrak{A}$ collapsing the pair $\langle a, b\rangle$; it is so called *principal tolerance on* $\mathfrak{A}$ generated by $a, b$. If $B \subseteq A$ and $T \in LT(\mathfrak{A})$, denote by $T|_B$ the restriction of $T$ onto $B$:

$$T|_B = T \cap (B \times B).$$

If $\mathfrak{B}$ is a subalgebra of $\mathfrak{A}$, $T \in LT(\mathfrak{A})$ and $T' \in LT(\mathfrak{B})$, then $T$ is called an *extension* of $T'$ provided $T|_B = T'$.

If $p$ is an $(n+m)$-ary polynomial over $\mathfrak{A}$ and $a_1, \ldots, a_m \in A$, by an $n$-ary *algebraic function* $\varphi$ over $\mathfrak{A}$ generated by $p, a_1, \ldots, a_m$ is meant a mapping of $A^n$ into $A$ given by

$$\varphi(x_1, \ldots, x_n) = p(x_1, \ldots, x_n, a_1, \ldots, a_m).$$

Let $\mathscr{V}$ be a variety of algebras. By $F_n(x_1, \ldots, x_n)$ a free algebra of $\mathscr{V}$ generated by the set of free generators $\{x_1, \ldots, x_n\}$ will be denoted. If $\mathfrak{A} = (A, F)$ and $a_1, \ldots, a_k \in A$, denote by $Gen\,(a_1, \ldots, a_k)$ a subalgebra of $\mathfrak{A}$ generated by the set $\{a_1, \ldots, a_k\}$.

**Definition.** A class $\mathscr{C}$ of algebras is said to satisfy the *(Principal) Tolerance Extension Property* if for each $\mathfrak{A} \in \mathscr{C}$ and each subalgebra $\mathfrak{B}$ of $\mathfrak{A}$ every (principal) tolerance on $\mathfrak{B}$ is the restriction of a tolerance on $\mathfrak{A}$.

We abbreviate the Principal Tolerance Extension Property by (PTEP) and Tolerance Extension Property by (TEP).

**Lemma 1.** *Let $\mathfrak{A} = (A, F)$ be an algebra and $a_i, b_i \in A$ for $i = 1, \ldots, n$. The following conditions are equivalent:*

(i) $\langle x, y\rangle \in \bigvee_A\{T_A(a_i, b_i);\, i = 1, \ldots, n\}$;

(ii) *there exists a $2n$-ary algebraic function $\varphi$ over $\mathfrak{A}$ with*

$$x = \varphi(a_1, \ldots, a_n, b_1, \ldots, b_n),$$
$$y = \varphi(b_1, \ldots, b_n, a_1, \ldots, a_n).$$

For the proof, see e.g. [2].

**Theorem 1.** *Let $\mathscr{V}$ be a variety of algebras. The following two conditions are equivalent:*

(1) *$\mathscr{V}$ satisfies* (PTEP);

(2) *for every $(2+n)$-ary polynomial $p$ over $\mathscr{V}$ there exists a 6-ary polynomial $q$ over $\mathscr{V}$ such that*

$$p(x, y, \vec{z}_i) = q(x, y, x, y, p(x, y, \vec{z}_i), p(y, x, \vec{z}_i)),$$
$$p(y, x, \vec{z}_i) = q(y, x, x, y, p(x, y, \vec{z}_i), p(y, x, \vec{z}_i)).$$

Proof. Clearly $\mathscr{V}$ satisfies (PTEP) if and only if for each $\mathfrak{A} \in \mathscr{V}$ and every subalgebra $\mathfrak{B}$ of $\mathfrak{A}$ and each $a, b \in B$ we have

$$T_A(a, b)\,|_B = T_B(a, b).$$

The inclusion $T_A(a, b)|_B \supseteq T_B(a, b)$ is evident for every two $\mathfrak{A}$,, $\mathfrak{B}$, thus (PTEP) is equivalent only to the converse inclussion.

(1) $\Rightarrow$ (2): Let $\mathfrak{A} = F_{2+n}(x, y, z_1, \ldots, z_n)$ and $p$ be a $(2+n)$-ary polynomial over $\mathscr{V}$. Denote

$$c = p(x, y, \vec{z}_i), \qquad d = p(y, x, \vec{z}_i).$$

Let $\mathfrak{B} = Gen(x, y, c, d)$. Then clearly $\langle c, d\rangle \in T_A(x, y)\,|_B$ and, by (1), also $\langle c, d\rangle \in$

$\in T_B(x, y)$. Hence, by Lemma 1, $c = \psi(x, y)$, $d = \psi(y, x)$ for some binary algebraic function $\psi$ over $\mathfrak{B}$, i.e. there exists a 6-ary polynomial $q$ over $\mathscr{V}$ such that

$$\psi(\xi_1, \xi_2) = q(\xi_1, \xi_2, x, y, c, d),$$

i.e.

$$c = q(x, y, x, y, c, d), \qquad d = q(y, x, x, y, c, d),$$

whence (2) is evident.

(2) $\Rightarrow$ (1): Let $\mathfrak{A}, \mathfrak{B} \in \mathscr{V}$, $\mathfrak{B}$ be a subalgebra of $\mathfrak{A}$, $x, y \in \mathfrak{B}$ and

$$\langle c, d\rangle \in T_A(x, y)\,|_B.$$

Then, by Lemma 1, $c = p(x, y, a_1, \ldots, a_n)$, $d = p(y, x, a_1, \ldots, a_n)$ for some $a_1, \ldots, a_n \in A$ and some polynomial $p$. By (2), $c, d \in B$ implies also $q(x, y, x, y, c, d) \in B$ and $q(y, x, x, y, c, d) \in B$, thus, also by Lemma 1,

$$\langle c, d\rangle \in T_B(x, y)$$

proving (PTEP).

**Example 1.** *The variety of semigroups satisfying the identity $xzy = xy$ satisfies* (PTEP).

Proof. Let $c = p(x, y, \vec{z}_i)$, $d = p(y, x, \vec{z}_i)$ for some $(2 + n)$-ary polynomial $p$. Clearly it suffices only to investigate the case $c \neq d$. Without the loss of generality, we can try only the case $c = xz_i$, $d = yz_i$ (for some $i \in \{1, \ldots, n\}$). We can choose a 6-ary polynomial $q$ as follows:

$$q(x_1, x_2, x_3, x_4, x_5, x_6) = x_1 x_6.$$

Clearly

$$q(x, y, x, y, c, d) = xyz_i = xz_i = c,$$
$$q(y, x, x, y, c, d) = yyz_i = yz_i = d,$$

which implies (PTEP).

**Example 2.** *The non-trivial variety of semilattices does not satisfy* (PTEP).

Proof. Put $n = 1$, and $p(x_1, x_2, x_3) = x_1 \vee x_3$. Then there evidently does not exist a 6-ary semilattice polynomial $q$ such that

$$x \vee z = q(x, y, x, y, x \vee z, y \vee z),$$
$$y \vee z = q(y, x, x, y, x \vee z, y \vee z).$$

**Remark 1.** Example 2 shows that there is a different situation as in the case of congruences. Since every equationally complete variety of semigroups satisfies (PCEP), see [5], it is not true for (PTEP).

**Remark 2.** A variety $\mathscr{V}$ is called *Principal Tolerance Trivial* if for each $\mathfrak{A} \in \mathscr{V}$, every $T_A(a, b)$ is a congruence on $\mathfrak{A}$, i.e. for each $a, b \in A$ we have $T_A(a, b) = \Theta_A(a, b)$. The well-known examples of such varieties are:

(I) congruence-permutable varieties (see [10]),

(II) the variety of distributive lattices (see [4] or [1]). For such varieties, we obtain immediately from Theorem 1 and Day's Theorem [5]:

**Theorem 2.** *Let $\mathscr{V}$ be a Principal Tolerance Trivial variety. The following conditions are equivalent:*

(A) *$\mathscr{V}$ satisfies* (CEP);

(B) *it holds* (2) *of Theorem* 1.

Theorem 2 enables us to characterize varieties of groups and quasigroups satisfying (CEP).

**Example 3.** *The variety of all abelian groups satisfies* (CEP).

Proof. Let $p$ be a $(2 + n)$-ary group polynomial. Thanks to the commutativity, it has the cannonical form

$$p(x_1, x_2, \dots, x_{2+n}) = x_1^a \,.\, x_2^b \,.\, x_{2+1}^{e_1} \dots x_{2+n}^{e_n}.$$

Put

$$q(x_1, x_2, x_3, x_4, x_5, x_6) = x_1^a \,.\, x_2^b \,.\, x_3^{-a} \,.\, x_4^{-b} \,.\, x_5 .$$

Evidently, (2) of Theorem 1 is satisfied.

It was proved in [7] (Theorem 6.1) that for congruence distributive varieties generated by a finite algebra, (CEP) is equivalent to the existence of *Universal Restricted Congruence Scheme.* Our Theorem 2 enables us to give a similar condition in the case of Principal Tolerance Trivial (e.g. congruence permutable) varieties:

**Theorem 3.** *Let $\mathscr{V}$ be a Principal Tolerance Trivial variety. The following two conditions are equivalent:*

(A) *$\mathscr{V}$ satisfies* (CEP);

(B) *$\langle c, d\rangle \in \Theta_A(x, y)$ if and only if there exists a* 6-*ary polynomial $q$ over $\mathscr{V}$ such that*

$$c = q(x, y, x, y, c, d),$$
$$d = q(y, x, x, y, c, d).$$

Proof. (A) $\Rightarrow$ (B): If $\mathscr{V}$ satisfies (CEP), it satisfies also (PTEP) since it is Principal Tolerance Trivial. Let $\langle c, d\rangle \in \Theta_A(x, y) = T_A(x, y)$. By Lemma 1, there exists a $(2 + n)$-ary polynomial $p$ with $c = p(x, y, \vec{z}_i)$, $d = p(y, x, \vec{z}_i)$. Theorem 2 implies (B) immediately.

(B) $\Rightarrow$ (A) is clear by Corollary 1 in [5].

**Example 4.** *The variety $\mathscr{D}$ of all distributive lattices satisfies* (CEP) (see also [4], [10])

Proof. It is well-known that $\langle c, d\rangle \in \Theta_A(x, y)$ for $\mathfrak{A} \in \mathscr{D}$ if and only if

$$(*) \qquad c \vee d = [(x \vee y) \vee (c \wedge d)] \wedge (c \vee d)$$
$$c \wedge d = [(x \wedge y) \vee (c \wedge d)] \wedge (c \vee d).$$

Since $\langle c, d\rangle \in \Theta_A(x, y)$ if and only if $\langle c\wedge d, c\vee d\rangle \in \Theta_A(x, y)$ and $\Theta_A(x, y) = \Theta_A(x\wedge y, x\vee y)$, it suffices to investigate only the case $d \leqq c$, $y \leqq x$. In this case, the foregoing identities have a form

$$c = (x\vee d)\wedge c,$$
$$d = (y\vee d)\wedge c,$$

which is the form desired in Theorem 3 ($\mathscr{D}$ is Principal Tolerance Trivial, see Remark 2).

**Remark 3.** Contrary to the case of congruences, (PTEP) and (TEP) are not equivalent on varieties:

The variety $\mathscr{D}$ of distributive lattices satisfies (PTEP) since it is Principal Tolerance Trivial and it satisfies (CEP), however, it does not satisfy (TEP) since $F_2(x, y) \in \mathscr{D}$ has not this property (see the Proposition in [1]).

Therefore, these two conditions will be investigated separately and we give a characterization of varieties satisfying the equivalence (PTEP) $\Leftrightarrow$ (TEP).

**Lemma 2.** *Let $\mathfrak{B} = (B, F)$ be a subalgebra of $\mathfrak{A}$. The following two conditions are equivalent:*

(a) *For every 2n-ary algebraic function $\varphi$ over $\mathfrak{A}$ and for each $x_1, \ldots, x_n$, $y_1, \ldots, y_n \in B$ with*

$$\varphi(x_1, \ldots, x_n, y_1, \ldots, y_n) \in B, \quad \varphi(y_1, \ldots, y_n, x_1, \ldots, x_n) \in B,$$

*there exists a 2n-ary algebraic function $\psi$ over $\mathfrak{B}$ such that*

$$\varphi(\vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i) = \psi(\vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i), \qquad \varphi(\vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i) = \psi(\vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i);$$

(b) *For each $x_1, \ldots, x_n, y_1, \ldots, y_n \in B$ we have*

$$\mathbf{V}_B\{T_B(x_i, y_i); i = 1, \ldots, n\} = (\mathbf{V}_A\{T_A(x_i, y_i); i = 1, \ldots, n\})|_B.$$

Proof. (a) $\Rightarrow$ (b): The inclusion $\subseteq$ in (b) is evident in any case. Prove the converse inclusion. Let

$$\langle x, y\rangle \in (\mathbf{V}_A\{T_A(x_i, y_i); i = 1, \ldots, n\})|_B.$$

By Lemma 1, there exists a $2n$-ary algebraic function $\varphi$ over $\mathfrak{A}$ with

$$x = \varphi(\vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i) \in B, \qquad y = \varphi(\vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i) \in B.$$

By (a), there exists a $2n$-ary algebraic function $\psi$ over $\mathfrak{B}$ with

$$\varphi(\vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i) = \psi(\vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i), \qquad \varphi(\vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i) = \psi(\vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i)$$

and, by Lemma 1,

$$\langle x, y\rangle \in \mathbf{V}_B\{T_B(x_i, y_i); i = 1, \ldots, n\}.$$

(b) $\Rightarrow$ (a): Let $\varphi$ and $x_1, \ldots, x_n, y_1, \ldots, y_n$ be as in (a). By Lemma 1, $\langle x, y\rangle \in \in (\bigvee_A\{T_A(x_i, y_i); i = 1, \ldots, n\})\,|_B$ for

$$x = \varphi(x_1, \ldots, x_n, y_1, \ldots, y_n),\ y = \varphi(y_1, \ldots, y_n, x_1, \ldots, x_n).$$

By (b), $\langle x, y\rangle \in \bigvee_B\{T_B(x_i, y_i); i = 1, \ldots, n\}$ and Lemma 1 yields (a).

**Lemma 3.** *Let $\mathscr{C}$ be a class of algebras closed under the formation of subalgebras. The following conditions are equivalent:*

(i) *$\mathscr{C}$ satisfies* (TEP);

(ii) *for each $\mathfrak{A} \in \mathscr{C}$ and every subalgebra $\mathfrak{B}$ of $\mathfrak{A}$ it is valid* (b) *of Lemma* 2.

Proof. (i) $\Rightarrow$ (ii): If $\mathscr{C}$ satisfies (TEP), then clearly

$$\bigvee_A\{T_A(x_i, y_i); i = 1, \ldots, n\}$$

is an extension of

$$\bigvee_B\{T_B(x_i, y_i); i = 1, \ldots, n\} \qquad \text{for } x_i, y_i \in B$$

and (b) of Lemma 2 is evident.

(ii) $\Rightarrow$ (i): Let $T \in LT(\mathfrak{B})$. By Theorem 14 in [3],

$$T = \bigvee_B\{T_B(c, d); \langle c, d\rangle \in T\}.$$

Put $T^* = \bigvee_A\{T_A(c, d); \langle c, d\rangle \in T\}$ and prove that $T^*$ is an extension of $T$. Clearly $T^*\,|_B \supseteq T$. Conversely, let $\langle x, y\rangle \in T^*\,|_B$. By Theorem 2 in [3], there exists an $n$-ary polynomial $p$ over $\mathfrak{A}$ such that

$$x = p(x_1, \ldots, x_n), \qquad y = p(y_1, \ldots, y_n),$$

where $\langle x_i, y_i\rangle \in T_A(c_i, d_i)$ for some $\langle c_i, d_i\rangle \in T$. Combining it with Lemma 1,

$$x = \varphi(c_1, \ldots, c_n, d_1, \ldots, d_n),$$
$$y = \varphi(d_1, \ldots, d_n, c_1, \ldots, c_n),$$

for some $2n$-ary algebraic function $\varphi$ over $\mathfrak{A}$. By (ii) and Lemma 2,

$$x = \psi(c_1, \ldots, c_n, d_1, \ldots, d_n),$$
$$y = \psi(d_1, \ldots, d_n, c_1, \ldots, c_n),$$

for some $2n$-ary algebraic function $\psi$ over $\mathfrak{B}$, thus, by Lemma 1,

$$\langle x, y\rangle \in \bigvee_B\{T_B(c_i, d_i); \langle c_i, d_i\rangle \in T \text{ for } i = 1, \ldots, n\} \subseteq T$$

and $T^*\,|_B \subseteq T$ is proved.

**Theorem 4.** *Let $\mathscr{V}$ be a variety of algebras. The following two conditions are equivalent:*

(3) *$\mathscr{V}$ satisfies* (TEP),

(4) *for every* $(2n + k)$*-ary polynomial* $p$ *over* $\mathscr{V}$ *there exists a* $(4n + 2)$*-ary polynomial* $q$ *over* $\mathscr{V}$ *such that*

$$p(\vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i, \vec{\mathbf{z}}_j) = q(\vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i, p(\vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i, \vec{\mathbf{z}}_j), p(\vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i, \vec{\mathbf{z}}_j)),$$
$$p(\vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i, \vec{\mathbf{z}}_j) = q(\vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i, \vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i, p(\vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i, \vec{\mathbf{z}}_j), p(\vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i, \vec{\mathbf{z}}_j)).$$

The proof is analogous to that of Theorem 1 only the equality (b) of Lemma 2 is used instead of the equality $T_A(a, b)|_B = T_B(a, b)$ and $\mathfrak{A}$, $\mathfrak{B}$ are chosen as follows:

$$\mathfrak{A} = F_{2n+k}(x_1, \ldots, x_n, y_1, \ldots, y_n, z_1, \ldots, z_k),$$
$$\mathfrak{B} = Gen(x_1, \ldots, x_n, y_1, \ldots, y_n, p(\vec{\mathbf{x}}_i, \vec{\mathbf{y}}_i, \vec{\mathbf{z}}_j), p(\vec{\mathbf{y}}_i, \vec{\mathbf{x}}_i, \vec{\mathbf{z}}_j)).$$

**Remark 4.** Although it looks rather hard to satisfy (4) of Theorem 4, it is actually fulfilled e.g. in every congruence permutable variety satisfying (CEP) as it follows from the Theorem of Werner [10].

**Example 5.** *Every variety of unary algebras satisfies* (TEP).

It follows directly from Theorem 4. As a consequence of Theorems 1 and 4, we infer a characterization of varieties satisfying (PTEP) ⇔ (TEP):

**Theorem 5.** *For a variety* $\mathscr{V}$, *the following conditions are equivalent:*
(A) (PTEP) ⇔ (TEP) *in* $\mathscr{V}$;
(B) (2) *implies* (4) *in* $\mathscr{V}$.

**Example 6.** *In every variety of commutative semigroups, the conditions* (PTEP) *and* (TEP) *are equivalent.*

Proof. Let $p$ be a $(2n + k)$-ary semigroup polynomial and

$$a = p(x_1, \ldots, x_n, y_1, \ldots, y_n, \vec{\mathbf{z}}_j),$$
$$b = p(y_1, \ldots, y_n, x_1, \ldots, x_n, \vec{\mathbf{z}}_j).$$

Thanks to the commutativity, we can express it in the form

$$a = x \,.\, z, \qquad b = y \,.\, z,$$

where

$$x = x_1^{e_1} \ldots x_n^{e_n} \,.\, y_1^{f_1} \ldots y_n^{f_n}$$
$$y = y_1^{e_1} \ldots y_n^{e_n} \,.\, x_1^{f_1} \ldots x_n^{f_n}$$

and $z$ is a product of all other occuring variables which has the same value in these both formulas. If $\mathscr{V}$ satisfies (PTEP), there exists a 6-ary polynomial $q$ such that

$$a = q(x, y, x, y, x \,.\, z, y \,.\, z),$$
$$b = q(y, x, x, y, x \,.\, z, y \,.\, z),$$

i.e., by replacing $x, y, z$ and $a, b$, we obtain easily (4) and Theorem 5 yields the statement.

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