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The Osculating Hyperquadrics of the Three-Component Distribution in Affine Space

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Abstract

Three-component distributions of multidimensional affine space are discussed. The geometrical objects, which determine the normal of the first kind of the equipping hyperdistribution of affine space, are constructed in the differential neighbourhood of the second order. The fields of the invariant osculating hyperquadrics are constructed in the different differential neighbourhoods.

Key words: Osculating hyperquadric, three-component distribution, equipping hyperdistribution, affine space.

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1 Introduction

The real article applies to differential geometry of (n+1)-dimensional affine space A_{n+1}.

The three-component distributions ($H(M(\Lambda))$-distributions) of affine space are discussed. The objects Q^α, Q^i, Q^p are introduced in the second differential neighbourhood. They determine the normal of the first kind of the H-distribution—analog of the normal, which was constructed by E. D. Alshibaj [1] for the hyperplane distribution of affine space. Their geometrical characteristics are taken. The fields of the invariant osculating hyperquadrics are constructed in the different differential neighbourhoods, which depend on choice of the normal of the first kind of the equipping H-distribution.

The fields of the invariant osculating hyperquadrics are constructed in the first and in the third differential neighborhoods of the forming element of the three-component distribution.

We found the conditions of the tangency of the second order of the osculating hyperquadrics with the curves, which belong to the different distributions.

We use results, which we have got in the articles [2, 3].

During all summary indexes take following meanings:

$$\sigma, \rho, \tau = \overline{1, n}; \qquad p, q, r = \overline{1, r}; \qquad \alpha, \beta, \gamma = \overline{m+1, n};$$
$$i, j, k = \overline{r+1, m}; \qquad u, v, w = \overline{r+1, n}; \qquad I, J, K = \overline{1, n+1};$$
$$a, b, c = \overline{1, m}; \qquad \hat{u}, \hat{v}, \hat{w} = \overline{r+1, n+1}; \qquad \hat{\alpha}, \hat{\beta}, \hat{\gamma} = \overline{m+1, n+1}.$$

2 Definition of the three-component distribution

Let us consider $(n+1)$-dimensional affine space A_{n+1}, which is taken to movable frame $R = \{A, \overline{e}_I\}$. Differential equations of infinitesimal transference of frame R look as follows:

$$dA = \omega^I \overline{e}_I, \qquad d\overline{e}_I = \omega_I^K \overline{e}_K,$$

where ω_I^K, ω^I – invariant forms of affine group, which satisfy equations of the structure:

$$d\omega^I = \omega^K \Lambda \omega_K^I, \qquad d\omega_I^K = \omega_I^J \Lambda \omega_J^K.$$

Structural forms of current point $X = A + x^I \overline{e}_I$ of space A_{n+1} look as follows:

$$\Delta X^I = dx^I + x^K \omega_K^I + \omega^I.$$

Combination of current point X and point of frame A leads to the following equation:

$$\Delta X^I = \omega^I.$$

Immobility condition of the point A is written down as follows: $\omega^I = 0$. Let the frame chosen by this way be called as frame $\tilde{R}$.

Let Π_r-r-dimensional plane in A_{n+1}, given by the following way:

$$\Pi_r = \left[A, \overline{L}_p\right],$$

where $\overline{L}_p = \overline{e}_p + \Lambda_p^{\hat{u}} \overline{e}_{\hat{u}}$.

Let m-dimensional plane Π_m was set by this following way:

$$\Pi_m = \left[A, \overline{M}_a\right],$$

where $\overline{M}_a = \overline{e}_a + M_a^{\hat{\alpha}} \overline{e}_{\hat{\alpha}}$.

And hyperplane Π_n is set

$$\Pi_n = \left[A, \overline{T}_\sigma\right],$$

where $\overline{T}_\sigma = \overline{e}_\sigma + H_\sigma^{n+1} \overline{e}_{n+1}$.

The $(n+1)$-dimensional manifolds in spaces of notion $\{\Delta\Lambda_p^{\hat{u}}, \omega^I\}$, $\{\Delta M_a^{\hat{\alpha}}, \omega^I\}$, $\{\Delta H_\sigma^{n+1}, \omega^I\}$, which are determined by differential equations

$$\Delta\Lambda_p^{\hat{u}} = \Lambda_{pK}^{\hat{u}}\omega^K, \qquad \Delta M_a^{\hat{\alpha}} = M_{aK}^{\hat{\alpha}}\omega^K, \qquad \Delta H_\sigma^{n+1} = H_{\sigma K}^{n+1}\omega^K \tag{1}$$

are called distributions of the first kind accordingly of: r-dimensional linear elements (Λ-distribution), m-dimensional linear elements (M-distribution) and hyperplanes (H-distribution). Equations of system (1) to each point A (center of distribution) is set according to planes Π_r, Π_m, Π_n. Let consider, that manifolds (1) are distributions of tangent elements: center A belongs to planes Π_r, Π_m, Π_n.

We demand, that in some area of space A_{n+1} for any center A the following condition take place:

$$A \in \Pi_r \subset \Pi_m \subset \Pi_n.$$

The three of distributions of affine space A_{n+1}, consisting of basic distribution of the first kind r-dimensional linear elements $\Pi_r \equiv \Lambda$ (Λ-distribution), equipping distribution of the first kind of m-dimensional linear elements $\Pi_m \equiv M$ (M-distribution) and equipping distribution of the first kind of hyperplane elements $\Pi_n \equiv H$ $(r < m < n)$ (H-distribution) with relation of incidence of their corresponding elements in common center A of the following view: $A \in \Lambda \subset M \subset H$ are called $H\,(M(\Lambda))$-distribution.

Let us make the following canonization of frame $\tilde{R}$: we will place vectors $\overline{e}_p$ in the plane Π_r, vectors $\overline{e}_i$—in plane Π_m, and vectors $\overline{e}_\sigma$—in plane Π_n. Such frame will be called frame of the null order R^0. This definition leads to the following equations:

$$\Lambda_p^{\hat{u}} = 0, \qquad M_a^{\hat{\alpha}} = 0, \qquad H_\sigma^{n+1} = 0.$$

In frame R^0 $H\,(M(\Lambda))$-distribution is defined by the differential equations:

$$\omega_p^{\hat{u}} = \Lambda_{pK}^{\hat{u}}\omega^K, \qquad \omega_i^{\hat{\alpha}} = M_{iK}^{\hat{\alpha}}\omega^K, \qquad \omega_\alpha^{n+1} = H_{\alpha K}^{n+1}\omega^K.$$

It is possible partial zero-order frame R^0 canonization, where $M_{iq}^{n+1} = 0$, $H_{\alpha q}^{n+1} = 0$. We will call it frame of the first order R^1.

In the chosen frame R^1 manifold $H\,(M(\Lambda))$ is determined by the following system of differential equations:

$$\omega_p^{\hat{u}} = \Lambda_{pK}^{\hat{u}}\omega^K, \qquad \omega_i^{n+1} = M_{i\hat{u}}^{n+1}\omega^{\hat{u}}, \qquad \omega_i^{\alpha} = M_{iK}^{\alpha}\omega^K,$$

$$\omega_\alpha^{n+1} = H_{\alpha\hat{u}}^{n+1}\omega^{\hat{u}}, \qquad \omega_u^p = A_{uK}^p\omega^K.$$

3 The normal Q of the H-distribution

The geometrical objects of the second order were constructed using results, which we have got in the articles [2, 3]:

$$Q^\alpha = -q_{n+1}^\beta\tilde{q}_\beta^\alpha, \qquad Q^i = -\left(Q^\alpha q_\alpha^K + q_{n+1}^K\right)\tilde{q}_K^i,$$

$$Q^p = -\left(\overline{A}_i^q Q^i + \overline{A}_\alpha^q Q^\alpha + \overline{A}_{n+1}^q\right)\tilde{A}_q^p.$$

The quasitensor of the second order $\{Q^\sigma\}$ determines invariant equipment—the normal of the first kind of the H-distribution, which innerly connected with $H\,(M(\Lambda))$-distribution.

The theorem has been formulated, which generalized the corresponding theorem proved by E. D. Alshibaja [1] for the hyperplane elements.

Theorem 1 *The normal $\overline{L}$, where*

$$\overline{L} = L^p \overline{e}_p + L^u \overline{e}_u + \overline{e}_{n+1},$$

moves parallel along the curves, which belong to distribution of the normal Q.

Really along the curves $\omega^\sigma = Q^\sigma \omega^{n+1}$ we have:

$$d\overline{L} = \left(L^p \omega_p^{n+1} + L^u \omega_u^{n+1} + \omega_{n+1}^{n+1}\right) \overline{L}.$$

Remarks

1. The normal of the first kind Q, introduced for the $H\,(M(\Lambda))$-distribution, is analog the normal, which was constructed E. D. Alshibaja [1] for the hyperplane distribution of affine space.
2. The field of the quasitensor $\{Q^a\}$ creates the field of the normals of the first kind of the M-distribution, the field of the quasitensor $\{Q^\alpha\}$ creates the field of the normal of the first kind of the Φ-distribution, the field of the quasitensor $\{Q^p\}$ creates the field of the normal of the first kind of the Λ-distribution, and the field of the quasitensor $\{Q^u\}$ creates the field of the normal of the first kind of the X-distribution.

4 The osculating hyperquadrics of the three-component distribution

Let us construct sequentially systems of the values:

$$M^i_{pq} = a^i_{pq} - a^i a_{pq}, \qquad M^\alpha_{pq} = a^\alpha_{pq} - a^\alpha a_{pq},$$

$$b^{pq}_i = A^p_{ir} a^{rq} - \hat{a}_i a^{pq}, \qquad b^p_{is} = b^{pt}_i a_{ts}, \qquad b^\alpha_i = b^{ps}_i M^\alpha_{ps},$$

$$b_{ij} = b^p_{is} b^s_{jp}, \qquad b^j_i = b^{ps}_i M^j_{ps}.$$

For the tensor $\{M^\alpha_{ps}\}$ let us consider directed tensor of the first order $\{\tilde{M}^{ps}_\alpha\}$, which satisfies equations:

$$\tilde{M}^{ps}_\alpha M^\beta_{ps} = r\delta^\beta_\alpha, \qquad \tilde{M}^{pt}_\alpha M^\alpha_{ps} = (n-m)\delta^t_s$$

and satisfies differential equations:

$$\nabla \tilde{M}^{ps}_\alpha = \tilde{M}^{ps}_{\alpha K} \omega^K.$$

Let us consider relative tensor of the second order $\{Q_i^J\}$:

$$t_\alpha^i = \frac{1}{r} M_{ps}^i \tilde{M}_\alpha^{ps}, \qquad \bar{b}_i^j = t_\alpha^j b_i^\alpha, \qquad Q_i^j = \bar{b}_i^j - b_i^j.$$

Will bring in the directed tensor $\{Q_i^j\}$ of the second order for the tensor $\{\tilde{Q}_i^j\}$:

$$Q_K^i \tilde{Q}_j^K = Q_j^K \tilde{Q}_K^i = \delta_j^i.$$

With the help of the tensors $\{\tilde{Q}_i^j\}$ and $\{b_{ij}\}$ will determine the symmetric tensor of the second order:

$$B_{ij} = \frac{1}{2}\left(\tilde{Q}_i^K b_{Kj} + \tilde{Q}_j^K b_{Ki}\right).$$

In the general case the tensor $\{B_{ij}\}$ is nondegenerated. The quasitensor $\{t_\alpha^i\}$ and the tensor $\{B_{ij}\}$ give possibility to construct the object $\{B_{ij},\ B_{i\alpha},\ B_{\alpha\beta}\}$, where

$$B_{i\alpha} = -B_{ij} t_\alpha^j, \qquad B_{\alpha\beta} = -B_{i\alpha} t_\beta^i.$$

Let us consider the absolute tensor in the differential neighborhood of the second order of the forming element of the three-component distribution :

$$b_{pq} = a^{st} a^{rf} B_{srp} B_{tfq}.$$

In the general case the tensor $\{b_{pq}\}$ is nondegenerated, so making it possible to bring in the absolute tensor $\{b^{qr}\}$, which opposite to it:

$$b_{pq} b^{qr} = \delta_p^r.$$

Then we will obtain subsequently the following tensors of the second order:

$$b_p^{\alpha q} = M_{pr}^\alpha a^{rq}, \qquad \ell_p = B_{pqs} b^{qs}, \qquad \Lambda_{\alpha p} = \ell_p \hat{a}_\alpha, \qquad N_q = \Lambda_{\alpha p} b_q^{\alpha p}.$$

The equation of the osculating hyperquadric relatively to the local reper looks as follows:

$$A_{JK} x^J x^K + 2A_J x^J + A = 0, \qquad A_{JK} = A_{KJ}.$$

Coefficients of the field of the osculating hyperquadrics of the three-component distribution may be captured by components of the sequence of fundamental geometric objectives of distribution in different ways.

Following the work of A. Stoliarov [4], we have the following field of the invariant osculating hyperquadrics:

$$\begin{aligned} &a_{pq} x^p x^q + a_{ij} x^i x^j + a_{\alpha\beta} x^\alpha x^\beta + 2\Lambda_{pi} x^p x^i + 2\Lambda_{p\alpha} x^p x^\alpha + 2M_{i\alpha} x^i x^\alpha \\ &+ 2\overline{p}_{a,n+1} x^a x^{n+1} + 2\overline{p}_{\alpha,n+1} x^\alpha x^{n+1} + \overline{p}_{n+1,n+1} x^{n+1} x^{n+1} - 2x^{n+1} = 0, \end{aligned}$$

where

$$\begin{aligned} \overline{p}_{p,n+1} &= -\left(a_{pq}\nu^q - \nu_p + \Lambda_{p\alpha}\nu^\alpha + \Lambda_{pi}\nu^i\right), \\ \overline{p}_{i,n+1} &= -\left(a_{ij}\nu^j + \tfrac{1}{2} M_{i\alpha}\nu^\alpha + \hat{a}_i + \Lambda_{pi}\nu^p\right), \\ \overline{p}_{\alpha,n+1} &= -\left(a_{\alpha\beta}\nu^\beta + \tfrac{1}{2} M_{i\alpha}\nu^i + \hat{a}_\alpha + \Lambda_{p\alpha}\nu^p\right), \\ \overline{p}_{n+1,n+1} &= 2\left(\hat{a}_i\nu^i + \hat{a}_\alpha\nu^\alpha - \nu_p\nu^p + \Lambda_{p\alpha}\nu^p\nu^\alpha + \Lambda_{pi}\nu^p\nu^i\right) + a_{pq}\nu^p\nu^q \\ &\quad + a_{ij}\nu^i\nu^j + a_{\alpha\beta}\nu^\alpha\nu^\beta + \tfrac{1}{2} M_{i\alpha}\nu^i\nu^\alpha. \end{aligned}$$

This field of the osculating hyperquadrics is determined in the second differential neighbourhood of the forming element of the three-component distribution.

Differently from the captures are constructed in the work [4], where the quasitensor $\{a^u\}$ is fixed, we apply quasitensor $\{\nu^i,\ \nu^\alpha\}$ designating liberally in the internal invariant normal of first order X-distribution.

One more field of the osculating hyperquadrics will be obtained in similar ways:

$$a_{pq}x^px^q + a_{ij}x^ix^j + a_{\alpha\beta}x^\alpha x^\beta + 2\Lambda_{pi}x^px^i + 2\Lambda_{p\alpha}x^px^\alpha + 2M_{i\alpha}x^ix^\alpha$$
$$+ 2\tilde{p}_{a,n+1}x^ax^{n+1} + 2\tilde{p}_{\alpha,n+1}x^\alpha x^{n+1} + \tilde{p}_{n+1,n+1}x^{n+1}x^{n+1} - 2x^{n+1} = 0, \qquad (2)$$

where

$$\begin{aligned}
\tilde{p}_{p,n+1} &= -\left(a_{pq}\nu^q + \Lambda_{p\alpha}\nu^\alpha + \Lambda_{pi}\nu^i\right),\\
\tilde{p}_{i,n+1} &= -\left(a_{ij}\nu^j + M_{i\alpha}\nu^\alpha + \Lambda_{pi}\nu^p\right),\\
\tilde{p}_{\alpha,n+1} &= -\left(a_{\alpha\beta}\nu^\beta + M_{i\alpha}\nu^i + \Lambda_{p\alpha}\nu^p\right),\\
\tilde{p}_{n+1,n+1} &= 2\left(M_{i\alpha}\nu^i\nu^\alpha + \Lambda_{p\alpha}\nu^p\nu^\alpha + \Lambda_{pi}\nu^p\nu^i\right) + a_{pq}\nu^p\nu^q + a_{ij}\nu^i\nu^j + a_{\alpha\beta}\nu^\alpha\nu^\beta.
\end{aligned}$$

Will emphasize that the field of the invariant osculating hyperquadrics (2) is determined in the first differential neighbourhood of the forming element of the three-component distribution. Really, if the normal $\{L^\rho\}$ in the captures $\tilde{p}_{p,n+1}$, $\tilde{p}_{\alpha,n+1}$, $\tilde{p}_{i,n+1}$ will be taken as the normal $\{\nu^\rho\}$, that the field of the invariant osculating hyperquadrics (2) will be determined in the first differential neighbourhood.

The constructed hyperquadrics by virtue of

$$A_{pq} = a_{pq}, \qquad A_{ij} = a_{ij}, \qquad A_{\alpha\beta} = a_{\alpha\beta}, \qquad (3)$$

are the osculating ones not as regards to the Λ-distriburion only but to the $M\Lambda$-distribution and the Φ-distribution too.

The following theorems are proved:

Theorem 2 *The hyperquadric Q_n has the tangency of the second order with the curves, which belong to the H-distribution, if and only if, when the conditions (3) and following conditions*

$$A_{pi} = A_{p\alpha} = A_{i\alpha} = A_{ip} = A_{\alpha p} = A_{\alpha i} = 0,$$

are satisfied.

Remarks

1. If the hyperquadric Q_n has the tangency of the second order with the curves, which belong to the H-distribution, that it is the tangency of the second order with any curve of the Λ-distribution, of the $M\Lambda$-distribution, M-distribution and Φ-distribution;
2. If the hyperquadric Q_n has the tangency of the second order with the curves, which belong to the M-distribution, that it is the tangency of the second order with any curve of the Λ-disrtibution and $M\Lambda$-distribution.

5 The condition that the zero subtenzor of tenzor of inholonomicity of the basic distribution

Let us consider the values:

$$c^{\alpha}_{ij} = M^{\alpha}_{ij} - \nu^{\alpha} M_{ij}, \quad \nabla_{\delta} c^{\alpha}_{ij} = 0,$$

$$c^{i}_{pq} = \Lambda^{i}_{pq} - \nu^{i} \Lambda_{pq}, \quad \nabla_{\delta} c^{i}_{pq} = 0,$$

$$c^{\alpha}_{pq} = \Lambda^{\alpha}_{pq} - \nu^{\alpha} \Lambda_{pq}, \quad \nabla_{\delta} c^{\alpha}_{pq} = 0.$$

Then we will obtain the following geometrical objects of the second order of the three-component distribution of affine space:

$$\begin{aligned}
& b^{i}_{\alpha k} = b^{ij}_{\alpha} a_{jk}, && \nabla_{\delta} b^{i}_{\alpha k} = 0,\\
& b^{\beta}_{\alpha} = b^{pq}_{\alpha} c^{\beta}_{pq}, && \nabla_{\delta} b^{\beta}_{\alpha} = b^{\beta}_{\alpha} \Pi^{n+1}_{n+1},\\
& \overset{1}{b}{}^{\beta}_{\alpha} = b^{ij}_{\alpha} c^{\beta}_{ij}, && \nabla_{\delta} \overset{1}{b}{}^{\beta}_{\alpha} = \overset{1}{b}{}^{\beta}_{\alpha} \Pi^{n+1}_{n+1},\\
& b^{i}_{j} = b^{pq}_{j} c^{\beta}_{ij}, && \nabla_{\delta} b^{i}_{j} = b^{i}_{j} \Pi^{n+1}_{n+1},\\
& \overset{1}{b}{}_{\alpha\beta} = b^{j}_{\alpha k} b^{k}_{\beta j}, && \nabla_{\delta} \overset{1}{b}{}_{\alpha\beta} = 0,\\
& b^{\beta}_{\alpha} \tilde{b}^{\alpha}_{\gamma} = \delta^{\beta}_{\gamma}, && \nabla_{\delta} \tilde{b}^{\beta}_{\alpha} = -\tilde{b}^{\beta}_{\alpha} \Pi^{n+1}_{n+1},\\
& \overset{1}{b}{}^{\beta}_{\alpha} \overset{1}{\tilde{b}}{}^{\alpha}_{\gamma} = \delta^{\beta}_{\gamma}, && \nabla_{\delta} \overset{1}{\tilde{b}}{}^{\beta}_{\alpha} = -\overset{1}{\tilde{b}}{}^{\beta}_{\alpha} \Pi^{n+1}_{n+1},\\
& b^{i}_{j} \tilde{b}^{j}_{k} = \delta^{i}_{k}, && \nabla_{\delta} \tilde{b}^{i}_{j} = -\tilde{b}^{i}_{j} \Pi^{n+1}_{n+1},\\
& B_{ij} = -\tfrac{1}{2}\left(\tilde{b}^{k}_{i} b_{kj} + \tilde{b}^{k}_{j} b_{ki}\right), && \nabla_{\delta} B_{ij} = -B_{ij} \Pi^{n+1}_{n+1},\\
& B_{\alpha\beta} = -\tfrac{1}{2}\left(\tilde{b}^{\gamma}_{\alpha} b_{\gamma\beta} + \tilde{b}^{\gamma}_{\beta} b_{\gamma\alpha}\right), && \nabla_{\delta} B_{\alpha\beta} = -B_{\alpha\beta} \Pi^{n+1}_{n+1},\\
& \overset{1}{B}{}_{\alpha\beta} = -\tfrac{1}{2}\left(\overset{1}{\tilde{b}}{}^{\gamma}_{\alpha} \overset{1}{b}{}_{\gamma\beta} + \overset{1}{\tilde{b}}{}^{\gamma}_{\beta} \overset{1}{b}{}_{\gamma\alpha}\right), && \nabla_{\delta} \overset{1}{B}{}_{\alpha\beta} = -\overset{1}{B}{}_{\alpha\beta} \Pi^{n+1}_{n+1},\\
& \hat{B}_{\alpha} = -\left(B_{\alpha\beta} \nu^{\beta} + \hat{a}_{\alpha}\right), && \nabla_{\delta} B_{\alpha} = B_{\alpha\beta} \Pi^{\beta}_{n+1},\\
& \overset{1}{\hat{B}}{}_{\alpha} = -\left(\overset{1}{B}{}_{\alpha\beta} \nu^{\beta} + \hat{a}_{\alpha}\right), && \nabla_{\delta} \overset{1}{\hat{B}}{}_{\alpha} = \overset{1}{B}{}_{\alpha\beta} \Pi^{\beta}_{n+1},\\
& \overset{2}{\hat{B}}{}_{\alpha} = -\left(B_{\alpha\beta} \nu^{\beta} + \hat{b}_{\alpha}\right), && \nabla_{\delta} \overset{2}{\hat{B}}{}_{\alpha} = B_{\alpha\beta} \Pi^{\beta}_{n+1},\\
& \overset{3}{\hat{B}}{}_{\alpha} = -\left(\overset{1}{B}{}_{\alpha\beta} \nu^{\beta} + \hat{b}_{\alpha}\right), && \nabla_{\delta} \overset{3}{\hat{B}}{}_{\alpha} = \overset{1}{B}{}_{\alpha\beta} \Pi^{\beta}_{n+1},\\
& \hat{B}_{i} = -\left(B_{ij} \nu^{j} + \hat{a}_{i}\right), && \nabla_{\delta} \hat{B}_{i} = B_{ij} \Pi^{j}_{n+1},
\end{aligned}$$

$$t_p = \frac{1}{r+2} a_{pqr} a^{pq}, \qquad \nabla_{\delta} t_p = \left(a_{pr} + \frac{1}{r+2} a^{qs} r_{qs} a_{pr}\right) \Pi^{r}_{n+1}. \tag{4}$$

Let us construct the values in the third differential neighbourhood of the forming element of the three-component distribution.

We have the differential equations:

$$\nabla t_{p\mathcal{K}} - t_r \Lambda_{p\mathcal{K}} \omega^r_{n+1} - \Big(a_{pr\mathcal{K}} - \tfrac{1}{r+2} a^{fs} a_{tf\mathcal{K}} a_{pr} r_{qs} a^{qt} + \tfrac{1}{r+2} a^{ts} r_{ts\mathcal{K}} a_{pr}$$
$$+ \tfrac{1}{r+2} a^{qs} r_{qs} a_{pr\mathcal{K}}\Big)\omega^r_{n+1} + \Big(a_{pr} + \tfrac{1}{r+2} a^{qs} r_{qs} a_{pr}\Big)\mathcal{A}^r_{u\mathcal{K}} \omega^u_{n+1} = t_{p\mathcal{K}\mathcal{L}} \omega^{\mathcal{L}},$$

which were obtained with the help of differentiation of the equations: (4)

Let us construct systems of the values of the third order on the condition that the zero subtenzor $\{r_{qs}\}$ of the tenzor of inholonomicity of the basic distribution:

$$T = (t_{pq} - t_p t_q)\, a^{pq}, \qquad \delta T = 2 r t_p \Pi^p_{n+1} - \mathcal{A}^r_{ur} \Pi^u_{n+1} + T \Pi^{n+1}_{n+1},$$

$$T_0 = \tfrac{T}{r} - \hat{B}_\alpha \nu^\alpha - \hat{B}_i \nu^i, \qquad \delta T_0 = T_0 \Pi^{n+1}_{n+1} + 2\hat{B}_\alpha \Pi^\alpha_{n+1} + 2\hat{B}_i \Pi^i_{n+1} + 2 t_r \Pi^r_{n+1},$$

$$\overset{1}{T}_0 = \tfrac{T}{r} - \overset{1}{\hat{B}}_\alpha \nu^\alpha - \hat{B}_i \nu^i, \qquad \delta \overset{1}{T}_0 = \overset{1}{T}_0 \Pi^{n+1}_{n+1} + 2 \overset{1}{\hat{B}}_\alpha \Pi^\alpha_{n+1} + 2\hat{B}_i \Pi^i_{n+1} + 2 t_r \Pi^r_{n+1},$$

$$\overset{2}{T}_0 = \tfrac{T}{r} - \overset{2}{\hat{B}}_\alpha \nu^\alpha - \hat{B}_i \nu^i, \qquad \delta \overset{2}{T}_0 = \overset{2}{T}_0 \Pi^{n+1}_{n+1} + 2 \overset{2}{\hat{B}}_\alpha \Pi^\alpha_{n+1} + 2\hat{B}_i \Pi^i_{n+1} + 2 t_r \Pi^r_{n+1},$$

$$\overset{3}{T}_0 = \tfrac{T}{r} - \overset{3}{\hat{B}}_\alpha \nu^\alpha - \hat{B}_i \nu^i, \qquad \delta \overset{3}{T}_0 = \overset{3}{T}_0 \Pi^{n+1}_{n+1} + 2 \overset{3}{\hat{B}}_\alpha \Pi^\alpha_{n+1} + 2\hat{B}_i \Pi^i_{n+1} + 2 t_r \Pi^r_{n+1}.$$

On the conditions:

$$\mathcal{A}_{ij} = a_{ij}, \quad \mathcal{A}_{\alpha\beta} = a_{\alpha\beta}, \quad \mathcal{A}_{pi} = \Lambda_{pi}, \quad \mathcal{A}_{p\alpha} = \Lambda_{p\alpha}, \quad \mathcal{A}_{i\alpha} = M_{i\alpha},$$

$$\begin{array}{lllll}
1) & \mathcal{A}_{pn+1} = t_p, & \mathcal{A}_{in+1} = \hat{B}_i, & \mathcal{A}_{\alpha n+1} = \hat{B}_\alpha, & \mathcal{A}_{n+1\, n+1} = T_0; \\
2) & \mathcal{A}_{pn+1} = t_p, & \mathcal{A}_{in+1} = \hat{B}_i, & \mathcal{A}_{\alpha n+1} = \overset{1}{\hat{B}}_\alpha, & \mathcal{A}_{n+1\, n+1} = \overset{1}{T}_0; \\
3) & \mathcal{A}_{pn+1} = t_p, & \mathcal{A}_{in+1} = \hat{B}_i, & \mathcal{A}_{\alpha n+1} = \overset{2}{\hat{B}}_\alpha, & \mathcal{A}_{n+1\, n+1} = \overset{2}{T}_0; \\
4) & \mathcal{A}_{pn+1} = t_p, & \mathcal{A}_{in+1} = \hat{B}_i, & \mathcal{A}_{\alpha n+1} = \overset{3}{\hat{B}}_\alpha, & \mathcal{A}_{n+1\, n+1} = \overset{3}{T}_0;
\end{array}$$

we have four more fields of the osculating hyperquadrics of the three component distribution.

6 The conditions of the tangency of the second order of the hyperquadric with the curves

Let us consider the conditions of the tangency of the second order of the hyperquadric Q_n with the curves, which belong to the different distributions of the three-component distribution:

The distributions to which belong the curves	The conditions of the tangency of the second order of the hyperquadric with the curves
Λ-distribution	$\mathcal{A}_{pq} = a_{pq},$
$M\Lambda$-distribution	$\mathcal{A}_{ij} = a_{ij},$
M-distribution	$\mathcal{A}_{pq} = a_{pq}, \mathcal{A}_{ij} = a_{ij}, \mathcal{A}_{pi} = \mathcal{A}_{ip} = 0,$
$\mathcal{X}_{n-m}$-distribution	$\mathcal{A}_{\alpha\beta} = a_{\alpha\beta},$
H-distribution	$\mathcal{A}_{pq} = a_{pq},\ \mathcal{A}_{ij} = a_{ij},\ \mathcal{A}_{\alpha\beta} = a_{\alpha\beta},$ $\mathcal{A}_{pi} = \mathcal{A}_{p\alpha} = \mathcal{A}_{i\alpha} = \mathcal{A}_{ip} = \mathcal{A}_{\alpha p} = \mathcal{A}_{\alpha i} = 0$

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