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ON THE RECEPTION OF BINARY PHASE SHIFT KEYED SEQUENCES BY AN AUTOCORRELATION RECEIVER

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The autocorrelation reception of signals, used in the instantaneous frequency measurement (IFM) receiver, is investigated in connection with receiving binary phase shift keyed (BPSK) sequences.

1. INTRODUCTION

Recently, some interest is concentrated on the so-called autocorrelation receiver. This interest is motivated partly by "instantaneous" frequency measurement of pulsed radar signals ([4], [6]), partly by "without-search" reception of communication binary coded signals ([5], p. 192).

The idea of autocorrelation reception is old ([2], pp. 50, 74), but the real-time applications, described in detail in other cited references, are relatively new.

In the present article, attention will be concentrated on what is to be expected from this reception method in receiving BPSK sequences in noise.

2. AUTOCORRELATION MULTIPLYING

Let

$$A\,e^{i[\omega t + \Phi(t) + \varphi(t)]} + n(t) \tag{1}$$

be a signal at the output of a wideband filter (see e.g. the schemas in [4]). In (1), A is the amplitude, ω the frequency, $\Phi(t) + \varphi(t)$ the phase of the wanted component of the signal, $n(t)$ is its noise component the properties of which will be specified later on. Let $0 \leqq t \leqq N\tau_i = T$, let $\Phi(t) = 0, \pi$ and its value can be changed by jumps only in $t = k\tau_i$ $(k = 1, 2, \ldots, N - 1)$ according to a given BPSK sequence (of the time length T).

The bandwidth of the filter is dictated by the need to detect signals the frequency of which can lie actually in a wide band.

Let the signal (1) be delayed by $\tau > 0$ and its complex conjugate be multiplied by (1),

$$(2) \qquad v(t) = [A\, e^{i[\omega t + \Phi(t) + \varphi(t)]} + n(t)] \,.\, [A\, e^{-i[\omega(t-\tau) + \Phi(t-\tau) + \varphi(t-\tau)]} + \bar{n}(t-\tau)]\,.$$

Physically, this operation is executed with the aid of frequency independent quadrature channels (see Fig. 1 in [4]). $v(t)$ is well defined for t fulfilling $\tau \leqq t \leqq T$. Supposing that the arbitrary phase term is not changing in the given time interval $(0, T)$, $\varphi(t) = \varphi$, one gets

$$(3) \qquad v(t) = A^2\, e^{i[\omega\tau + \Phi(t) - \Phi(t-\tau)]} + A\, e^{i[\omega t + \Phi(t) + \varphi]} \,.\, \bar{n}(t-\tau) + \\ + A\, e^{-i[\omega(t-\tau) + \Phi(t-\tau) + \varphi]} \,.\, n(t) + n(t)\,\bar{n}(t-\tau)\,.$$

Using the common definition of the correlation function (E being the expectation symbol)

$$(4) \qquad R_s(t, \tau) = \mathsf{E}[s(t)\, \bar{s}(t + \tau)]\,,$$

one gets

$$(5) \qquad \mathsf{E}[v(t)] = \bar{R}_s(t, \tau)\,.$$

Hence the name "autocorrelation reception".

In what follows there will be seen that the respective correlation function is real, thus τ or $-\tau$ are irrelevant in (4), (5).

If the noise were absent, only the first term would remain in (3). Normalizing the absolute value $A^2 = 1$ and mapping the resulting unit length vector in the complex plane (an operation that can be executed technically on a cathode ray tube), it is seen that the vector endpoint is given by $\omega\tau$ if $\Phi(t) - \Phi(t - \tau) = 0$ (this situation is considered in [4]), and by $\omega\tau + \pi$ if $\Phi(t) - \Phi(t - \tau) = \pi$.

It is evident that

(a) ω can be read from the angle $\omega\tau$, τ being known,

(b) the fine structure of the BPSK sequence is not seen from the vector mapping,

(c) for unambiguous mapping, it is necessary $\tau \leqq \tau_i$.

The condition (c) has practical sense for communication applications, and in radar for the longest delay line, supposed that more than one delay lines are used (see [4]). For the shortest one, denoting ω_1, ω_2 the lower and upper bound angular frequencies of the wideband filter, there follows from (a) and the unambiguity postulate, that

$$(6) \qquad (\omega_2 - \omega_1)\,\tau \leqq \pi$$

and $\tau \ll \tau_i$ can result.

Some sort of time expansion of the cathode ray tube image is necessary to see BPSK sequence structure. Even in this case, only the difference sequence is seen, but, since two sum sequences of a given sequence differ only by a constant, no further error is introduced in signal recognition process.

3. NOISE INFLUENCE

In reality, noise is always present and the endpoints of the signal vectors will fluctuate around their "correct" values.

Let us suppose that the wideband filter is ideal and (without loss of generality) of the amplitude characteristic of magnitude 1 in the whole band $\omega_1 < \omega < \omega_2$.

Let the input noise of the filter be white Gaussian, this means each of its two independent real and imaginary components being Gaussian and possessing mean value 0 and spectral density $N_0/2$.

Then at the output of the filter the correlation functions of both (real and imaginary) components $x(t)$, $y(t)$ are

$$R_x(\tau) = R_y(\tau) = R(\tau) = \frac{N_0}{2\pi}\left(\omega_2 \frac{\sin \omega_2\tau}{\omega_2\tau} - \omega_1 \frac{\sin \omega_1\tau}{\omega_1\tau}\right). \tag{7}$$

From the independence of $x(t)$, $y(t)$, the crosscorrelation function $R_{xy}(\tau) = 0$ for every τ, and

$$R_n(\tau) = R_x(\tau) + R_y(\tau) = 2\,R(\tau)\,, \tag{8}$$

thus, it is real. Especially for $\tau = 0$

$$\sigma_n^2 = 2\sigma^2 \tag{9}$$

and from (7)

$$\sigma^2 = N_0(f_2 - f_1)\,, \tag{10}$$

$$\sigma_n^2 = 2\,N_0(f_2 - f_1)\,, \tag{11}$$

where f_1, f_2 are the lower and upper bound frequencies (Hz) of the filter. These formulas are well known.

In practice (see [4], [8], [9]), there can be e.g. $f_1 = 2$ GHz, $f_2 = 4$ GHz.

The first term in (3), representing the wanted information, is a step function of time and is constant in time intervals of the minimum possible length τ. Thus $v(t)$ can be passed through a lowpass filter of the bandwidth $f_3 = \varkappa/\tau$, where the constant $\varkappa$ is of the order of unity. In practice f_3 is only some Megahertz or tens of Megahertz.

The input noise to the lowpass filter, comparing the bands f_3 and $f_2 - f_1$, may be considered again as white Gaussian one.

Now, the meaning of the second, third, and fourth term in (3) will be investigated more thoroughly.

In the second term, $A \exp \mathrm{i}[\Phi(t) + \varphi]$ may be interpreted as an arbitrary complex constant. By $\exp \mathrm{i}\omega t$, both filter passbands, $(-\omega_2, -\omega_1)$, (ω_1, ω_2), are shifted to the right, the left one to the baseband, and the right one to frequencies about 2ω.

This is filtered out. Analogous reasoning can be made about the third term of (3).

Supposing again the lowpass filter ideal with the amplitude characteristic 1, one gets for its output signal

$$(12) \qquad w(t) = A^2\, e^{i[\omega\tau + \Phi(t) - \Phi(t-\tau)]} + A\, \bar{n}_0(t-\tau) + A\, n_0(t) + n_0(t)\, \bar{n}_0(t-\tau)\,,$$

and, substituting $f_1 = 0$ and $f_2 = f_3$ in (7),

$$(13) \qquad R_{x0}(\tau) = R_{y0}(\tau) = R_0(\tau) = \frac{N_0}{2\pi}\, \omega_3\, \frac{\sin \omega_3 \tau}{\omega_3 \tau} = N_0 f_3\, \frac{\sin \omega_3 \tau}{\omega_3 \tau}\,,$$

where $x_0(t)$ and $y_0(t)$ are the quadrature components of $n_0(t)$ and may be again supposed independent Gaussian, with mean values 0.

From (9), (11)

$$(14) \qquad \sigma_{n0}^2 = 2\sigma_0^2 = 2\sigma^2\, \frac{f_3}{f_2 - f_1} = 2N_0 f_3\,.$$

And for the second and third term in (12), the r.m.s. values are

$$(15) \qquad A^2 \sigma_{n0}^2 = 2A^2 N_0 f_3\,.$$

The fourth term of (12) is a complex random variable with

$$(16) \qquad \mathsf{E}[x_0(t)\, x_0(t-\tau)] = \mathsf{E}[y_0(t)\, y_0(t-\tau)] = R_0(\tau)\,.$$

Further, there is

$$(17) \qquad \sigma^2_{x_0(t)x_0(t-\tau)} = \mathsf{E}[(x_0(t)\, x_0(t-\tau))^2] - R_0^2(\tau)\,.$$

Denoting for a moment $x_0(t) = X_1$, $x_0(t-\tau) = X_2$, using the Gaussianness supposition and the formula (13, 36) in [2], p. 99, one gets

$$(18) \qquad \mathsf{E}[(x_0(t)\, x_0(t-\tau))^2] = \mathsf{E}[X_1^2 X_2^2] = 2\, R_0^2(\tau) + R_0^2(0)\,.$$

For the whole complex variable $n_0(t)\, \bar{n}(t-\tau)$, one gets

$$(19) \qquad \mathsf{E}[n_0(t)\, \bar{n}_0(t-\tau)] = R_{n0}(\tau) = 2\, R_0(\tau)$$

and by an easy calculation

$$(20) \qquad \mathsf{E}[|n_0(t)\, \bar{n}_0(t-\tau)|^2] = 4[R_0^2(\tau) + R_0^2(0)]\,,$$

so that the variance of $n_0(t)\, \bar{n}_0(t-\tau)$ is

$$(21) \qquad \sigma^2_{n_0(t)\bar{n}_0(t-\tau)} = 4[R_0^2(\tau) + R_0^2(0)] - 4\, R_0^2(\tau) = 4\, R_0^2(0) = 4\sigma_0^4$$

independently of τ. With (21), (14), (15), one may write

$$(22) \qquad \frac{4\sigma_0^4}{A^2 \sigma_{n0}^2} = \frac{4\sigma_0^4}{A^2 \,.\, 2\sigma_0^2} = \frac{2\sigma_0^2}{A^2} = \frac{2\sigma^2}{A^2}\, \frac{f_3}{f_2 - f_1}\,.$$

The first term on the right is the input noise/signal ratio of the lowpass filter.

By expression (22), the influence of the fourth term in (12) relative to the second and third one is shown.

4. OUTPUT SIGNAL/NOISE RATIO

From (12)

$$\mathsf{E}[w(t)] = A^2\, e^{i[\omega\tau + \Phi(t) - \Phi(t-\tau)]} + 2\, R_0(\tau)\,. \tag{23}$$

One will compute

$$\begin{aligned} &\mathsf{E}[|w(t) - \mathsf{E}[w(t)]|^2] = \\ &= \mathsf{E}[|A\, \bar{n}_0(t-\tau) + A\, n_0(t) + n_0(t)\, \bar{n}_0(t-\tau) - 2\, R_0(\tau)|^2] = \\ &\mathsf{E}[(A\, \bar{n}_0(t-\tau) + A\, n_0(t) + n_0(t)\, \bar{n}_0(t-\tau) - 2\, R_0(\tau))\,. \\ &.\, (A\, n_0(t-\tau) + A\, \bar{n}_0(t) + \bar{n}_0(t)\, n_0(t-\tau) - 2\, R_0(\tau))]\,. \end{aligned} \tag{24}$$

From the expected values of 16 product terms, some are clear at the first sight. It remains to compute (25), (26)

$$\begin{aligned} \mathsf{E}[n_0(t)\, n_0(t-\tau)] &= \mathsf{E}[(x_0(t) + i\, y_0(t))\,.\,(x_0(t-\tau) + i\, y(t-\tau))] = \\ &= \mathsf{E}[x_0(t)\, x_0(t-\tau)] - \mathsf{E}[y_0(t)\, y_0(t-\tau)] = 0 \end{aligned} \tag{25}$$

(since $x_0(t)$ and $y_0(t)$ are identically distributed),

$$\begin{aligned} &\mathsf{E}[(n_0(t) + \bar{n}_0(t))\, n_0(t-\tau)\, \bar{n}_0(t-\tau)] = \\ &= \mathsf{E}[2x_0(t)\,.\,(x_0^2(t-\tau) + y_0^2(t-\tau))] = 0\,. \end{aligned} \tag{26}$$

This result follows from $\mathsf{E}[x_0(t)\, y_0^2(t-\tau)] = 0$, which is seen immediately, and from

$$\mathsf{E}[x_0(t)\, x_0^2(t-\tau)] = 0\,. \tag{27}$$

This last result can be obtained as by-result in computing (13, 35), and (13, 36) from (13, 33) in [2], p. 99.

Finally, one gets

$$\mathsf{E}[|w(t) - \mathsf{E}[w(t)]|^2] = 4\sigma_0^4 \left(1 + 2\frac{A^2}{2\sigma_0^2}\right). \tag{28}$$

Thus

$$\frac{|\mathsf{E}[w(t)]|^2}{4\sigma_0^4\left(1 + 2\dfrac{A^2}{2\sigma_0^2}\right)} = \frac{A^4 + 4\, R_0(\tau)\, [A^2 \cos(\omega\tau + \Phi(t) - \Phi(t-\tau)) + R_0(\tau)]}{4\sigma_0^4\left(1 + 2\dfrac{A^2}{2\sigma_0^2}\right)} \tag{29}$$

and integrating in $\omega\tau$ over $(0, \pi)$, one gets

$$(30) \qquad (s/n)_o = \frac{\left(\frac{A^2}{2\sigma_0^2}\right)^2 + \frac{R_0^2(\tau)}{R_0^2(0)}}{1 + 2\frac{A^2}{2\sigma_0^2}}.$$

Denoting $R_0(\tau)/R_0(0) = \varrho(\tau)$, $A^2/2\sigma^2 = A^2/2N_0(f_2 - f_1) = q$, $(f_2 - f_1)/f_3 = Q$, one gets

$$(31) \qquad (s/n)_o = \frac{(qQ)^2 + \varrho^2(\tau)}{1 + 2qQ}.$$

Suppose τ chosen so that

$$(32) \qquad \omega_3\tau = 2\pi f_3\tau = K\pi\,, \quad (K = 1, 2, \ldots)\,.$$

Then

$$(33) \qquad f_3 = \frac{K}{2}\frac{1}{\tau}$$

and from (13)

$$(34) \qquad \varrho(\tau) = 0$$

This is the worst case. Practically, K somewhere between 2 and 4 must be chosen.

5. ANGLE ERROR DISTRIBUTION

Let the expression in (22) be much smaller than 1. Then the last term in (12) may be neglected, and, also approximately, (34) holds.

Dividing (12) by A^2, one gets

$$(35) \qquad z(t) = e^{i[\omega\tau + \Phi(t) - \Phi(t-\tau)]} + \frac{n_0(t)}{A} + \frac{\bar{n}_0(t-\tau)}{A}\,.$$

Both error (noise) terms are bivariate Gaussian, with mean values 0, independent mutually and on $\omega\tau$, and so is their sum, and the r.m.s. value of their quadrature components is from (15), (22) and (31) with $qQ \gg 1$

$$(36) \qquad \sigma_{xe}^2 = \sigma_{ye}^2 = \sigma_e^2 = \frac{2N_0 f_3}{A^2} = \frac{1}{qQ}\,.$$

And the probability that the endpoint of the unit lengtht vector lies in the angle interval $\langle \omega\tau - \varepsilon, \omega\tau + \varepsilon \rangle$ if $\Phi(t) - \Phi(t - \tau) = 0$ ($\langle \omega\tau - \varepsilon + \pi, \omega\tau + \varepsilon + \pi \rangle$ if $\Phi(t) - \Phi(t - \tau) = \pi$ is the same as the probability that the vector $z(t)$ falls in the angle $\langle \omega\tau - \varepsilon, \omega\tau + \varepsilon \rangle$ ($\langle \omega\tau - \varepsilon + \pi, \omega\tau + \varepsilon + \pi \rangle$).

For this probability, a formula ((5) in [7]), useful for numerical integration, has been derived by the author of the present article.

There is

$$(37)\quad \mathsf{P}(z(t) \in \langle \omega\tau - \varepsilon, \omega\tau + \varepsilon \rangle) = \sqrt{\frac{2}{\pi}} \int_0^\infty \mathrm{e}^{-(x-\sqrt{(qQ)})^2/2} \left(\frac{1}{\sqrt{(2\pi)}} \int_0^{x \operatorname{tg} \varepsilon} \mathrm{e}^{-y^2/2} \, \mathrm{d}y \right) \mathrm{d}x \,.$$

This probability is independent of $\omega\tau$, thus the relative error of the vector endpoint position is smaller for greater $\omega\tau$. However, practically, τ is limited from the top by τ_i (property (c)), and ω by other considerations.

An approximation to (37) has been proposed in [4] using the marginal one-dimensional Gaussian distribution from the error terms in (35), in the direction perpendicular to that given by the principal term in (35).

Since the original distribution is circularly symmetric, this approximation is

$$(38)\quad \mathsf{P}(z(t) \in \langle \omega\tau - \varepsilon, \omega\tau + \varepsilon \rangle) = 2[\Phi(\operatorname{tg} \varepsilon / \sigma_e) - \tfrac{1}{2}] = \\ = 2[\Phi(\sqrt{(qQ)} \,.\, \operatorname{tg} \varepsilon) - \tfrac{1}{2}] \,,$$

where Φ is the standard table Gaussian distribution function.

For ε not too great, the approximation is fairly good, as is seen from the following table.

Table 1. Probabilities from (37), (38).

ε°	qQ					
	16 (12 dB)		4 (6 dB)		1 (0 dB)	
	(37)	(38)	(37)	(38)	(37)	(38)
10	0·50	0·52	0·28	0·28	0·15	0·14
20	0·82	0·86	0·51	0·54	0·29	0·28
30	0·94	0·98	0·69	0·74	0·41	0·44

From (6), (31), (33), (36), (38) there is seen that the difficulties in receiving BPSK sequences are due to the reduction of the transmitted power in comparison with that used in the single pulse case.

6. REMARKS TO COMMUNICATION APPLICATION

In the communication case, ω is often known (with some small Doppler effect), thus there may be chosen $f_2 - f_1 = f_3$. And from (31), (34)

$$(39)\quad (s/n)_o = \frac{q^2}{1 + 2q} \,.$$

This is the principal term of the formula on page 193 in [5]. Often synchronization is possible or, as in the DPSK, τ is made dependent on ω so that $\cos \omega\tau = 0$ (or

$\sin \omega\tau = 0$), and only one channel reception can be used, a gain of 3 dB against the case of quadrature channel reception resulting.

Further, τ_i being known, $\tau = \tau_i$ may be chosen, resulting in smaller error according to (33), (36).

Finally, since each information bit of the signal is superposed on the whole sequence period of the length T, a further filtering and amelioration of the s/n ratio by the factor $T/\tau_i = N$ is possible.

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