Abstracts of Ph.D. theses in mathematics

Commentationes Mathematicae Universitatis Carolinae, Vol. 47 (2006), No. 1, 175--187

Persistent URL: http://dml.cz/dmlcz/119584

ABSTRACTS OF Ph.D. THESES IN MATHEMATICS

defended recently at Charles University, Prague

TEACHING PROBABILITY AT SECONDARY SCHOOLS USING COMPUTERS

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(March 7, 2005; supervisor J. Antoch)

This dissertation thesis is an attempt to combine teaching probability with elements of algorithmization and computer programming. The main idea is rather simple. Through modification and execution of simulation algorithms students perform random experiments such as rolling a die, tossing a coin, spinning a roulette wheel or taking a card from a deck. They watch the results and try to answer the questions concerning various random events. With this method the students can quickly acquire the ability to handle probabilistic problems of daily life or scientific research. And the students understand what they are doing, with little danger of formula-juggling which often afflicts conventional teaching methods.

Computer simulation method is not offered as a successor to conventional teaching methods. Rather, it can enrich traditional education, it can give the students enough experience and motivation to study probability theory more deeply.

Creating or modifying the simulation algorithms are great means to familiarize the students with essential programming principles. However, careful choice of suitable programming language and development environment is very important. The author chose computer algebra system MuPAD (`http://www.mupad.de/`) especially for its simple Pascal-like programming language syntax, quality graphical output and congenial help tool. MuPAD Help Tool consists of DVI (TeX output format) file viewer with hypertext functionality and direct linking to MuPAD. Since the thesis is written in TeX, there also exists its electronic version. When studying it, the students can execute all included algorithms simply by a mouse-click.

LIFE AND WORK OF MILOŠ KÖSSLER

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(March 7, 2005; supervisor M. Rokyta)

Professor Miloš Kössler (19.6.1884–8.2.1961) was a Czech mathematician whose active work could be situated in the first half of the 20th century. He was not only

a brilliant specialist in the theory of analytic functions and the number theory, but first of all he was an excellent teacher. As a person, he was very modest and tolerant man, always ready to help anyone. It would not be very precise to say that he achieved grandiose and definite results in his treatises every time, as his importance consisted mainly in his educational mastership. The aim of the thesis was to give a detailed summary of his life and his scientific activities.

The thesis is divided into seven chapters. In the first chapter, we present Kössler's personality, description of his career and comments on the situation at the university (especially concerning teaching of mathematics) during the first half of the 20th century. In the next three chapters we discuss the main results obtained by Kössler together with some historical remarks for clarifying the background of development of problems he was interested in. The fifth chapter is connected to Kössler's textbooks written for his students at the university. We also present some interesting ideas of Kössler from his "diaries", unpublished till this time, in the sixth chapter. At the end of the thesis, we give a full review of Kössler's work — his scientific publications together with their references in databases Mathematical Reviews, Zentralblatt für Mathematik und ihre Grenzgebiete and Jahrbuch über die Fortschritte der Mathematik, the list of his pedagogical activities at the university and a survey of doctoral thesis that he reviewed.

INTEGRODIFFERENTIAL EQUATIONS IN BANACH SPACES

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(March 24, 2005; supervisor J. Milota)

This thesis deals with various problems in linear integrodifferential equations in Chapters 1, 2 and 3 and with nonlinear integrodifferential equations in Chapter 4.

The first chapter studies parabolic Volterra integrodifferential equations with perturbed scalar kernels. In particular, various types of stability results are obtained and applied to Miller's model of heat conduction in materials with memory.

Chapter 2 is devoted to comparison of two different approaches to integrodifferential equations: via semigroups and via the Laplace transform. This chapter contains comparison of the abstract results obtained by the two methods, as well as their application to a simple viscoelastic model. The conclusion is that uniform exponential stability can be obtained by either of the two methods but only the Laplace transform approach allows to obtain some weaker forms of stability if the kernel does not tend to zero exponentially.

In Chapter 3 analyticity of the solutions and maximal L^p-regularity are investigated. These are the first results of this type obtained via semigroups. We work with spaces of holomorphic functions instead of usual L^p. The left translation semigroups on the used spaces are analytic, thus we overcome the main difficulty.

The last chapter is devoted to nonlinear equations. Kato's results on abstract quasilinear problems are extended to a larger class of right-hand side functions de-

pending not only on the present value of the unknown function u but on its whole history. This allows the results to be applied to quasilinear integrodifferential equations with convolution kernels of special type. The abstract results on local and global existence are finally applied to MacCamy's model of heat conduction in materials with memory.

ASYMPTOTIC BEHAVIOR OF THE REGULAR ORBITS OF STRONGLY CONTINUOUS SEMIGROUP

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In Hilbert spaces, many asymptotic properties of strongly continuous semigroups of operators can be characterized in terms of the resolvent properties of the generator. This is well known Gearhart-Prüss theorem. In particular, the semigroup in Hilbert space is exponentially stable if and only if the resolvent of the generator is bounded in the right complex half-plane. In Banach spaces, however, the situation is much more complicated since Gearhart-Prüss theorem does not remain valid.

In our thesis, we show that some Hilbert space results can be reformulated in Banach spaces assuming additional regularity of the orbits. The thesis consists of two parts. In the first part, we discuss the spectral properties of the semigroup $(T(t))_{t\geq 0}$. We introduce a new notion of Y-spectrum $\sigma_Y(T(t))$ and we show that this spectrum is connected with the growth of the orbits $T(.)x$ where $x \in Y$. Further, we prove that Banach space version of Gearhart-Prüss spectral theorem holds for the Y-spectrum if Y is an *admissible interpolation space* between the state space X and the generators domain $D(A)$.

In the second part, we discuss the dichotomy of the semigroup $(T(t))_{t\geq 0}$. It is shown that the boundedness of the generators resolvent along the imaginary axis implies the dichotomy of the orbits from the admissible interpolation space. This is on one hand an analogy of Prüss's characterization of dichotomy in Hilbert spaces and on the other hand a generalization of deLaubenfels-Latushkin theorem where much larger regularity of the orbits is needed. As a technical tool, we develop the theory of the unbounded $\mathcal{H}^\infty$-functional calculus on the vertical strip. These results may be of an independent interest.

ON CONVEX FUNCTIONS, D.C. FUNCTIONS AND BOUNDARY STRUCTURE OF CONVEX SETS

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(April 7, 2005; supervisor L. Zajíček)

The thesis consists of five chapters each concerning convex or delta convex (d.c.)

function. The first chapter completes the result of Zajíček by giving a precise characterization of sets of points of non-differentiability of convex functions on $\mathbb{R}^n$. We show that each F_σ-subset of countable union of d.c. hypersurfaces is the set of points of non-differentiability for some convex function. In the second chapter it is proved that there exists a d.c. function on $\mathbb{R}^2$ of class C^1 which cannot be represented as difference of convex functions of class C^1. There is also given a simple example of d.c. function on $\mathbb{R}^2$ differentiable everywhere but not strictly differentiable at the origin.

The third chapter is concerned with a generalization of the theorem of Landis which says that for a d.c. function f on $\mathbb{R}^2$, the image of the set $\{f' = 0\}$ has zero Lebesgue measure. Using the method of Landis and recent results of Ambrosio, Caselles, Masnou and Morel on sets of finite perimeter we proved that, for a Lipschitz function $f\colon \mathbb{R}^2 \to \mathbb{R}^k$ having first partial derivatives in $BV_{\mathrm{loc}}(\mathbb{R}^2)$, the image of the set $\{f' = 0\}$ has zero 1-dimensional Hausdorff measure. Moreover, we generalize Landis' result to the case of a d.c. mapping $f : \mathbb{R}^2 \to X$, where X is a Banach space. Also some results on Lipschitz BV_2 functions on $\mathbb{R}^n$ are proved.

In the fourth chapter we prove that the set of directions of $(n-2)$-dimensional balls which are contained in the boundary ∂K of a convex body $K \subset \mathbb{R}^n$ but in no $(n-1)$-dimensional convex subset of ∂K is σ-1-rectifiable. We also show that there exists a close connection between smallness of the set of directions of line segments on ∂K and smallness of the set of tangent hyperplanes to the graph of a d.c. (delta-convex) function on $\mathbb{R}^{n-2}$. Using this connection, we construct $K \subset \mathbb{R}^3$ such that the set of directions of segments on ∂K cannot be covered by countably many simple Jordan arcs having half-tangents at all points. Also new results on directions of r-dimensional balls in ∂K parallel to a fixed linear subspace are proved. The fifth chapter improved the result of fourth chapter on the covering directions of segments on ∂K for $K \subset \mathbb{R}^3$.

DUALITY IN MULTISTAGE STOCHASTIC PROGRAMMING AND ITS APPLICATION TO ARBITRAGE THEORY

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(May 10, 2005; supervisor P. Lachout)

The theory of duality brings a better understanding of optimization problems behaviour. It can propose necessary and sufficient conditions for optimality, or it can at least give bounds for the optimal value. It also leads to algorithmic approaches to find the optimal solution. The optimal solutions to dual problem propose stability results for the optimal value and can have, in some cases, an interesting import.

The fundamental theorem of asset pricing states that a financial market admits no arbitrage opportunity if and only if there is an equivalent probability measure

for which the price process is a martingale. The equivalence of no arbitrage with the existence of an equivalent martingale measure is at the basis of the entire theory of "pricing by arbitrage". The aim of the theory of arbitrage-free pricing is to assess, in arbitrage-free market, each contingent claim with an initial fair price such that the market model augmented by this contingent claim as a new possible investment still admits no risk-free profit. The proof of these pricing results requires a separation theorem.

The goal of this thesis is to relate the arbitrage theory to the theory of duality in optimization. The thesis proceeds as follows.

In the first chapter we summarize without proofs the relevant material on the theory of duality in optimization. In the second chapter we proceed with the study of the duality theory for multistage stochastic programming. First we compound the competitive ways of modeling the multistage information structure. Then the abstract multistage program is formulated and some popular forms of the cost and the constraint functions are described. Finally we exhibit the duality results for various multistage programs.

In the third chapter we indicate how the theory of conjugate duality may be used to derive results of arbitrage theory and the technique of arbitrage-free pricing of contingent claims. First we consider a single-period model of a financial market on a finite sample space. We show the classical results of arbitrage-free pricing in this model as a consequence of the duality in linear programming. Then we proceed with the analysis of a multi-period market model on a general sample space. Extending the pricing results to this general model requires some sort of separation theorem for infinite-dimensional spaces. We follow the recently-made attempt of King and Korf (2001) to analyze the involved pricing problems in an infinite-dimensional multistage stochastic programming setting from the perspective of conjugate duality. Nevertheless, the work of King and Korf had serious gaps in the proof of the fundamental theorem of asset pricing and consequently it is possible to improve the pricing results stated there.

STOCHASTIC PROGRAMMING APPROACH TO ASSET-LIABILITY MANAGEMENT

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(May 10, 2005; supervisor J. Dupačová)

We have elaborated on ALM problems for pension funds. Main focus is on scenario-based stochastic programming methodology to ALM. Important role in scenario based stochastic programs plays scenario generation. Relatively low level of information available to us implies need for appropriate one. Here we employed a method based on moment fitting extended to include no-arbitrage conditions. Scenario generation model for liabilities of a pension fund working under defined

contribution setting is developed.

A single currency multi-stage stochastic problem is delineated and several investigation topics covering sensitivity to underlying economic factors and parameter distribution inputs are considered. We give theoretical background for out-of-sample stress-testing of the optimal objective of a pension fund or its final wealth at the end of a planning horizon.

Important aspect of scenario generation via moment fitting is stability of stochastic programs with respect to estimation errors of moments of underlying stochastic factors. We laid down theory for two-stage stochastic programs. Asymptotic distribution of the optimal value and the unique optimal solution is derived. Inference about impact of estimation errors of underlying stochastic factors on the unique optimal solution is possible.

SOME PROBLEMS IN ROTUND RENORMINGS OF BANACH SPACES AND IN OPERATOR THEORY

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The thesis consists of four chapters. The first two of them investigate the connection between rotund renormings and topological properties of compact sets. The main result of the first chapter characterizes Banach spaces with unconditional Schauder bases whose dual admits an equivalent URED norm as those whose dual unit ball is uniformly Eberlein compact in weak* topology.

The second chapter discusses renorming of $C(K)$ spaces. It is shown that a property "having a dual URED norm" is a three-space-like property; and that $C(K)$ spaces have this property for a large class of compacts K.

The third chapter deals with invariant subspaces. A Banach space X is called simple if X is the only nontrivial invariant subspace of X^{**} with respect to biconjugates of all bounded linear operators on X. Two core results are that ℓ_1 is not simple and that c_0 is simple. It follows that if X has an unconditional Schauder basis, then X is simple if and only if $c_0 \subset X$, if and only if $\ell_1 \not\subset X$. Moreover, $C(K)$ is simple if and only if K is one point compactification of natural numbers.

The fourth chapter investigates the connection between rotund renormings and nonlinear analysis. The main result is that the James tree space can be renormed to be Lipschitz separated. It presents a negative answer to the problem of Borwein, Giles and Vanderwerff whether every Lipschitz separated Banach space is an Asplund space.

WEAK PIGEONEHOLE PRINCIPLE AND RANDOMIZED COMPUTATION

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We study the extension of the theory S^1_2 by instances of the dual (onto) weak pigeonhole principle for p-time functions, $dWPHP(PV)^x_{x^2}$. We propose a natural framework for formalization of randomized algorithms in bounded arithmetic, and use it to provide a strengthening of Wilkie's witnessing theorem for $S^1_2 + dWPHP(PV)$.

Then we show that $dWPHP(PV)$ is (over S^1_2) equivalent to a statement asserting the existence of a family of Boolean functions with exponential circuit complexity. Building on this result, we formalize the Nisan-Wigderson construction (conditional derandomization of probabilistic p-time algorithms) in a conservative extension of $S^1_2 + dWPHP(PV)$. We also develop in S^1_2 the algebraic machinery needed for implicit list-decoding of Reed-Muller error-correcting codes (including some results on a modification of Soltys' theory $\forall LAP$), and use it to formalize the Impagliazzo-Wigderson strengthening of the Nisan-Wigderson theorem.

We construct a propositional proof system WF (based on a reformulation of Extended Frege in terms of Boolean circuits), which captures the $\forall\Pi^b_1$-consequences of $S^1_2 + dWPHP(PV)$. As an application, we show that WF and G_2 p-simulate the Unstructured Extended Nullstellensatz proof system.

We also consider two theories which have explicit counting facilities in their language. The first one is the Impagliazzo-Kapron logic; we propose a modification of the theory, and prove a generalization of the Impagliazzo-Kapron soundness theorem to $\forall\exists$-consequences of the theory. The second one is a feasible theory of approximate counting, formulated in a variant of Kleene's 3-valued logic. We introduce the theory, and prove a witnessing theorem for its existential consequences.

GEOMETRY AS CREATION

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(June 28, 2005; supervisor F. Kuřina)

The presented dissertation is the result of searching possible attitudes towards geometry, especially in stereometry and mapping. The work is divided into three main chapters: Mapping and children's drawings, Perspective and general education, Geometry as creation.

Over a series of works it was documented, that living through sensation of space leads to intuitive assumption of elements, not only front viewed and plan viewed (parallel) projection, but also with stress to central projection. The child is spontaneously arriving at the same principles as those of the study of projection methods in descriptive geometry.

In the second chapter it was implemented a geometric reconstruction of *Dürers Melancholy* into linear single-point vanishing perspective. It was found out possi-

ble construction mistakes in the projections of spheres, cuboctaedrs and ladders on copperplate using stereometrical analysis.

The entire third chapter is dedicated to original work in the fields of: geometric creation for children, constructive attitudes in education for future teachers of maths and primary pedagogy, creation itself as related to professional interests. In the presented dissertation there is also pointed out the problem related to the volume conservation of flexible polyhedrons in my definition of polyhedron. It was also described original methodology of work with teaching aids, created-folding models for teaching the techniques of projection methods. A special chapter is dedicated to optical illusions and anamorphosis. The third part of the chapter is dedicated to three geometric topics and their solutions: There was created paper model of Klein's bottle, there is prove that the so-called *impossible stairs* exist and there was also created models for these and there is constructed model of space illusion inside a cylindrical surface with a center of projection which is not on the cylinder's axis.

COMPARATIVE ANALYSIS OF GEOMETRIC SOFTWARE PACKAGES BASED ON SOLVING SELECTED PROBLEMS

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The thesis consists of five chapters: "Elementary constructions", "Solved problems", "Groebner bases", "Circle tangent to three objects" and "On the history of computer geometry".

In the first chapter, the differences between the elementary constructions in classical geometry and the ones generated using a particular software are discussed.

The second chapter represents the core of the thesis. In detail, the methods of solution of particular geometrical problems are analyzed; this analysis is based on that each problem is solved independently using different approaches and the results are compared and discussed. In all the problems, surfaces are to be constructed from given objects. Each problem is solved in three different ways. The synthetic solution is represented by two approaches: classical solution in Monge's projection (the final drawing is produced by AutoCAD) and modern solution by the software Rhinoceros. Finally, the analytic solution is realized with the help of software Maple.

Groebner bases, which the third chapter is devoted to, are a tool for finding the solution to a set of polynomial equations. In addition to Groebner bases, other methods how to solve these sets are known, let us remind for example the methods of resultants or eigenvalues of matrices. Polynomial equations appear when solving spatial problems in an analytic way. In Chapter 3, first of all the

theoretical basis of the method is reviewed briefly. Then the library "Groebner" included in the software Maple is mentioned. Finally, the method is demonstrated on solving particular problems.

In Chapter 4, the problem of finding a circle (in space) tangent to three given objects is solved. The software Rhinoceros is able to solve this problem by a single command *Curve/Circle/Tangent to 3 Curves*. On the contrary, the classical solutio is by far not trivial.

The last chapter deals with relatively short histories of CAD (Computer Aided Design) systems and of curves and surfaces important for geometric modelling. Curves most often used in computer geometry are defined there.

NUMERICAL METHODS FOR SOLVING COMPRESSIBLE FLOW PROBLEMS

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(August 30, 2005; supervisor M. Feistauer)

Investigation of flow and problems connected with this topic involves many present fields of science and technology. As an example we can bring up aviation, automobile industry, turbine development. Other fields are biology, meteorology, oceanography, medicine and environmental protection. From this short list we can see, that study of flow involves not only theoretical fields, but we can use our theoretical results in practical life.

Now directly to the contents of this dissertation. We concern with numerical solution of nonlinear convection-diffusion problems and compressible flow. Our goal is to derive, describe and test suitable numerical methods for solving compressible flow problems. In the first part of thesis we describe basic equations and mathematical notation.

Second chapter describes the finite volume method applied to the solution of the Euler equations. We mention there some finite volume meshes, a semiimplicit scheme of the finite volume method and an explicit scheme of the finite volume method with the proof of stability of the scheme for a scalar linear equation and the Euler equations. Numerical solution of Euler equations is described in many articles and monographs. Let us mentions some of them. Famous books were written by E. Godlewski and P. Raviart, D. Kröner, R.J. Le Veque, E.F. Toro or M. Feistauer.

Third chapter is concerned with the combined finite element-finite volume method for the solution of a scalar linear equation and the Navier-Stokes equations. Very interesting part is the theoretical analysis of error estimates for triangular finite elements combined with triangular finite volumes, which was firstly published in this dissertation and it is one of the main assets. The combined finite element-finite volume method takes advantages of both approaches. The

finite element method is a modern method for solving partial differential equations. It comes out from variational formulation and uses polynomial functions for approximation. Extensive theoretical description of this method can be found in works by I. Babuška, T. Strouboulis, S. Brenner, or M. Křížek and P. Neittaanmäki. We are going to deal with the Navier-Stokes equations. The finite volume method represents a high-performance method for solving inviscid compressible flow problems. Idea of the combined finite element-finite volume method was published in the work by M. Feistauer, J. Felcman and M. Lukáčová. Basic idea is to approximate inviscid terms by the finite volume method and viscous terms by finite element method.

In the last chapter, the discontinuous Galerkin method is described. This method is applied to a scalar equation in 1D and 2D and also the Navier-Stokes equations, including numerical examples. There is a need to construct schemes over one mesh only and using advantages of the FV and FE methods. Such technique is the *discontinuous Galerkin finite element* (DG FE) *method* representing a generalization of the FV approach and allowing naturally high order of accuracy. It is based on the idea to approximate the solution of an initial-boundary value problem by piecewise polynomial functions over a FE mesh, without any requirement on interelement continuity. The original DG FE method was introduced by Reed and Hill for the solution of the neutron transport equation. The first analysis of this method was made by Le Saint and Raviart, later an improvement was achieved by Johnson and Pitkäranta. The DG FE method was applied to nonlinear conservation laws already in 1989 by Cockburn and Shu. Their approach uses advantages of the FE method and FV schemes with an approximate Riemann solver. During several recent years the DG FE schemes have been extensively developed. An important question is the discretization of diffusion terms in the framework of the DG FE methods. There exist various treatments of this problem. One possibility is to apply a mixed formulation. Its disadvantage is a large number of unknowns. Another method is a direct discretization.

SYSTEMS OF ALGEBRAIC EQUATIONS AND THEIR SOLUTION IN ANTIQUITY AND THE MIDDLE AGES

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The dissertation thesis is devoted to problems we would write today as a system of polynomial equations such that one or more of them is nonlinear. Included are also systems of equations containing radicals or expressions involving unknowns in the denominator, since they can be easily converted to systems of algebraic equations. In particular methods of solutions of specified problems are studied throughout the thesis. Solution procedures are showed by many examples taken from original works. More than one hundred problems is presented together with

some commentary (analysis of solution, presumable explanation of procedures used by original author or in the original work etc.).

The work is divided to four parts. Besides the first introductory and last closing section it involves passages entitled *Systems in Antiquity* and *Systems in the Middle Ages*. The former deals with problems contained in Egyptian mathematical papyri, Mesopotamian cuneiform texts, Euclid's *Data*, Heron's *Geometry*, Diophantus's *Arithmetics*, and Chinese *Nine Chapters on the Mathematical Art*. The latter is concerned with systems in Brahmagupta's *Brahma Correct System*, Mahāvīra's *Compendium of Calculation*, Bhāscara's *Head Jewel of Accuracy*, al-Chwārizmī's *The Condensed Book on the Calculation of al-Jabr wa al-Muqābala*, Abū Kāmil's *Book on Algebra* and *Book of Rare Things in the Art of Calculation*, al-Karajī's *al-Fakhrī* and *al-Badī*, Qin Jiushao's *Mathematical Treaties in Nine Sections*, Li Yeh's *Sea Mirror of Circle Measurements* and *New Steps in Computation*, Yang Hui's *Yang Hui suanfa*, and Zhu Shije's *Jade Mirror of the Four Elements*.

DECOMPOSITIONS OF MODULES

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This work contains several results on direct sum decompositions of serial modules. Recall that a module is said to be *uniserial* if the lattice of its submodules is a chain. Serial modules are (possibly infinite) direct sums of uniserial modules. The work was motivated by several results of Facchini, Dung and Puninski achieved in late 90's.

After a short introduction we investigate serial modules of finite Goldie dimension, that is finite direct sums of uniserial modules. We prove that any direct summand of such a module is serial. This result together with Facchini's weak Krull-Schmidt theorem give a description of all direct summands of an arbitrary serial module of finite Goldie dimension.

The rest of the work brings several results on infinite direct sums of uniserial modules. We prove that for any uniserial modules U, V and for any non-zero cardinal κ, $U^{(\kappa)} \simeq V^{(\kappa)}$ implies $U \simeq V$ (so called κ-th root property). Further we prove that the class of direct sums of so called non quasi-small uniserial modules is closed under direct summands, we describe objects in $\mathrm{Add}(U)$, where U is an arbitrary uniserial module, and extending results of Dung and Facchini we give a criterion when two families of non-zero uniserial modules have isomorphic direct sums (so called infinite version of weak Krull-Schmidt theorem). The final part of the work brings a module theoretic view on Puninski's results — the example of a uniserial module that is not quasi-small and an example of a serial module containing a non-serial direct summand.

VALUATION OF LIFE INSURANCE USING DIFFUSION MODEL OF INTEREST RATE

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(September 19, 2005; supervisor P. Mandl)

The thesis presents the methodology how to use diffusion interest rate models, e.g. Hull-White or Vasicek model for discounting cash flows. The methodology is based on Markovian character of diffusion processes and is generally usable in actuarial computations. We do not use the assumption that investment yields in particular accounting periods are non-correlated, which is common in approximative methods that can be found in literature. Described method is applied to liability valuation and liability adequacy testing for an endowment contract with profit sharing and interest rate guarantee. The thesis is concerned also with other topical issues as profit, release from risk and items of income statement of Life Insurance Company.

EQUILIBRIUM BEHAVIOUR OF ZERO RANGE PROCESSES ON BINARY TREE

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(September 20, 2005; supervisor M. Janžura)

We focus on describing an interacting particle system in the case when the set of sites, on which the particles move, has a structure different from the usually considered set $\mathbb{Z}^d$. We have chosen the tree structure with the dynamics that leads to one of the classical particle systems, called the zero range process. The reason for this choice was given by a motivation from queueing systems and networks, since the zero range process corresponds to an infinite system of queues and the arrangement of servers in the tree structure is natural in a number of situations.

The main result of this work is a characterisation of invariant measures for some important cases of site-disordered zero range processes on a binary tree. Namely, the case when the single particle law is a simple random walk on a binary tree, and the case of arbitrary totally asymmetric single particle law on a binary tree. Another result is connected with the speed of convergence to equilibrium for the first-mentioned zero range process.

The work follows up the known results of [1], [2] and [3] for the zero range process on $\mathbb{Z}^d$.

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RANDOM COEFFICIENT MOVING AVERAGE MODELS

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(September 20, 2005; supervisor J. Anděl)

A linear moving average model with random coefficients (RCMA) is proposed as more general alternative to usual linear MA models. This class of models includes many well known time series models as well as models which have not been investigated yet. The basic properties of this model are obtained. Although some model properties are similar to linear case the RCMA model class is too general to find general invertibility and stationarity conditions. Some sufficient conditions are obtained and the invertibility of some special examples of RCMA(1) model are investigated.

The estimation of model order is also investigated. The method based on the sample autocovariance function and the method based on the sample distribution function are adopted. The method based on Spearman correlation coefficient is proposed and comparison of all used methods is given.

We also deal with parameter estimation. There are used some moment methods and maximum likelihood method. Two modifications of maximum likelihood method are proposed and the review of asymptotic theory is also presented. These methods are demonstrated in the case of linear MA(1) process $X_t = \varepsilon_t + \alpha\varepsilon_{t-1}$ and the case of simple non-linear moving average process $X_t = \varepsilon_t + \alpha\varepsilon_{t-1} + \beta\varepsilon_t\varepsilon_{t-1}$. Particularly the non-linear model shows some theoretical and computational problems we meet with during parameter estimation procedure. The simulation study of asymptotic properties of investigated estimates is presented.