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A Comparison of Three Optimality Criteria for Observation Channels

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A general observation channel $\{P, Q\}$ with two input symbols is considered. New estimates of the discrimination information $I(P, Q)$ in term of average probability of error are found. These estimates and previous estimated of the same kind as well as previous estimates of Hellinger's integral $H_\alpha(P, Q)$ of order $\alpha \in (0, 1)$ in term of average probability of error are numerically evaluated. Results are presented in a graphical form.

1. PROBLEM STATEMENT

Suppose that simple hypothesis $H_1, H_2, \ldots$ are tested on the basis of a statistics distributed by a probability distribution P_i on a measurable space $(X, \mathscr{X})$ provided H_i is true. In accordance with [1], the family $\{P_i\}$ can be interpreted as a general observation channel with the input or output space $\{1, 2, \ldots\}$ or X respectively. Thus the output $x \in X$ of the channel is supposed to be distributed by P_i iff (if and only if) the i-th symbol has been realized at the input.

In communication, radar detection, search for a location of an object, experiments design, and many other situations we are faced with a possibility to specify the channel within certain range given by a priori known circumstances, i.e. we can choose P_is arbitrarily from given distribution classes $\mathscr{P}_i$, $i = 1, 2, \ldots$

As optimum observation channel is usually considered that minimizing the probability of error corresponding to a priori given probabilities $\pi_1, \pi_2, \ldots$ of the inputs $1, 2, \ldots$ and to the best algorithm for identification of the inputs at the output.

However, simultaneously with the probability of error optimality criterion, another criteria have also been considered in the literature. This is motivated by frequent difficulties with minimization of the error probability over the Cartesian product $\mathscr{P}_1 \times \mathscr{P}_2 \times \ldots$ Interpretation of these criteria is similar (even if not so evident) as that of the probability of error and, at the same time, they are appreciably simpler for analytical treatment and numerical calculations than the probability of error.

In the present paper discrimination information and Hellinger's integral of order $0 < \alpha < 1$ are considered as optimality criteria and relations between them and the probability of error are studied.

2. OPTIMALITY CRITERIA

Throughout this paper we shall restrict ourselves to the two hypothesis case only, i.e. we shall consider observation channels of the form $\{P, Q\}$ only, where P, Q are probability distributions given on $(X, \mathscr{X})$ by Radon-Nikodym densities p, q with respect to a dominating finite measure μ on $(X, \mathscr{X})$. The P or Q is governing the distribution of the channel output depending on whether the hypothesis H or its alternative K is true respectively. The H and K are a priori expected with probabilities π and $1 - \pi \in (0, 1)$.

The probability of error $e_\pi(P, Q)$ is defined by

$$(1) \qquad e_\pi(P, Q) = \pi P(E) + (1 - \pi)\, Q(X - E) = \int_X \min\left[\pi p, (1 - \pi)\, q\right] \mathrm{d}\mu\,,$$

where $E \in \mathscr{X}$ denotes the subset of the channel outputs for which H is less a posteriori probable than K, $E = \{x \in X : \pi p < (1 - \pi)\, q\}$.

Remark 1. In what follows we shall write simply $e(P, Q)$ instead of $e_{1/2}(P, Q)$. In [2] it has been proved that

$$(2) \qquad 2 \min\left[\pi, 1 - \pi\right] e(P, Q) \leqq e_\pi(P, Q) \leqq 2 \max\left[\pi, 1 - \pi\right] e(P, Q)\,.$$

Using this inequality relations between various optimality criteria and $e_\pi(P, Q)$ can be reduced to the relations with the more simple $e(P, Q)$ even in the case $\pi \neq 1/2$.

Remark 2. Since the monotone relation

$$(3) \qquad V(P, Q) = 2[1 - 2e(P, Q)]$$

holds between $e(P, Q)$ and the total variation*

$$(4) \qquad V(P, Q) = \int_X |p - q|\, \mathrm{d}\mu = 2[Q(E) - P(E)]\,,$$

the total variation is usually used instead of $e(P, Q)$ in the literature. Recall that the range of values of $e_\pi(P, Q)$ or $V(P, Q)$ is the interval $[0, \min(\pi, 1 - \pi)]$ or $[0, 2]$ respectively, and that $e_\pi(P, Q) = 0$ or $V(P, Q) = 2$ iff P and Q are orthogonal on $(X, \mathscr{X})$ and $e_\pi(P, Q) = \min(\pi, 1 - \pi)$ or $V(P, Q) = 0$ iff P and Q are identical on $(X, \mathscr{X})$.

* The same E as in (1).

Another functional which has been used in the literature (see, for example, [3, 4]) as an optimality measure for the channel $\{P, Q\}$ is the I-divergence (or generalized Shannon's relative entropy) of P with respect to Q defined by

$$I(P, Q) = \int_X p \log p/q \, d\mu \tag{5}$$

or J-divergence $J(P, Q) = I(P, Q) + I(Q, P)$.

In [5, 6] Hellinger's integral $H_{1/2}(P, Q)$ and in [2, 7, 8] generalized Hellinger's integral or order α

$$H_\alpha(P, Q) = \int_X p^\alpha q^{1-\alpha} \, d\mu\,, \quad \alpha \in (0, 1)\,, \tag{6}$$

has been used as an optimality criterion for $\{P, Q\}$.

Remark 3. It is well-known that $I(P, Q)$ or $H_\alpha(P, Q)$ takes on values from the interval $[0, +\infty]$ or $[0, 1]$, where $I(P, Q) = 0$ and $H_\alpha(P, Q) = 1$ iff $P = Q$ on $\mathscr{X}$ and $H_\alpha(P, Q) = 0$ iff P and Q are orthogonal on $\mathscr{X}$. If P is not absolutely continuous with respect to Q on $\mathscr{X}$, then $I(P, Q) = +\infty$.

Since all three functionals $e_\pi(\cdot, \cdot)$, $I(\cdot, \cdot)$ and $H_\alpha(\cdot, \cdot)$ should serve for the same purpose one can ask into which extend the corresponding optimality criteria are equivalent or, in other words, which is the relation between quantities $e_\pi(P, Q)$, $I(P, Q)$, and $H_\alpha(P, Q)$. This problem, however, has many particular aspects, depending mainly on if or which restrictions are imposed on the a priori considered classes $\mathscr{P}$, $\mathscr{Q}$ of distributions P, Q.

In the present paper upper and lower bounds for I-divergence and Hellinger's integral of order $\alpha \in (0, 1)$ in term of $e_\pi(P, Q)$ or total variation $V(P, Q)$ are investigated provided no restrictions are imposed on $\mathscr{P}$, $\mathscr{Q}$.

3. NOTATION

Denote

$$U(V) = \sup_{V(P,Q)=V} I(P, Q)\,, \quad U(e \mid \pi) = \sup_{e_\pi(P,Q)=e} I(P, Q)\,, \tag{7}$$

$$U(e \mid \pi, \alpha) = \sup_{e_\pi(P,Q)=e} H_\alpha(P, Q)\,, \tag{8}$$

$$L(V) = \inf_{V(P,Q)=V} I(P, Q)\,, \quad L(e \mid \pi) = \inf_{e_\pi(P,Q)=e} I(P, Q)\,, \tag{9}$$

$$L(e \mid \pi, \alpha) = \inf_{e_\pi(P,Q)=e} H_\alpha(P, Q)\,, \tag{10}$$

for every real $\alpha, \pi \in (0, 1)$, $V \in [0, 2]$, and $w \in [0, \min(\pi, 1 - \pi)]$. The constraint infima and suprema are considered for all possible pairs P, Q on a non-trivial measurable space $(X, \mathscr{X})$. (It can be shown that the quantities in (7)–(10) are the same for any $(X, \mathscr{X})$ such that that there exists non-empty $E \in \mathscr{X}$ different from X.)

It follows from (3) that

$$U(V) = U\left(\frac{1}{2}\left[1 - \frac{V}{2}\right] \middle| \frac{1}{2}\right) \quad \text{or} \quad U(e \mid 1/2) = U(2(1 - 2e)) \tag{11}$$

and

$$L(V) = L\left(\frac{1}{2}\left[1 - \frac{V}{2}\right] \middle| \frac{1}{2}\right) \quad \text{or} \quad L(e \mid 1/2) = L(2(1 - 2e)). \tag{12}$$

4. RELATION BETWEEN $I(P, Q)$ AND $V(P, Q)$ OR $e_\pi(P, Q)$

As to the upper bounds $U(V)$, $U(e \mid \pi)$, they are not too interesting: $U(e \mid \pi) = +\infty$ (*i.e.* $U(V) = +\infty$) for $e \in [0, \min(\pi, 1 - \pi))$ (i.e. $V \in (0, 2]$) and $U(\min[\pi, 1 - \pi] \mid \pi) = 0$ (i.e. $U(0) = 0$). As to the lower bounds $L(V)$, $L(e \mid \pi)$, they have not been explicitely evaluated so far but in [9–14] attention has been paid to obtaining satisfactory lower estimates of the function $L(V)$ on the interval $V \in [0, 2]$. In this section new upper and lower estimates of $L(V)$ will be found and, using Remark 1, the corresponding estimates of $L(e \mid \pi)$ for $e \in [0, \min(\pi, 1 - \pi)]$, where $\pi \neq 1/2$, will be established, too.

For applications mentioned in Sec. 1 above, lower estimates of $L(V)$ are interesting in the first place. In [9] is has been shown that $L(V) \geqq L_1(V)$ for $V \in [0, 2]$, where

$$L_1(V) = V - \log(1 + V). \tag{13}$$

In [10] the inequality $L(V) \geqq L_2(V)$ on $[0, 2]$ has been proved with $L_2(V) = V^2/\Gamma$ and $\Gamma > 0$.

This result has been sharpened in [11, 12], where the same inequality has been proved with $\Gamma = 2$, i.e. with

$$L_2(V) = \frac{V^2}{2}. \tag{14}$$

The following lower estimate

$$L_3(V) = \frac{V^2}{2} + \frac{V^4}{36}, \tag{15}$$

which has been first stated in [13], is a corrected version of a lower estimate from [14]. In [13] the following estimate has also been stated:

$$L_4(V) = \log\frac{2 + V}{2 - V} - \frac{2V}{2 + V}. \tag{16}$$

The curves for all these estimates can be seen from Fig. 1. From the same figure one can also see the following upper estimates of $L(V)$ obtained in [13]:

$$L^1(V) = L_3(V) + \frac{V^4(14 + V^2)}{36(4 - V^2)}, \tag{17}$$

$$L^2(V) = L_4(V) + \frac{2V^3}{(2 + V)^2}, \tag{18}$$

$$L^3(V) = \frac{V}{2} \log \frac{2 + V}{2 - V}. \tag{19}$$

Remark 4. For all numerical calculations in this work we use Hewlett-Packard calculator model 9100 B. When using iterative procedures (for example for solving transcendental equations) we use the accuracy of about 10^{-4}.

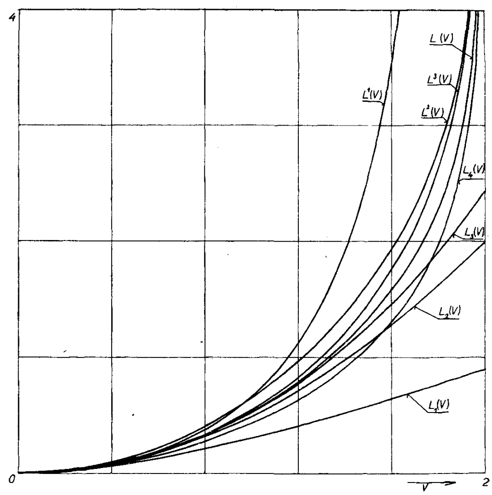

Fig. 1.

All the results mentioned above are based on the fact that

$$I(P, Q) \geqq P(E) \log \frac{P(E)}{Q(E)} + (1 - P(E)) \log \frac{1 - P(E)}{1 - Q(E)},$$

for every $E \in \mathscr{X}$. Setting here the same E as in (4) or (1) and applying (4) we obtain

$I(P, Q) \geqq \psi(P(E), V(P, Q))$, where $\psi(x, V)$ is defined for $V \in [0, 2]$ and $x \in [0, 1 - V/2]$ by

$$\psi(x, V) = x \log \frac{x}{x + V/2} + (1 - x) \log \frac{1 - x}{1 - x - V/2}.$$

The curves for $\psi(x, V)$ are given in Fig. 2. One can see that for any $V \in [0, 2)$ there exists exactly one $x = x(V)$ minimizing $\psi(x, V)$ on $[0, 1 - V/2]$, and that

$$L(V) = \min_{x \in [0, 1 - V/2]} \psi(x, V) = \psi(x(V), V). \tag{20}$$

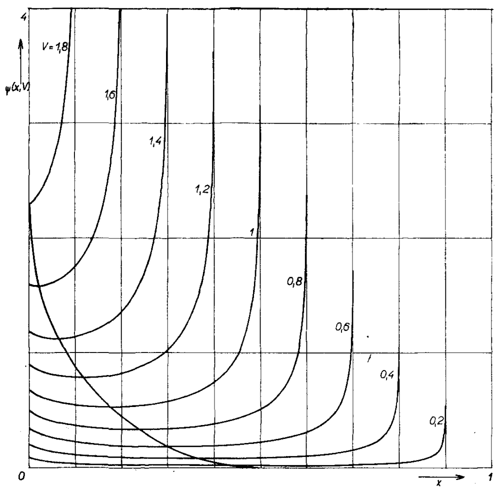

Fig. 2.

Unfortunately, the function $x(V)$ of the minimizing x's has not been explicitely evaluated so far because of the equation

$$\frac{\partial \psi(x, V)}{\partial x} = 0 \quad \text{for fixed} \quad V \in [0, 2)$$

is transcendental. Numerical evaluation of this function can be seen in Fig. 2 and Fig. 3. Using this numerical evaluation $L(V) = \psi(x(V), V)$ has been evaluated in Fig. 1.

In this situation we can proceed by the following manner. We shall try to find out estimates $l(V) \leqq x(V) \leqq u(V)$, $V \in [0, 2)$, and then, since $\psi(x, V)$ is convex on

$[0, 1 - V/2]$, to estimate $\psi(x(V), V)$ by the point of intersection of the tangents $t_{l(V)}(x)$, $t_{u(V)}(x)$ of the curve $\psi(x, V)$ in the points $x = l(V)$, $x = u(V)$. If we denote by $\psi'(x, V)$ the partial derivative $\partial\psi(x, V)/\partial x$, then the tangent $t_{x_0}(x)$ of $\psi(x, V)$ in the point $x = x_0$ is given by the equation

$$t_{x_0}(x) = \psi'(x_0, V)\, x + \psi(x_0, V) - \psi'(x_0, V)\, x_0 .$$

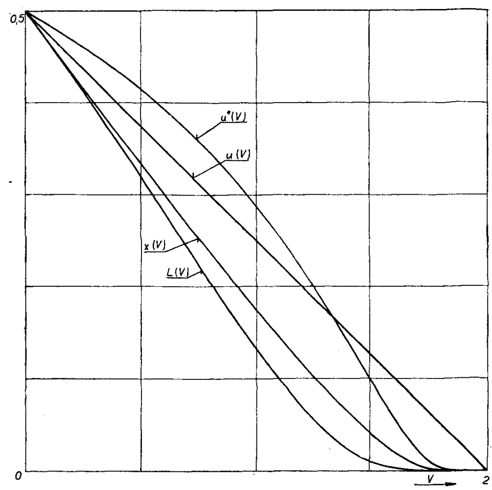

Fig. 3.

Therefore the x-coordinate $\tilde{x}(V)$ of the point of intersection of $t_{l(V)}(x)$ and $t_{u(V)}(x)$ is given by

$$\tilde{x}(V) = \frac{\psi(l(V), V) - \psi(u(V), V) + \psi'(u(V), V)\, u(V) - \psi'(l(V), V)\, l(V)}{\psi'(u(V), V) - \psi'(l(V), V)}$$

and it holds

(21) $$t_{l(V)}(\tilde{x}(V)) \leqq L(V) \leqq \psi(\tilde{x}(V), V) .$$

Hence we obtain by this way the following lower or upper estimates of $L(V)$:

(22) $$L_5(V) = t_{l(V)}(\tilde{x}(V)) = t_{u(V)}(\tilde{x}(V))$$

or

(23) $$L^4(V) = \psi(\tilde{x}(V), V)$$

respectively.

In order to obtain the corresponding explicite formulas for $L_5(V)$ and $L^4(V)$ we shall prove the following

Lemma. *It holds* $l(V) \leqq x(V) \leqq u(V)$ *on* $[0, 2)$ *for*

$$l(V) = \frac{1}{2} \exp\left[-\frac{4V}{3(2-V)}\right], \tag{24}$$

$$u(V) = \frac{1}{2}\left(1 - \frac{V}{2}\right). \tag{25}$$

Proof. The inequality $x(V) \leqq u(V)$ for $u(V)$ given by (25) has been proved in [13] (this inequality has been used to obtain the lower estimate (16)).

As to the inequality $l(V) \leqq x(V)$ it is easy to see that

$$\lim_{V \to 0^+} \psi'(l(V), V) = 0 \tag{24}$$

for $\psi'(x, V) = \partial\psi(x, V)/\partial x$ and for $l(V)$ given by (24). We shall prove that $\psi'(l(V), V)$ is decreasing for $V \in (0, 2)$ i.e. that

$$\frac{\mathrm{d}}{\mathrm{d}V} \psi'(l(V), V) < 0\,, \quad V \in (0, 2)\,. \tag{25}$$

This together with (24) implies that $\psi'(l(V), V) \leqq 0$ for $V \in [0, 2]$. Since $\psi(x, V)$ is convex on $[0, 1 - V/2]$ and $\psi'(x(V), V) = 0$, the inequality $\psi'(l(V), V) \leqq 0$ implies $l(V) \leqq x(V)$, q.e.d.

Thus it remains to prove (25) only. Instead of (25) we shall prove equivalent relation

$$\frac{\mathrm{d}}{\mathrm{d}u} \psi'(l(2u), 2u) < 0\,, \quad u \in (0, 1)\,. \tag{26}$$

Evaluating the derivative, after obvious manipulations we obtain, that (25) is equivalent to the following inequality

$$\mathscr{L}(u) = -Al^2 + Bl + C < 0\,, \quad u \in (0, 1)\,, \tag{27}$$

where

$$l = l(2u) = \frac{1}{2} \exp\left[-\frac{4}{3}\frac{u}{1-u}\right],$$

$$A = A(u) = 6 + 6u^2\,,$$

$$B = B(u) = 9 - 12u + 5u^2 - 6u^3\,,$$

$$C = C(u) = -3 + 8u - 7u^2 + 2u^3\,.$$

 Denote

$$I_1 = \{u \in (0, 1): B > 0, B^2 + 4AC \geqq 0\},$$

$$I_2 = \{u \in (0, 1): B < 0, B^2 + 4AC \geqq 0\}.$$

We shall prove that

$$l < \frac{B - \sqrt{(B^2 + 4AC)}}{2A} \quad \text{on} \quad I_1, \tag{28}$$

$$l > \frac{B + \sqrt{(B^2 + 4AC)}}{2A} \quad \text{on} \quad I_2. \tag{29}$$

These two statements imply (27) because, since $\mathscr{L}(0) = \mathscr{L}(1) = 0$, $\mathscr{L}(1/2) < 0$, $\mathscr{L}(u) > 0$ for some $u \in (0, 1)$ implies that there exists $\tilde{u} \in (0, 1)$ such that $\mathscr{L}(\tilde{u}) = 0$, i.e. that $l(2\tilde{u})$ is a root of the equation

$$-A(\tilde{u})\, x^2 + B(\tilde{u})\, x + C(\tilde{u}) = 0$$

or, in other words, that

$$l(2\tilde{u}) \in \left\{\frac{B(\tilde{u}) - \sqrt{(B(\tilde{u})^2 + 4A(\tilde{u})\, C(\tilde{u}))}}{2A(\tilde{u})}, \frac{B(\tilde{u}) + \sqrt{(B(\tilde{u})^2 + 4A(\tilde{u})\, C(\tilde{u}))}}{2A(\tilde{u})}\right\}.$$

Proof of (28): By an application of the inequality $\exp(y) \leqq (1 - y)^{-1}$, which holds for all $y \in (-\infty, +\infty)$, we get

$$l(2u) \leqq \frac{1}{2} \frac{1}{1 + \frac{4}{3}\frac{u}{1 - u}} = \frac{3}{2}\frac{1 - u}{3 + u} = \Phi(u).$$

We shall prove that

$$\Phi < \frac{B - \sqrt{(B^2 - 4AC)}}{2A} \quad \text{on} \quad I_1.$$

Since $A\Phi^2 + C$ has no zero point on I_1, and $A > 0$ on $(0, 1)$, it holds on I_1 that $B^2 > -4AC > 4A^2\Phi^2$. Under this condition the last relation is equivalent to the following series of relations:

$$B - 2A\Phi > \sqrt{(B^2 + 4AC)} \quad \text{on} \quad I_1,$$

$$\Phi^2 A > \Phi B + C \quad \text{on} \quad I_1,$$

$$9(1 - u)^2 A > 6(1 - u)(3 + u) B + 4(3 + u)^2 C > 0 \quad \text{on} \quad I_1,$$

$$52u^3 - 8u^4 + 44u^5 > 0 \quad \text{on} \quad I_1.$$

The last of this relations evidently holds.

Proof of (29): Since, evidently, $A > 0$, $C < 0$ on $(0, 1)$, $\sqrt{(B^2 + 4AC)} < |B|$.
Hence, on I_2, $B + \sqrt{(B^2 + 4AC)} < |B| + B = 0$, and, consequently,

$$\frac{B + \sqrt{(B^2 + 4AC)}}{2A} < 0 .$$

Thus (29) holds.

Thus, we have proved the following

Theorem 1. *It holds* $L_5(V) \leqq L(V) \leqq L^4(V)$ *on the interval* $[0, 2]$ *for functions defined by* (22), (23) *with* $l(V)$, $u(V)$ *given by* (24), (25).

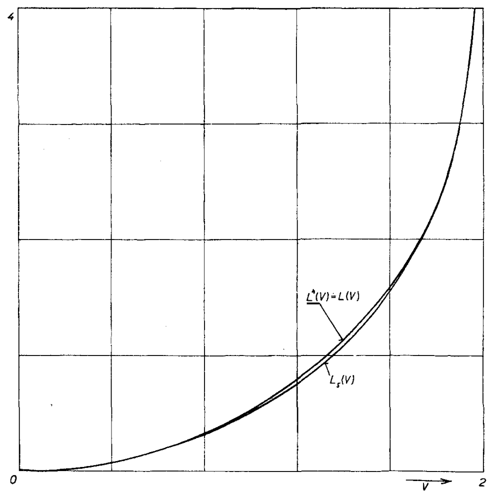

Fig. 4.

The curves for $L_5(V)$, $L^4(V)$ are given in Fig. 4 ($L^4(V)$ is in this figure identical with the numerically calculated $L(V)$. One can see from this figure that the analytic expressions $L_5(V)$, $L^4(V)$ are better than any of the estimates $L_i(V)$, $L^i(V)$ listed above and that $L^4(V)$ can be used as a satisfactory approximation for $L(V)$.

Remark 5. Instead of (25)

$$u^*(V) = \frac{1}{2} \exp\left[-\frac{11V}{20(2 - V)}\right]$$

might be used in Theorem 1. This substitution to (22), (23) yields similar estimates

$L_6(V)$, $L^5(V)$ as $L_5(V)$, $L^4(V)$ in Theorem 1. The behaviour of $u^*(V)$ is shown in Fig. 3. The proof of the inequality $x(V) \leqq u^*(V)$ can be based on the same idea as that used in the proof of (24).

Applying Remark 1 and taking into account the fact all the estimates $L_i(V)$, $L^i(V)$ above are monotonic functions of V we obtain the following result:

Theorem 2. *For every* $\pi \in (0, 1)$ *and* $0 \leqq e \leqq \min(\pi, 1 - \pi)$ *it holds*

$$(26)\quad L_i\left(2\left[1 - \frac{2e}{\min(\pi, 1-\pi)}\right]\right) \leqq L(e \mid \pi) \leqq L^j\left(2\left[1 - \frac{2e}{\max(\pi, 1-\pi)}\right]\right),$$

where $L_i(V)$, $L^j(V)$ *are arbitrary of the estimates considered above.*

5. RELATION BETWEEN $H_\alpha(P, Q)$ AND $e_\pi(P, Q)$

This relation which will be here expressed in terms of $U(e \mid \pi, \alpha)$, $L(e \mid \pi, \alpha)$ introduced in (8), (10), has been previously studied in [5, 6] and the most recent results have been published in [7, 8, 2]. The following result follows from [2] (as to the inequality (28), it can be deduced from [8] as well).

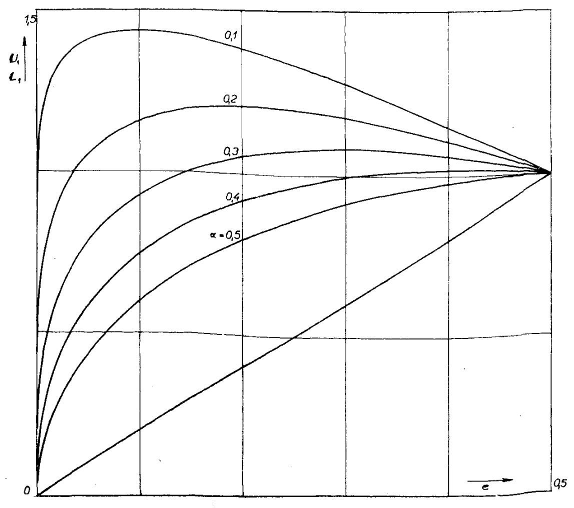

Fig. 5. $\pi = 0{\cdot}5$

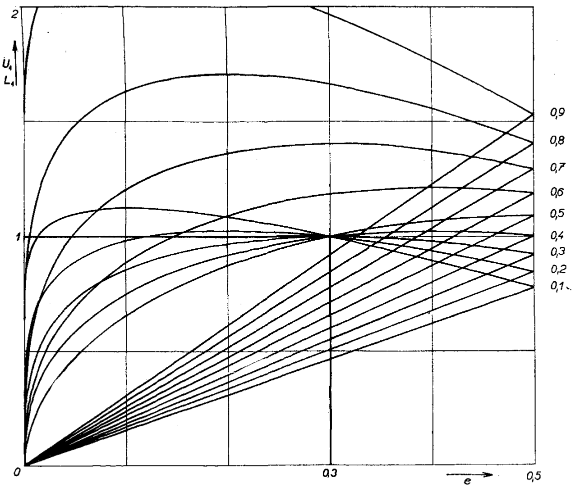

Fig. 6. $\pi = 0{\cdot}3$

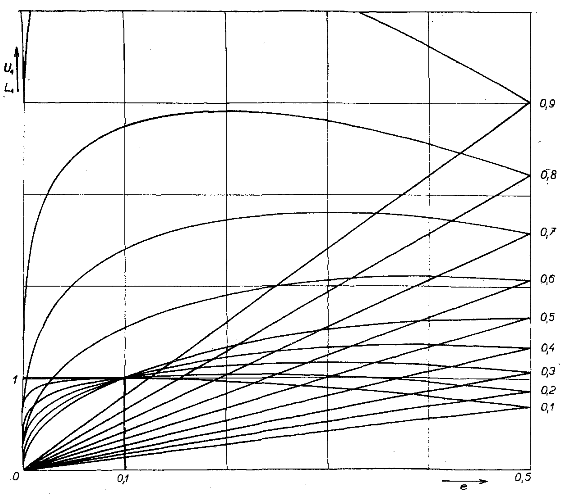

Fig. 7. $\pi = 0{\cdot}1$

Theorem 3. *For every $\alpha, \pi \in (0, 1)$ and every $0 \leqq e \leqq \min(\pi, 1 - \pi)$ it holds*

$$U(e \mid \pi, \alpha) \leqq U_1(e \mid \pi, \alpha) = \pi^{-\alpha}(1 - \pi)^{\alpha - 1}\, e^{\min(\alpha, 1-\alpha)}(1 - e)^{\max(\alpha, 1-\alpha)}, \tag{27}$$

$$L(e \mid \pi, \alpha) \geqq L_1(e \mid \pi, \alpha) = \pi^{-\alpha}(1 - \pi)^{\alpha - 1}\, e. \tag{28}$$

The curves $U_1(e \mid \pi, \alpha)$ and $L_1(e \mid \pi, \alpha)$ are shown in the following three figures for $\pi = 0{\cdot}5$, $0{\cdot}3$, and $0{\cdot}1$, and α varying between 0 and 1.

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