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NATURAL AFFINORS ON r-JET PROLONGATION OF THE TANGENT BUNDLE

W. M. Mikulski

Abstract. We deduce that for $n \geq 2$ and $r \geq 1$, every natural affinor on J^rT over n-manifolds is of the form $\lambda\delta$ for a real number λ, where δ is the identity affinor on J^rT.

0. We fix two nonnegative integers $n \geq 1$ and r.

The r-jet prolongation J^rTM of the tangent bundle TM of an n-manifold M is the space of all r-jets of vector fields on M, i.e.

$$J^rTM = \{j^r_xX \mid X \text{ is a vector field on } M,\ x \in M\} .$$

It is a vector bundle over M with respect to the source projection $\pi^r : J^rTM \to M$, $j^r_xX \to x$. Every embedding $\varphi : M \to N$ of two n-manifolds induces a vector bundle mapping $J^rT\varphi : J^rTM \to J^rTN$ given by $J^rT\varphi(j^r_xX) = j^r_{\varphi(x)}(T\varphi \circ X \circ \varphi^{-1})$.

An affinor on J^rTM is a tensor field of type $(1,1)$ on J^rTM. It can be interpreted as a vector bundle homomorphism $T(J^rTM) \to T(J^rTM)$ over the identity map $id_{J^rTM} : J^rTM \to J^rTM$.

A *natural affinor A on J^rT over n-manifolds* is a system of affinors

$$A_M : T(J^rTM) \to T(J^rTM)$$

for every n-manifold M such that $A_N \circ T(J^rT\varphi) = T(J^rT\varphi) \circ A_M$ for every embedding $\varphi : M \to N$ of two n-manifolds.

For example, the family δ of affinors $\delta_M = id_{T(J^rTM)} : T(J^rTM) \to T(J^rTM)$ for any n-manifold M is a natural affinor on J^rT over n-manifolds. In this paper we prove.

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Theorem 0.1. *If $n \geq 2$ and $r \geq 1$ are two fixed integers, then any natural affinor on J^rT over n-manifolds is of the form $\lambda\delta = \{\lambda\delta_M\}$, where δ is the identity affinor on J^rT and λ is a real number.*

Classifications of natural affinors on some other natural bundles are given in [1]—[5] e.t.c. For example, in [3] all natural affinors on the Weil bundle of A-velocities are described. In particular, any natural affinor on $J^0T = T$ over n-manifolds is of the form $\lambda\delta + \mu J$, where δ is the identity affinor on T, J is the tangent structure on T and λ, μ are real numbers.

Natural affinors play a very impotrant role in the differential geometry. For example, they can be used to define torsions of a connection, see [4].

All manifolds and mappings in this paper are assumed to be smooth, i.e. infinitely differentiable.

1. In the proof of Theorem 0.1 we shall use some facts proved in this item.

Let $n \geq 1$ and r be two nonnegative integers. From now on we shall use the following notations and observations.

The set of all natural affinors on J^rT over n-manifolds will be denoted by $\mathcal{A}(n,r)$. It is a vector space over $\mathbf{R}$. For any $A, B \in \mathcal{A}(n,r)$ and any $\alpha, \beta \in \mathbf{R}$ the natural affinor $\alpha A + \beta B \in \mathcal{A}(n,r)$ is defined by $(\alpha A + \beta B)_M = \alpha A_M + \beta B_M$, where M is an n-manifold.

The usual coordinates on $\mathbf{R}^n$ will be denoted by $x^1, ..., x^n$. The canonical vector fields $\frac{\partial}{\partial x^i}$ on $\mathbf{R}^n$ will be denoted by ∂_i.

For a vector field X on an n-manifold M, the complete lift of X to J^rTM will be denoted by J^rTX.

Let us denote

$$\mathbf{N}_r = \{\alpha = (\alpha_1, ..., \alpha_n) \in (\mathbf{N} \cup \{0\})^n \mid |\alpha| = \alpha_1 + ... + \alpha_n \leq r\} . \tag{1.1}$$

Then the elements

$$J^rT\partial_{i|j^r_0\partial_1} \ , \quad j^r_0(x^\alpha\partial_j) \in (J^rT)_0\mathbf{R}^n \tilde{=} V_{j^r_0\partial_1}J^rT\mathbf{R}^n \subset T_{j^r_0\partial_1}J^rT\mathbf{R}^n \tag{1.2}$$

for $i, j = 1, ..., n$ and $\alpha \in \mathbf{N}_r$ form a basis of the vector space $T_{j^r_0\partial_1}J^rT\mathbf{R}^n$.

From the definition of the complete lift of vector fields it follows that

$$J^rT(x^{\alpha+1_1}\partial_j)_{|j^r_0\partial_1} = \frac{d}{dt}_{|t=0}(J^rT\varphi_t(j^r_0\partial_1)) = \frac{d}{dt}_{|t=0}j^r_0((\varphi_t)_*\partial_1) =$$
$$= j^r_0(\frac{d}{dt}_{|t=0}(\varphi_t)_*\partial_1) = -j^r_0([x^{\alpha+1_1}\partial_j, \partial_1]) = (\alpha_1 + 1)j^r_0(x^\alpha\partial_j)$$

for any $\alpha \in \mathbf{N}_r$ and $j = 1, ..., n$, where φ_t is the flow of $x^{\alpha+1_1}\partial_j$. Then

$$J^rT(x^{\alpha+1_1}\partial_j)_{|j^r_0\partial_1} = (\alpha_1 + 1)j^r_0(x^\alpha\partial_j) \tag{1.3}$$

for any $\alpha \in \mathbf{N}_r$ and $j = 1, ..., n$.

Consequently, the elements $J^rT(x^\alpha\partial_j)_{|j^r_0\partial_1}$ for $\alpha \in \mathbf{N}_{r+1}$ and $j = 1, ..., n$ generate $T_{j^r_0\partial_1}J^rT\mathbf{R}^n$.

After these preparations we prove the following lemma.

Lemma 1.1. *Let $A \in \mathcal{A}(n,r)$ be such that*

$$A_{\mathbf{R}^n}(J^rT((x^1)^s\partial_n)_{|j_0^r\partial_1}) = 0 \tag{1.4}$$

for $s = 0, ..., r+1$. If $n \geq 2$ and $r \geq 0$, then $A = 0$.

Proof. By the Frobenius theorem $j_0^r\partial_1 \in J^rT\mathbf{R}^n$ has dense orbit in $J^rT\mathbf{R}^n$ with respect to the lifted embeddings. Then from the naturality of A with respect to charts it follows the following implication.

If $A_{\mathbf{R}^n}(v) = 0$ for any $v \in T_{j_0^r\partial_1}J^rT\mathbf{R}^n$, then $A = 0$.

Since $A_{\mathbf{R}^n}(v)$ depends linearly on v and the elements $J^rT(x^\alpha\partial_j)_{|j_0^r\partial_1}$ for $\alpha \in \mathbf{N}_{r+1}$ and $j = 1, ..., n$ generate $T_{j_0^r\partial_1}J^rT\mathbf{R}^n$, it remains to prove that

$$A_{\mathbf{R}^n}(J^rT(x^\alpha\partial_j)_{|j_0^r\partial_1}) = 0 \tag{1.5}$$

for any $\alpha \in \mathbf{N}_{r+1}$ and $j = 1, ..., n$.

We consider two cases.

(I) Let $\alpha \in \mathbf{N}_{r+1}$ and $j \in \{2, ..., n\}$. Then $\alpha_1 \leq r+1$, and consequently we have (1.4) for $s = \alpha^1$.

By the Frobenius theorem there exists a diffeomorphism $\varphi : \mathbf{R}^n \to \mathbf{R}^n$ of the form $id_{\mathbf{R}} \times \psi$ such that $\varphi_*\partial_n = \partial_n + (x^2)^{\alpha_2}...(x^n)^{\alpha_n}\partial_j$ on some neighbourhood of 0. Then $\varphi_*\partial_1 = \partial_1$ and $\varphi_*((x^1)^{\alpha_1}\partial_n) = (x^1)^{\alpha_1}\partial_n + x^\alpha\partial_j$ on some neighbourhood of 0. Now, using the invariancy of A and of the complete lifting of vector fields with respect to φ we obtain

$$A_{\mathbf{R}^n}(J^rT((x^1)^{\alpha_1}\partial_n + x^\alpha\partial_j)_{|j_0^r\partial_1}) = TJ^rT\varphi(A_{\mathbf{R}^n}(J^rT((x^1)^{\alpha_1}\partial_n)_{|j_0^r\partial_1})) = 0$$

because of (1.4) for $s = \alpha^1$. Then we have (1.5) because of (1.4) for $s = \alpha^1$ and the $\mathbf{R}$-linearity of the complete lifting of vector fields.

(II) Let $\alpha \in \mathbf{N}_{r+1}$ and $j = 1$. For any $\tau \in \mathbf{R}\backslash\{0\}$ we have the linear isomorphism $\varphi_\tau = (x^1 + \tau x^n, x^2, ..., x^n)$. Since $n \geq 2$, φ_τ preserves ∂_1 and sends $x^\alpha\partial_n$ into $(x^1 - \tau x^n)^{\alpha_1}(x^2)^{\alpha_2}...(x^n)^{\alpha_n}(\partial_n + \tau\partial_1)$. In (I) we have proved that

$$A_{\mathbf{R}^n}(J^rT(x^\alpha\partial_n)_{|j_0^r\partial_1}) = 0\ .$$

Now, using the naturality of A and of the complete lifting of vector fields with respect to φ_τ we obtain (similarly as in (I))

$$A_{\mathbf{R}^n}(J^rT((x^1 - \tau x^n)^{\alpha_1}(x^2)^{\alpha_2}...(x^n)^{\alpha_n}(\partial_n + \tau\partial_1))_{|j_0^r\partial_1}) = 0$$

for any $\tau \in \mathbf{R}\backslash\{0\}$. Both sides of the last equality are polynomials in τ. Considering the coefficients on τ of the polynomials we obtain

$$\begin{aligned} &A_{\mathbf{R}^n}(J^rT(x^\alpha\partial_j)_{|j_0^r\partial_1}) - \\ &\quad - \alpha_1 A_{\mathbf{R}^n}(J^rT((x^1)^{\alpha_1-1}(x^2)^{\alpha_2}...(x^{n-1})^{\alpha_{n-1}}(x^n)^{\alpha_n+1}\partial_n)_{|j_0^r\partial_1}) = 0\ . \end{aligned}$$

(If $\alpha_1 = 0$ the term $\alpha_1...$ does not occur.) In (I) we have proved that

$$\alpha_1 A_{\mathbf{R}^n}(J^rT((x^1)^{\alpha_1-1}(x^2)^{\alpha_2}...(x^{n-1})^{\alpha_{n-1}}(x^n)^{\alpha_n+1}\partial_n)_{|j_0^r\partial_1}) = 0\ .$$

Hence we have (1.5). □

The next lemma shows that any natural affinor sends vertical vectors into vertical ones.

Lemma 1.2. *Let $A \in \mathcal{A}(n,r)$, where $n \geq 1$ and $r \geq 0$. If $w \in TJ^rTM$ is vertical, then so is $A_M(w)$.*

In particular, $A_{\mathbf{R}^n}(J^rT((x^1)^s\partial_n)_{|j_0^r(\tau\partial_1)})$ is vertical for $s \geq 1$ and $\tau \in \mathbf{R}$.

Proof. By the Frobenius theorem $j_0^r\partial_1 \in J^rT\mathbf{R}^n$ has dense orbit with respect to lifted embeddings. Then by the naturality of A with respect to charts we can assume that $w \in V_{j_0^r\partial_1}J^rT\mathbf{R}^n$. Let

$$A_{\mathbf{R}^n}(w) = \sum_{i=1}^{n} \alpha_i J^rT\partial_{i|j_0^r\partial_1} + v \tag{1.6}$$

for some real numbers $\alpha_i \in \mathbf{R}$ and some vertical vector field $v \in V_{j_0^r\partial_1}J^rT\mathbf{R}^n$. It is sufficient to show that $\alpha_i = 0$ for $i = 1, ..., n$.

Let $i \in \{1, ..., n\}$. The mapping $\varphi_i = (x^1, ..., x^i + (x^1)^{r+1}x^i, ..., x^n)$ is a diffeomorphism near $0 \in \mathbf{R}^n$.

Since $j_0^{r+1}(\varphi_i) = id$, then (by the order argument) φ_i preserves $j_0^r\partial_1$ and any vertical vector from $V_{j_0^r\partial_1}J^rT\mathbf{R}^n$. In particular, φ_i preserves w and v.

On the other hand φ_i maps ∂_j into $\rho_i\delta_{ij}(x^1)^{r+1}\partial_i + \partial_j + \dots$, where δ_{ij} is the Kronecker delta, $\rho_1 = r+2$ and $\rho_i = 1$ for $i = 2, ..., n$ and the doots denote the vector field having the $(r+1)$-jet equal to 0.

Now, using the naturality of A with respect to φ_i, we get from (1.6)

$$A_{\mathbf{R}^n}(w) = A_{\mathbf{R}^n}(w) + \alpha_i\rho_i J^rT((x^1)^{r+1}\partial_i)_{|j_0^r\partial_1} \ .$$

But, by (1.3) $J^rT((x^1)^{r+1}\partial_i)_{|j_0^r\partial_1} = (r+1)j_0^r((x^1)^r\partial_i) \neq 0$. Hence $\alpha_i = 0$. □

Using Lemmas 1.1 and 1.2 we prove.

Lemma 1.3. *Let $A \in \mathcal{A}(n,r)$ be such that*

$$A_{\mathbf{R}^n}(J^rT\partial_{n|j_0^r\partial_1}) = 0 \ . \tag{1.7}$$

If $n \geq 2$ and $r \geq 0$, then $A = 0$.

Proof. Owing to Lemma 1.1 and the assumption (1.7) it is sufficient to verify formulas (1.4) for any $s = 1, ..., r+1$.

Of course, we can assume that $r \geq 1$. We consider two cases.

(I) $s = r+1$. Let $\alpha \in \mathbf{N}_{r+1}$ be such that $|\alpha| = r+1$. Then $j_0^r(\partial_n + x^\alpha\partial_n) = j_0^r(\partial_n)$. So, by the theorem of Zajtz [6] (see also [3]), there exists a diffeomorphism $\varphi : \mathbf{R}^n \to \mathbf{R}^n$ such that $j_0^{r+1}\varphi = j_0^{r+1}id$ and $\varphi_*\partial_n = \partial_n + x^\alpha\partial_n$ on some neighbourhood of 0. Because of the jets argument, φ preserves $j_0^r\partial_1$. Then using the naturalty of A with respect to φ from the assumption (1.7) we obtain $A_{\mathbf{R}^n}(J^rT\partial_{n|j_0^r\partial_1} + J^rTx^\alpha\partial_{n|j_0^r\partial_1}) = 0$. Then (we use (1.7) again)

$$A_{\mathbf{R}^n}(J^rTx^\alpha\partial_{n|j_0^r\partial_1}) = 0 \tag{1.8}$$

for any $\alpha \in \mathbf{N}_{r+1}$ with $|\alpha| = r+1$. In particular, $A_{\mathbf{R}^n}(J^rT(x^1)^{r+1}\partial_{n|j_0^r\partial_1}) = 0$.

(II) Assume that $s \in \{1, ..., r\}$.

The elements $j_0^r(x^\alpha\partial_i)$ for $\alpha \in \mathbf{N}_r$ and $i = 1, ..., n$ form a basis of the vector space $(J^rT)_0\mathbf{R}^n$. Then we have the basis $j_0^r(x^\alpha\partial_i)$ for $\alpha \in \mathbf{N}_r$ and $i = 1, ..., n$ of vector fields on $(J^rT)_0\mathbf{R}^n$.

According to Lemma 1.2 we can write

$$A_{\mathbf{R}^n}(J^rT((x^1)^s\partial_n)_{|j_0^r(\tau\partial_1)}) = \sum_{i=1}^{n}\sum_{\alpha\in\mathbf{N}_r} f_\alpha^{s,i}(\tau) j_0^r(x^\alpha\partial_i)_{|j_0^r(\tau\partial_1)} \tag{1.9}$$

for some uniquely determined smooth maps $f_\alpha^{s,i} : \mathbf{R} \to \mathbf{R}$.

The diffeomorphisms $\psi_t = (x^1, tx^2, ..., tx^n)$ for $t \in \mathbf{R}\setminus\{0\}$ preserve ∂_1 and they send $x^\alpha\partial_i$ into $t^{-(\alpha_2+\ldots+\alpha_n)+1-\delta_{1i}}x^\alpha\partial_i$. Then using the invariancy of A with respect to the ψ_t we obtain from (1.9)

$$tf_\alpha^{s,i}(\tau) = t^{-(\alpha_2+\ldots+\alpha_n)+1-\delta_{1i}}f_\alpha^{s,i}(\tau)\ .$$

Consequently, $f_\alpha^{s,1} = 0$ for any $\alpha \in \mathbf{N}_r$, and $f_\alpha^{s,i} = 0$ for any $\alpha \in \mathbf{N}_r$ with $\alpha_2 + ... + \alpha_n \geq 1$ and $i = 2, ..., n$.

The diffeomorphisms $\varphi_t = (x^1, tx^2, ..., tx^{n-1}, x^n)$ for $t \in \mathbf{R}\setminus\{0\}$ preserve ∂_1 and ∂_n and they send ∂_i into $t\partial_i$ for $i = 2, ..., n-1$. Then using the invariancy of A with respect to the φ_t we obtain from (1.9)

$$f_{(k,0,\ldots,0)}^{s,i}(\tau) = tf_{(k,0,\ldots,0)}^{s,i}(\tau)$$

for $i = 2, ..., n-1$, $k = 0, ..., r$, $\tau \in \mathbf{R}$ and $t \in \mathbf{R}\setminus\{0\}$. Consequently $f_{(k,0,\ldots,0)}^{s,i} = 0$ for any $k = 0, ..., r$ and $i = 2, ..., n-1$.

Then

$$A_{\mathbf{R}^n}(J^rT((x^1)^s\partial_n)_{|j_0^r(\tau\partial_1)}) = \sum_{k=0}^{r} g_k^s(\tau) j_0^r((x^1)^k\partial_n)_{|j_0^r(\tau\partial_1)} \tag{1.10}$$

for some uniquely determined smooth maps $g_k^s = f_{(k,0,\ldots,0)}^{s,n} : \mathbf{R} \to \mathbf{R}$.

Since $n \geq 2$, the diffeomorphisms $\mu_t = (tx^1, x^2, ..., x^n)$ for $t \in \mathbf{R}\setminus\{0\}$ preserve ∂_n and they send ∂_1 into $t\partial_1$. Then using the invariancy of A with respect to the μ_t we obtain from (1.10) that $t^{-s}g_k^s(t\tau) = t^{-k}g_k^s(\tau)$, i.e.

$$g_k^s(\tau) = t^{k-s}g_k^s(t\tau)$$

for any $k = 0, ..., r, t \in \mathbf{R}\setminus\{0\}$ and $\tau \in \mathbf{R}$. Consequently $g_k^s = 0$ for $k = s+1, ..., r$. Then

$$A_{\mathbf{R}^n}(J^rT((x^1)^s\partial_n)_{|j_0^r\partial_1}) = \sum_{k=0}^{s} a_k^s j_0^r((x^1)^k\partial_n)_{|j_0^r\partial_1} \tag{1.11}$$

for some uniquely determined real numbers $a_k^s = g_k^s(1) \in \mathbf{R}$.

Since $(s, 0, ..., 0, r+1-s) \in \mathbf{N}_{r+1}$, we get from (1.8)

$$A_{\mathbf{R}^n}(J^rT((x^1)^s(x^n)^{r+1-s}\partial_n)_{|j_0^r\partial_1}) = 0 \ . \tag{1.12}$$

Since $n \geq 2$, there exists a diffeomorphism ν_s preserving $0 \in \mathbf{R}^n$, x^1, ∂_1 and sending ∂_n into $\partial_n + (x^n)^{r+1-s}\partial_n$. Then using the invariancy of A with respect to the ν_s we obtain from (1.11) that

$$A_{\mathbf{R}^n}(J^rT((x^1)^s(\partial_n + (x^n)^{r+1-s}\partial_n))_{|j_0^r\partial_1}) = \\ = \sum_{k=0}^{s} a_k^s j_0^r((x^1)^k(\partial_n + (x^n)^{r+1-s}\partial_n))_{|j_0^r\partial_1} \ .$$

Consequently (we use (1.11) again)

$$A_{\mathbf{R}^n}(J^rT((x^1)^s(x^n)^{r+1-s}\partial_n)_{|j_0^r\partial_1}) = \sum_{k=0}^{s} a_k^s j_0^r((x^1)^k(x^n)^{r+1-s}\partial_n)_{|j_0^r\partial_1} \ . \tag{1.13}$$

Now, from (1.12) and (1.13) we get

$$\sum_{k=0}^{s} a_k^s j_0^r((x^1)^k(x^n)^{r+1-s}\partial_n)_{|j_0^r\partial_1} = 0 \ .$$

Hence if $k + r + 1 - s \leq r$, then $a_k^s = 0$. Then (by (1.11))

$$A_{\mathbf{R}^n}(J^rT((x^1)^s\partial_n)_{|j_0^r\partial_1}) = b^s j_0^r((x^1)^s\partial_n)_{|j_0^r\partial_1} \tag{1.14}$$

for some uniquely determined real number $b^s = a_s^s \in \mathbf{R}$.

Now, applying the invariancy of A with respect to the $\mu_t = (tx^1, x^2, ..., x^n)$ for $t \in \mathbf{R} \setminus \{0\}$ to both sides of (1.14), and next multiplying both sides of such obtained formula by t^s, and then taking the limit at $t = 0$ we obtain that

$$A_{\mathbf{R}^n}(J^rT((x^1)^s\partial_n)_{|j_0^r 0}) = b^s j_0^r((x^1)^s\partial_n)_{|j_0^r 0} \ . \tag{1.15}$$

The flow of $(x^1)^s\partial_n$ preserves both the zero vector field on $\mathbf{R}^n$ and $0 \in \mathbf{R}^n$. Then $J^rT((x^1)^s\partial_n)_{|j_0^r 0} = 0$. Hence $b^s j_0^r((x^1)^s\partial_n)_{|j_0^r 0} = 0$ by (1.15). Consequently $b^s = 0$ as $j_0^r((x^1)^s\partial_n)_{|j_0^r 0} \neq 0$.

The lemma is proved. □

2. Proof of Theorem 0.1. Consider $A \in \mathcal{A}(n, r)$. We can write

$$A_{\mathbf{R}^n}(J^rT\partial_{n\,|j_0^r(\tau\partial_1)}) = \sum_{i=1}^{n}\sum_{\alpha \in \mathbf{N}_r} f_\alpha^i(\tau) j_0^r(x^\alpha\partial_i)_{|j_0^r(\tau\partial_1)} + \\ + \sum_{i=1}^{n} f^i(\tau) J^rT\partial_{i\,|j_0^r(\tau\partial_1)} \tag{2.1}$$

for some uniquely determined smooth maps $f^i_\alpha, f^i : \mathbf{R} \to \mathbf{R}$.

The diffeomorphisms $\psi_t = (x^1, tx^2, ..., tx^n)$ for $t \in \mathbf{R} \setminus \{0\}$ preserve ∂_1 and they send $x^\alpha \partial_i$ into $t^{-(\alpha_2+\ldots+\alpha_n)+1-\delta_{1i}} x^\alpha \partial_i$. Then using the invariancy of A with respect to the ψ_t we obtain from (2.1)

$$tf^i_\alpha(\tau) = t^{-(\alpha_2+\ldots+\alpha_n)+1-\delta_{1i}} f^i_\alpha(\tau) \text{ and } tf^1(\tau) = f^1(\tau).$$

Consequently $f^1_\alpha = 0$ for any $\alpha \in \mathbf{N}_r$, $f^i_\alpha = 0$ for any $\alpha \in \mathbf{N}_r$ with $\alpha_2 + ... + \alpha_n \geq 1$ and any $i = 2, ..., n$, and $f^1 = 0$.

The diffeomorphisms $\varphi_t = (x^1, tx^2, ..., tx^{n-1}, x^n)$ for $t \in \mathbf{R} \setminus \{0\}$ preserve ∂_1 and ∂_n and they send ∂_i into $t\partial_i$ for $i = 2, ..., n-1$. Then using the invariancy of A with respect to the φ_t we obtain from (2.1)

$$f^i_{(k,0,\ldots,0)}(\tau) = tf^i_{(k,0,\ldots,0)}(\tau) \text{ and } f^i(\tau) = tf^i(\tau)$$

for $i = 2, ..., n-1$, $k = 0, ..., r$, $\tau \in \mathbf{R}$ and $t \in \mathbf{R} \setminus \{0\}$. Consequently $f^i_{(k,0,\ldots,0)} = 0$ for any $k = 0, ..., r$ and any $i = 2, ..., n-1$, and $f^i = 0$ for any $i = 2, ..., n-1$.

Then we have

$$A_{\mathbf{R}^n}(J^rT\partial_{n|j^r_0(\tau\partial_1)}) = \sum_{k=0}^{r} g_k(\tau) j^r_0((x^1)^k \partial_n)_{|j^r_0(\tau\partial_1)} + \\ + g(\tau) J^rT\partial_{n|j^r_0(\tau\partial_1)} \tag{2.2}$$

for some uniquely determined smooth maps $g_k = f^n_{(k,0,\ldots,0)} : \mathbf{R} \to \mathbf{R}$ and $g = f^n : \mathbf{R} \to \mathbf{R}$.

Since $n \geq 2$, the diffeomorphisms $\mu_t = (tx^1, x^2, ..., x^n)$ for $t \in \mathbf{R} \setminus \{0\}$ preserve ∂_n and they send ∂_1 into $t\partial_1$. Then using the invariancy of A with respect to the μ_t we obtain from (2.2) that

$$g_k(t\tau) = t^{-k} g_k(\tau)$$

for any $k = 0, ..., r$, $t \in \mathbf{R} \setminus \{0\}$ and $\tau \in \mathbf{R}$. Consequently $g_k = 0$ for $k = 1, ..., r$.

Then we have

$$A_{\mathbf{R}^n}(J^rT\partial_{n|j^r_0\partial_1}) = a_0 j^r_0(\partial_n)_{|j^r_0\partial_1} + aJ^rT\partial_{n|j^r_0\partial_1} \tag{2.3}$$

for some uniquely determined real numbers $a_0 = g_0(1)$ and $a = g(1)$.

Since $n \geq 2$, there exists a diffeomorphism ν preserving both $0 \in \mathbf{R}^n$ and ∂_1 and sending ∂_n into $\partial_n + x^n\partial_n$. Then using the invariancy of A with respect to ν we obtain from (2.3) that

$$A_{\mathbf{R}^n}(J^rT(\partial_n + x^n\partial_n)_{|j^r_0\partial_1}) = a_0 j^r_0(\partial_n + x^n\partial_n)_{|j^r_0\partial_1} + aJ^rT(\partial_n + x^n\partial_n)_{|j^r_0\partial_1} .$$

Consequently (we use (2.3) again)

$$A_{\mathbf{R}^n}(J^rT(x^n\partial_n)_{|j^r_0\partial_1}) = a_0 j^r_0(x^n\partial_n)_{|j^r_0\partial_1} + aJ^rT(x^n\partial_n)_{|j^r_0\partial_1} . \tag{2.4}$$

Since the flow of $x^n\partial_n$ preserves $j^r_0\partial_1$, then $J^rT(x^n\partial_n)_{|j^r_0\partial_1} = 0$. Hence $a_0 j^r_0(x^n\partial_n)_{|j^r_0\partial_1} = 0$ by (2.4). On the other hand, since $r \geq 1$, $j^r_0(x^n\partial_n)_{|j^r_0\partial_1} \neq 0$. Then $a_0 = 0$.

Then (by (2.3))

$$A_{\mathbf{R}^n}(J^rT\partial_{n|j^r_0\partial_1}) = aJ^rT\partial_{n|j^r_0\partial_1}$$

for some uniquely determined real number a.

Now, $A - a\delta$, where δ is the identity affinor on J^rT, satisfies the assumption of Lemma 1.3. Consequently (by Lemma 1.3) $A - a\delta = 0$, i.e. $A = a\delta$ for some $a \in \mathbf{R}$.

The proof of Theorem 0.1 is completed. □

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