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ON HOLOMORPHICALLY PROJECTIVE MAPPINGS FROM EQUIAFFINE GENERALLY RECURRENT SPACES ONTO KÄHLERIAN SPACES

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Abstract. In this paper we consider holomorphically projective mappings from the special generally recurrent equiaffine spaces A_n onto (pseudo-) Kählerian spaces $\bar{K}_n$. We proved that these spaces A_n do not admit nontrivial holomorphically projective mappings onto $\bar{K}_n$.

These results are a generalization of results by T. Sakaguchi, J. Mikeš and V. V. Domashev, which were done for holomorphically projective mappings of symmetric, recurrent and semisymmetric Kählerian spaces.

1. Introduction

In this paper we present some new results obtained for holomorphically projective mappings from equiaffine special spaces A_n onto Kählerian spaces $\bar{K}_n$.

These A_n are generally recurrent, including m-recurrent (K_n^m) in the sence of V. R. Kaygorodov [5, 6]. It is know that if the spaces K_n^m are (pseudo-) Riemannian spaces V_n (briefly – *Riemannian*) then they are semisymmetric.

An n-dimensional manifold A_n with affine connection ∇ is an *equiaffine space* if in A_n the Ricci tensor *Ric* is symmetric. These spaces are characterized by a coordinate system x such that $\Gamma^{\alpha}_{\alpha i}(x) = \partial f(x)/\partial x^i$, where $f(x)$ is a function on A_n, and $\Gamma^h_{ij}(x)$ are components of a connection ∇ [3, 13, 20, 23, 27].

A Riemannian space $\bar{K}_n$ is called a *Kählerian space* if it is endowed, besides a metric tensor $\bar{g}$, with an affinor structure F satisfying the following relations [4, 14, 23, 27]

$$F^2 = -\mathrm{Id}\,, \qquad \bar{g}(X, FX) = 0\,, \qquad \bar{\nabla} F = 0\,.$$

Here X are all tangent vectors of $T\bar{K}_n$ and $\bar{\nabla}$ is a connection of $\bar{K}_n$. The structure F is a complex structure.

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2. Holomorphically projective mappings

An *F-planar curve* of a space A_n with an affinor structure F is a curve $x = x(t)$ whose tangent vector $\lambda(t) = dx(t)/dt$, being translated, remains in the area element formed by the tangent vector λ and its conjugate vector $F\lambda$, i.e., the conditions

$$\nabla_\lambda \lambda = \rho_1(t)\,\lambda + \rho_2(t)\,F\lambda\,,$$

where ρ_1, ρ_2 are functions of the argument t, are fulfiled [14, 18].

In Kählerian and Hermitian spaces with a structure F these curves are called *analytically planar* [14, 19, 23, 27].

A diffeomorphism of A_n onto $\bar{A}_n$ is called an *F-planar mapping* if it maps all F-planar curve of A_n into $\bar{F}$-planar curve of $\bar{A}_n$ [14, 18].

If the structures F and $\bar{F}$ are (almost) complex structures then F-planar mappings are evidently *holomorphically projective mappings*. These mappings for Kählerian and Hermitian spaces have been studied by many authors, see [2, 8, 11, 14, 15, 16, 18, 19, 21, 23, 24, 27].

Consider a concrete mapping $f\colon\ A_n \to \bar{K}_n$, both spaces being referred to a common coordinate system x with respect to this mapping. This is a coordinate system where two corresponding points $M \in A_n$ and $f(M) \in \bar{K}_n$ have equal coordinates $x = (x^1, x^2, \dots, x^n)$; the corresponding geometric objects in $\bar{K}_n$ will be marked with a bar. For example, Γ^h_{ij} and $\bar{\Gamma}^h_{ij}$ are components of the affine connection ∇ on A_n and $\bar{\nabla}$ on $\bar{K}_n$, respectively.

An equiaffine space A_n admits a *holomorphically projective mapping* f onto a Kählerian space $\bar{K}_n$ if and only if

$$\bar{\nabla}_X Y = \nabla_X Y + \psi(X)Y + \psi(Y)X - \psi(FX)FY - \psi(FY)FX\,, \tag{1}$$

where $\forall X, Y \in TA_n$, ψ is a closed linear form on A_n, i.e. $\psi(X) = X\psi(x)$, $\psi(x)$ is a function on A_n.

If the linear form $\psi \not\equiv 0$, then a holomorphically projective mapping is called *nontrivial*; otherwise it is said to be *trivial* or *affine*. A complex structure F on a space A_n is necessary also covariantly constant, i.e. $\nabla F = 0$.

Further we will use local coordinates x on a chart $(x, U) \subset A_n$.

The formula (1) in this chart has the following expression:

$$\bar{\Gamma}^h_{ij}(x) = \Gamma^h_{ij}(x) + \delta^h_{(i}\psi_{j)} - F^h_{(i}F^\alpha_{j)}\psi_\alpha\,, \tag{2}$$

where Γ^h_{ij} and $\bar{\Gamma}^h_{ij}$ are components of ∇ and $\bar{\nabla}$, respectively, $\psi_i = \partial_i\psi(x)$ are components of linear form ψ, $F^h_i(x)$ are components of F, δ^h_i is the Kronecker symbol, and $(i\,j)$ denotes the symmetrization without division.

The following theorem holds [15]:

Theorem 1. *Let in an equiaffine space A_n exist the solution of the following system of linear differential equations with respect to the unknown functions $a^{ij}(x)$ and $\lambda^i(x)$:*

$$a^{ij}{}_{,k} = \lambda^i\delta^j_k + \lambda^j\delta^i_k + \lambda^\alpha F^i_\alpha F^j_k + \lambda^\alpha F^j_\alpha F^i_k\,, \tag{3}$$

where "," denotes the covariant derivative with respect to the connection ∇ *of the space* A_n*, the matrix* $\|a^{ij}\|$ *should further satisfy* $\det\|a^{ij}\| \neq 0$ *and the algebraic conditions* $a^{ij} = a^{ji}$ *and* $a^{ij} = a^{\alpha\beta} F^i_\alpha F^j_\beta$*.*

Then A_n *admits a holomorphically projective mapping onto a Kählerian space* $\bar{K}_n$*. The metric tensor* $\bar{g}_{ij}$ *of* $\bar{K}_n$ *and solutions of* (3) *are connected by the relations*

$$\text{(4)} \qquad \text{a) } a^{ij} = \mathrm{e}^{-2\psi(x)} \bar{g}^{ij}\,, \qquad \text{b) } \lambda^i = -a^{i\alpha} \partial_\alpha \psi(x)\,,$$

where $\bar{g}^{ij}$ *are components of inverse matrix of* $\|\bar{g}_{ij}\|$*.*

This theorem is a generalization of results in [2, 14, 23].

The question of existence of a solution of (3) leads to the study of integrability conditions and their differential prolongations. The general solution of (3) does not depend on more than $N_o = 1/4\,(n+1)^2$ parameters [15].

Let in an equiaffine space A_n the condition for Riemannian (curvature) tensor

$$\text{(5)} \qquad R^h_{ijk} = \delta^h_i \overset{1}{v}_{jk} + \delta^h_j \overset{2}{v}_{ik} - \delta^h_k \overset{2}{v}_{ij} + F^h_i \overset{3}{v}_{jk} + F^h_j \overset{4}{v}_{ik} - F^h_k \overset{4}{v}_{ij}$$

hold, where $\overset{\sigma}{v}$ are tensors.

Lemma 1. *If an equiaffine space* A_n *with the condition* (5) *admits a holomorphically projective mapping onto a Kählerian space* $\bar{K}_n$*, then* $\bar{K}_n$ *has constant holomorphic curvature.*

This space A_n is called a *holomorphically projective flat space*.

Proof. In [7, 12] an F-traceless decomposition of Riemannian tensor is studied. Formula (5) is this decomposition, in which F-traceless tensor vanishes.

The Riemannian tensor $\bar{R}^h_{ijk}$ of Kählerian space $\bar{K}_n$, onto which A_n is holomorphically projective mapped, satisfies an analogical form as (5). From [12], under this condition and the uniqueness of this decomposition one can show that a tensor of holomorphic projective curvature of $\bar{K}_n$ vanishes. This is a criterion for $\bar{K}_n$ to have constant holomorphic curvature, see [14, 23, 27]. □

3. Holomorphically projective mappings from semisymmetric equiaffine spaces

Hereafter we shall assume that in the equiaffine space A_n *the Ricci tensor will be preserved under the action of the structure* F, i.e.

$$Ric(FX, FY) = Ric(X, Y)\,.$$

We remind that the condition $\nabla F = 0$ implifies certain properties for the Riemannian tensor, for example:

$$F^h_\alpha R^\alpha_{ijk} = F^\alpha_i R^h_{\alpha jk}\,, \qquad F^h_\alpha F^\beta_i R^\alpha_{\beta jk} = -R^h_{ijk}\,.$$

These formulas naturally hold on Kählerian spaces.

The affine-connected spaces A_n are called *semisymmetric* if the condition $R\cdot R = 0$ holds, which, in coordinate notation, has the form $R^h_{ijk,[lm]} = 0$. According to the Ricci identity, this condition is written as follows

$$R^h_{\alpha jk} R^\alpha_{ilm} + R^h_{i\alpha k} R^\alpha_{jlm} + R^h_{ij\alpha} R^\alpha_{klm} - R^\alpha_{ijk} R^h_{\alpha lm} = 0\,.$$

Many investigations are devoted to the study of these spaces, see [1, 5, 6, 13, 14, 21, 23, 25].

Theorem 2 (M. Škodová, J. Mikeš, O. Pokorná [24])**.** *Let an equiaffine semisymmetric space A_n, where the Ricci tensor under the action of the structure F will be preserved, admit nontrivial holomorphically projective mappings onto a Kählerian space $\bar{K}_n$. If A_n is not a holomorphically projective flat space then the vector field λ^h from the equation* (3) *is convergent, i.e.* $\lambda^h_{,i} = \mathrm{const}\cdot\delta^h_i$ *is satisfied.*

Analogical results were proved by J. Mikeš for the geodesic mappings of semisymmetric Riemannian spaces and space with affine connection, see [10, 13, 23], and for holomorphically projective mappings of semisymmetric Kählerian spaces, see [14].

In the following we will study the holomorphically projective mapping A_n onto a Kählerian space $\bar{K}_n$, on the assumption that λ^h is a *concircular vector field* (in sense of K. Yano [26], see [13, 17, 23]), i.e. it holds

$$\lambda^h_{,i} = \varrho\,\delta^h_i\,, \tag{6}$$

where ϱ is a function.

The conditions of integrability (6) have the form: $\lambda^\alpha R^h_{\alpha jk} = \varrho_{,j}\delta^h_k - \varrho_{,k}\delta^h_j$. Because if in a space A_n $R^h_{i\alpha\beta}F^\alpha_j F^\beta_k = R^h_{ijk}$ holds then $\varrho = \mathrm{const}$, i.e. λ^h is *convergent*.

If we covariantly differentiate (4b) and (6) then we obtain the following formula

$$\psi_{ij} = \Delta\,\bar{g}_{ij}\,, \tag{7}$$

where Δ is a function and $\psi_{ij} = \psi_{i,j} - \psi_i\psi_j + \psi_\alpha\psi_\beta F^\alpha_i F^\beta_j$. We make sure by the analysis of the identity $\psi_\alpha R^\alpha_{ijk} = \psi_{i,jk} - \psi_{i,kj}$, that $\Delta \equiv \mathrm{const}$.

The formulas (6) and (7) are equivalent.

From equations (2) and (7) for the Riemannian and Ricci tensors of A_n and $\bar{K}_n$ this follows

$$\begin{aligned}\bar{R}^h_{ijk} &= R^h_{ijk} - \Delta(\delta^h_k\bar{g}_{ij} - \delta^h_j\bar{g}_{ik} + F^h_k F^\alpha_i\bar{g}_{\alpha j} - F^h_j F^\alpha_i\bar{g}_{\alpha k} + 2F^h_i F^\alpha_j\bar{g}_{\alpha k})\,,\\ \bar{R}_{ij} &= R_{ij} + (n+2)\Delta\bar{g}_{ij}\,.\end{aligned}$$

We can make sure that the integrability conditions of equations (3) and (6) and their prolongations have the following forms:

$$(8_0)\ \lambda^\alpha R^h_{\alpha jk} = 0,\ \dots,\ (8_m)\ \lambda^\alpha R^h_{\alpha jk,l_1\cdots l_m} = \varrho \overset{m}{T}{}^h_{jk\,l_1\cdots l_m}\,,$$
$$(9_0)\ a^{\alpha(i}R^{j)}_{\alpha kl} = 0,\ \dots,\ (9_m)\ a^{\alpha(i}R^{j)}_{\alpha kl,l_1\cdots l_m} = \lambda^{(i}\overset{m}{T}{}^{j)}_{kl\,l_1\cdots l_m} + \lambda^\alpha F^{(i}_\alpha F^{j)}_\beta\overset{m}{T}{}^\beta_{kl\,l_1\cdots l_m},$$

where the tensor $\overset{m}{T}$ is determined by the formulas

$$\overset{m}{T}{}^h_{jkl_1\cdots l_m} \overset{\text{def}}{=} -\sum_{s=1}^{m} R^h_{l_s jk,l_1\cdots l_{s-1}l_{s+1}\cdots l_m}\,.$$

4. Holomorphically projective mappings from generally recurrent equiaffine spaces onto Kählerian spaces

As it is known, *symmetric* and *recurrent* spaces A_n are characterized by differential conditions on the Riemannian tensor $R^h_{ijk,l} = 0$ and $R^h_{ijk,l} = \varphi_l R^h_{ijk}$, respectively, where $\varphi_l \neq 0$ is a covector.

Holomorphically projective mappings of symmetric and recurrent Kählerian spaces were studied by T. Sakaguchi [21], J. Mikeš and V. V. Domashev [2, 14, 23]. These results are generalized in the following theorem [24]:

Theorem 3. *Let an equiaffine symmetric* (*or semisymmetric recurrent*) *space A_n, where the Ricci tensor under the action of the structure F will be preserved, admit a nontrivial holomorphically projective mapping onto a Kählerian space $\bar{K}_n$. Then A_n is holomorphically projectively flat and the space $\bar{K}_n$ has constant holomorphic curvature.*

This theorem is possible to generalize for such A_n, which have more general recurrences of the Riemannian tensor.

Theorem 4. *Let A_n be an equiaffine space, where the Ricci tensor under the action of the structure F will be preserved, and one of these two conditions holds in A_n:*

$$R^h_{ijk,l} = R^h_{i\alpha\beta} S^{\alpha\beta}_{jkl}\,; \tag{10}$$

$$Q^{hi}_{(pr)jk,l} = Q^{\gamma\delta}_{(pr)\alpha\beta} S^{hi\alpha\beta}_{\gamma\delta jkl}|, \tag{11}$$

where S are certain tensors and

$$Q^{hi}_{prjk} \overset{\text{def}}{=} \delta^h_p R^i_{rjk} + \delta^i_p R^h_{rjk} + F^h_p F^i_\alpha R^\alpha_{rjk} + F^i_p F^h_\alpha R^\alpha_{rjk}\,.$$

If A_n admits a nontrivial holomorphically projective mapping onto a Kählerian space $\bar{K}_n$ and the condition (6) *holds then A_n is flat, and the space $\bar{K}_n$ has constant holomorphic curvature.*

Proof. Let A_n admit a nontrivial holomorphically projective mapping onto a Kählerian space $\bar{K}_n$ and assume that condition (6) holds. Hence the conditions (9) hold.

And it is easy to see that the conditions (11) follow from (10). We contract (11) with a^{pr}. According to formulas (9_0) and (9_1) we obtain

$$\lambda^{(h} R^{i)}_{ljk} + \lambda^\alpha F^{(h}_\alpha F^{i)}_\beta R^{\ \beta}_{ljk} = 0\,.$$

From these formulas on $\lambda^h \neq 0$ it follows that $R^h_{ijk} = 0$, i.e. A_n is flat, and according to Lemma 1 a space $\bar{K}_n$ has constant holomorphic curvature. □

5. Holomorphically projective mappings from m-recurrent equiaffine spaces onto Kählerian spaces

We mention the next definitions (V. R. Kaygorodov [5, 6]):

The space A_n is called an *m-recurrent space* (K_n^m) if

(12) $$R^h_{ijk,l_1\cdots l_m} = \Omega_{l_1\cdots l_m} R^h_{ijk}\,,$$

where Ω is a nonvanishing tensor.

All (pseudo-) Riemannian m-recurrent spaces K_n^m are semisymmetric [5, 6].

In the sequel we shall need the following Lemmas.

Lemma 2. *Let*

(13) $$A_{l_1}\omega_{l_2l_3\cdots l_m} + A_{l_2}\omega_{l_1l_3\cdots l_m} + \cdots + A_{l_m}\omega_{l_1l_2\cdots l_{m-1}} = 0,$$

hold for a covector A and a tensor ω. If the tensor ω is nonvanishing then $A = 0$.

Proof. Let formulas (13) hold and let the tensor ω be not vanishing.

We contract the tensor $\omega_{l_2l_3\cdots l_m}$ with a vector b^{l_m}, for which

$$\underset{2}{\omega}_{l_2l_3\cdots l_{m-1}} \overset{\text{def}}{=} b^\alpha \omega_{l_2l_3\cdots l_{m-1}\alpha} \neq 0\,,$$

holds. From (13) we obtain

$$A_{l_1}\underset{2}{\omega}_{l_2l_3\cdots l_{m-1}} + A_{l_2}\underset{2}{\omega}_{l_1l_3\cdots l_{m-1}} + \cdots + A_{l_{m-1}}\underset{2}{\omega}_{l_1l_2\cdots l_{m-2}}$$

(14) $$+\, b^\alpha A_\alpha\, \omega_{l_1l_2\cdots l_{m-1}} = 0\,.$$

Hence it follows that

$$b^\alpha A_\alpha = 0\,.$$

If $b^\alpha A_\alpha \neq 0$ held we would obtaine after the contraction (14) with $b^{l_{m-1}}$

$$A_{l_1}\underset{3}{\omega}_{l_2l_3\cdots l_{m-2}} + A_{l_2}\underset{3}{\omega}_{l_1l_3\cdots l_{m-2}} + \cdots + A_{l_{m-2}}\underset{3}{\omega}_{l_1l_2\cdots l_{m-3}}$$
$$+\, 2b^\alpha A_\alpha\, \underset{2}{\omega}_{l_1l_2\cdots l_{m-2}} = 0\,,$$

where

$$\underset{3}{\omega}_{l_2l_3\cdots l_{m-2}} \overset{\text{def}}{=} b^{l_{m-1}}\,\underset{2}{\omega}_{l_2l_3\cdots l_{m-1}}\,.$$

Because $\underset{2}{\omega}_{l_1l_2\cdots l_{m-2}} \neq 0$ and $b^\alpha A_\alpha \neq 0$ then $\underset{3}{\omega}_{l_1l_2\cdots l_{m-3}} \neq 0$.

We continue step-by-step, at last we obtain $A_{l_1}\underset{m}{\omega} + (m-1)\underset{m-1}{\omega}_{l_1} b^\alpha A_\alpha = 0$, where $\underset{m-1}{\omega} \neq 0$. We contract the last term with b^{l_1} and we can see that $m\,\underset{m}{\omega}\, b^\alpha A_\alpha = 0$, i.e. $b^\alpha A_\alpha = 0$ (because $\underset{m}{\omega} \neq 0$), i.e. it is a contradiction. Hence $b^\alpha A_\alpha = 0$.

Thus the formula (14) takes the form

$$A_{l_1}\underset{2}{\omega}_{l_2l_3\cdots l_{m-1}} + A_{l_2}\underset{2}{\omega}_{l_1l_3\cdots l_{m-1}} + \cdots + A_{l_{m-1}}\underset{2}{\omega}_{l_1l_2\cdots l_{m-2}} = 0\,,$$

where $\underset{2}{\omega} \neq 0$. The obtained formulas are of the form (14), with a difference, that the tensor $\underset{2}{\omega}$ has a valence lower by one than the tensor ω. We continue step by step until it is

$$A_{l_1}\tilde{\omega}_{l_2} + A_{l_2}\tilde{\omega}_{l_1} = 0\,,$$

where $\tilde{\omega}_l$ is a nonvanishing tensor. Hence it follows that $A_i = 0$. □

Lemma 3. *Let*

(15) $$R^h_{l_1jk}\omega_{l_2l_3\cdots l_m} + R^h_{l_2jk}\omega_{l_1l_3\cdots l_m} + \cdots + R^h_{l_mjk}\omega_{l_1l_2\cdots l_{m-1}} = 0\,,$$

hold for the Riemannian tensor of A_n. If the tensor ω is nonvanishing then A_n is flat.

Proof. Let A_n be non-flat then $R^h_{ijk} \neq 0$. Therefore a tensor ε^{jk}_h exists such that $A_i = \varepsilon^{jk}_h R^h_{ijk} \neq 0$. We contract (15) with ε^{jk}_h and obtain the formula (13) and Lemma 3 holds according to Lemma 2. □

The following holds

Theorem 5. *Let an equiaffine m-recurrent space K^m_n, where the Ricci tensor under the action of the structure F will be preserved, admit a nontrivial holomorphically projective mapping onto a Kählerian space $\bar{K}_n$ and the condition* (6) *holds.*

Then K^m_n is flat and the space $\bar{K}_n$ has constant holomorphic curvature.

Proof. Let the space K^m_n admit a nontrivial holomorphically projective mapping onto a Kählerian space $\bar{K}_n$ and assume that condition (6) holds. Contracting (12) with λ^i and using (8_0) and (8_m) we obtain

$$\varrho \overset{m}{T}{}^h_{kl\,l_1\cdots l_m} = 0\,.$$

We assume that $\varrho \neq 0$. Then $\overset{m}{T}{}^h_{kl\,l_1\cdots l_m} = 0$. We covariantly differentiate along x^l apply to (12) and obtain these formulas

(16) $$R^h_{l_1jk}\Omega_{l_2l_3\cdots l_ml} + R^h_{l_2jk}\Omega_{l_1l_3\cdots l_ml} + \cdots + R^h_{l_mjk}\Omega_{l_1l_2\cdots l_{m-1}l} = 0\,.$$

Because the tensor $\Omega \neq 0$ the vector ε^l exists such that

(17) $$\omega_{l_2l_3\cdots l_m} = \varepsilon^l\Omega_{l_2l_3\cdots l_ml} \neq 0\,.$$

Contracting (16) with ε^l we obtain the formula (15). Because A_n is not flat ($R^h_{ijk} \neq 0$), from Lemma 3 it follows that $\varrho = 0$, thus the vector λ^i is covariantly constant, i.e. $\lambda^h_{,i} = 0$.

Let us contract (12) with a^{ir} and after it let us alternate the indices r a h. After application of (9_0) and (9_m) we can obtain

(18) $$\lambda^{(i} \overset{m}{T}{}^{j)}_{kl\,l_1\cdots l_m} + \lambda^\alpha F^{(i}_\alpha F^{j)}_\beta \overset{m}{T}{}^\beta_{kl\,l_1\cdots l_m} = 0\,.$$

We covariantly differentiate (18) along x^l. Because F^h_i and λ^h are covariantly constant, after an application (12) we obtain

(19) $$A^{hi}_{l_1jk}\Omega_{l_2l_3\cdots l_ml} + A^{hi}_{l_2jk}\Omega_{l_1l_3\cdots l_ml} + \cdots + A^{hi}_{l_mjk}\Omega_{l_1l_2\cdots l_{m-1}l} = 0\,,$$

where

(20) $$A^{hi}_{ljk} \overset{\text{def}}{=} \lambda^{(h}R^{i)}_{ljk} + \lambda^\alpha F^{(h}_\alpha F^{i)}_\beta R^\beta_{ljk}\,.$$

If $A^{hi}_{ljk} \neq 0$ then exist a tensor ε^{jk}_{hi} such that $A_l = \varepsilon^{jk}_{hi}A^{hi}_{ljk} \neq 0$. Contracting (19) with ε^{jk}_{hi} and ε^l from (17) we obtain the term (13). $A_l = 0$ holds according to

Lemma 2, which is a contradiction. Thus $A^{hi}_{ljk} = 0$ it means with respect to (20) that

$$\lambda^{(h} R^{i)}_{ljk} + \lambda^{\alpha} F^{(h}_{\alpha} F^{i)}_{\beta} R^{\beta}_{ljk} = 0\,.$$

Hence it clearly follows from $\lambda^h \neq 0$ that $R^h_{ijk} = 0$, i.e. the space K^m_n is flat and according to Lemma 1 the space $\bar{K}_n$ has constant holomorphic curvature. □

According to Theorems 3, 4 and 5 it follows that generalized recurrent spaces A_n from these Theorems, which are not flat, do not admit the mentioned nontrivial holomorphically projective mappings onto Kählerian spaces.

These results are a generalization of results by T. Sakaguchi, J. Mikeš and V. V. Domashev, which were done for holomorphically projective mappings of symmetric, recurrent and semisymmetric Kählerian spaces [2, 14, 16, 21].

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