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Katedra matematické analýzy a numerické matematiky přírodovědecké fakulty University Palackého v Olomouci
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REMARK TO THE PAPER "ON SOME LIMIT PROPERTIES OF THE REWARD FROM A MARKOV REPLACEMENT PROCESS"

PAVLA KUNDEROVÁ
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In paper [1] the theory of point processes for studying the processes with replacements and rewards was used. There were proved two statements of limit behaviour of rewards (Theorems 1, 2) and one of the assumptions in both cases was that the common replacement policy F is a policy with a bounded intensity. We will show that the assumption about the bounded intensity can be weakened by passing to the Poisson process by changing time.

Let us denote ${}^lN_t$ the number of events with mark l ($l = 1, \dots, q$) occurred in time $\langle 0, t \rangle$, ${}^lA_t$ the compensator of the point process $\{{}^lN_t,\ t \geqq 0\}$ defined in [1] (not quite commonly) as an integral of the arrival intensity of the event with mark l, thus as a continuous nondecreasing process conformable to ${}^l\mathscr{F} = \{{}^l\mathscr{F}_t\}$, ${}^l\mathscr{F}_t = \sigma a\{{}^lN_s,\ s \in \langle 0, t \rangle\}$, such that $\{{}^lM_t = {}^lN_t - {}^lA_t,\ t \geqq 0\}$ is a martingale with respect to $\{{}^l\mathscr{F}_t,\ t \geqq 0\}$.
Further let us denote

$${}^l\sigma(t) = \inf\{s \geqq 0 : {}^lA_s > t\}, \qquad {}^l\mathscr{G}_t = {}^l\mathscr{F}_{{}^l\sigma(t)}, \qquad {}^l\pi_t = {}^lN_{{}^l\sigma(t)}.$$

In [2], page 280 is proved the following Lemma

Lemma.

If a) *the compensator* $\{{}^lA_t,\ t \geqq 0\}$ *is (almost sure) continuous process,*
b) $P(\lim_{t \to \infty} {}^lA_t = \infty) = 1$,

then

$\{{}^l\pi_t,\ t \geqq 0\}$ *is the Poisson process with parameter* $\lambda = 1$ *conformable to the system* $\{{}^l\mathscr{G}_t,\ t \geqq 0\}$.

Using the above Lemma we can prove that Theorems 1, 2 from [1] are valid in the following formulation:

Theorem 1.

Let $\{\underline{A}_t, t \geqq 0\} = \{({}^1A_t, \ldots, {}^qA_t), t \geqq 0\}$ *be a compensator of the process with replacements and with such replacement policy F that under this policy for every* $l = 1, \ldots, q$

$$P(\lim_{t \to \infty} {}^lA_t = \infty) = 1 \tag{1}$$

is valid. Further let $\{\tilde{\underline{A}}_t, t \geqq 0\}$ *be a compensator of the process with a stationary replacement policy of destination f such that there exists only one class of recurrent states under it. If for all* $l = 1, \ldots, q$

$$\lim_{t \to \infty} \frac{1}{t}({}^lA_t - {}^l\tilde{A}_t) = 0 \qquad \text{F-almost sure (F in probability)}$$

then

$$\lim_{t \to \infty} \frac{1}{t} R_t = \Theta \qquad \text{F-almost sure (F in probability)}$$

where Θ *is the mean reward from the process per a time unit and* R_t *is the reward up to time t.*

Theorem 2.

Let F be a replacement policy such that for any $l = 1, \ldots, q$ (1) *is true and let f be a stationary replacemert policy of destination such that there exists only one recurrent class under it. Let for any* $l = 1, \ldots, q$

$$\lim_{t \to \infty} \frac{1}{\sqrt{t}}({}^lA_t - {}^l\tilde{A}_t) = 0 \qquad \text{F-in probability}$$

and let for the total number O_t *of such replacements in interval* $\langle 0, t \rangle$ *which are different from* $i \to f(i)$

$$\lim_{t \to \infty} \frac{1}{\sqrt{t}} O_t = 0 \qquad \text{F-in probability,} \tag{2}$$

then

$$\frac{R_t - \Theta t}{\sqrt{t}}$$

has for $t \to \infty$ *assymptotically normal distribution* $N(0, \zeta)$, *where* ζ *is determined by aquations* (17) *ir* [1].

Prooves of both theorems are the same as in [1]. The interchange of assumption that F is a replacement policy with bounded intensity by the assumption (1) reveals only in such situations in which Lemma 2 of [1] was applied. From the proof of Lemma 2 in [1] it is obvious that it suffices the validity of the following statement to prove:

If for any $l = 1, \ldots, q$ the assumption (1) is true, then for any l

$$\lim_{t \to \infty} \frac{1}{t} {}^l M_t = 0 \qquad F\text{-almost sure.} \tag{3}$$

Actually, let l be an arbitrary firmly chosen mark. Let us set $t = {}^l\sigma(s)$ for all such $t \in \langle \sigma(0), \infty)$ for which

$${}^l\sigma(s - 0) < t \leqq {}^l\sigma(s).$$

Then

$${}^l M_t = {}^l N_t - {}^l A_t = {}^l N_{l_{\sigma(s)}} - s = {}^l \pi_s - s = {}^l M_s^*, \qquad s \geqq 0.$$

According to Lemma $\{\pi_s, s \geqq 0\}$ is the Poisson process with parameter $\lambda = 1$ and thus $\{{}^l M_s^* = {}^l\pi_s - s, \ s \geqq 0\}$ is a martingale with respect to $\{{}^l\mathscr{G}_s, \ s \geqq 0\}$. Using the fact that for all $p = 1, 2, \ldots$

$$E({}^l\pi_{n+1} - {}^l\pi_n)^p = \frac{1}{e} \sum_{k=0}^{\infty} \frac{k^p}{k!} = C(p)$$

we can show that analogous to the proof of Lemma 2 in [1] the statement

$$\lim_{s \to \infty} \frac{{}^l M_s^*}{s} = 0 \qquad F\text{-almost sure}$$

s valid and thus the validity of (3) is proved.

Remark. The statement given in [1] saying that assumption (2) in Theorem 2 of [1] may be omitted remains true even if the assumption about the bounded intensity of the replacement policy F is replaced by assumption (1). The proof remains the same, the modified assumption will occur in using Lemma 2 in [1] only.

I would like to express my appreciation to dr. P. Mandl, DrSc. for directing my attention to the fact that using the change of time we can weaken the assumptions of Theorems 1, 2 in [1].

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Souhrn

POZNÁMKA K ČLÁNKU „NĚKTERÉ LIMITNÍ VLASTNOSTI VÝNOSU Z MARKOVOVA PROCESU S OBNOVAMI“

PAVLA KUNDEROVÁ

V poznámce se ukazuje, že předpoklady vět 1, 2 z [1] o limitním chování výnosu z Markovova procesu s obnovami lze oslabit, užijeme-li přechodu k Poissonovu procesu záměnou času.

Резюме

ЗАМЕЧАНИЕ К СТАТЬЕ „НЕКОТОРЫЕ ПРЕДЕЛЬНЫЕ КАЧЕСТВА ДОХОДА ИЗ ПРОЦЕССА МАРКОВА С ВОССТАНОВЛЕНИЯМИ

ПАВЛА КУНДЕРОВА

В замечании показано, что предположения теорем 1,2 из [1] возможно сделать более слабыми при использовании перехода к процессу Пуассона изменением времени.