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DECENTRALIZED ADAPTIVE PI CONTROL STRATEGIES FOR FREQUENCY CONTROL OF INTERCONNECTED SYSTEMS

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Algorithms for load frequency control of interconnected power systems in the presence of known constant plus time-varying disturbances are investigated. First, we have developed a decentralized PI controller based on decentralized state estimation. Controller based estimator is designed by exploiting the combine advantage of linear quadratic problem and eigenvalue assignment problem. PI controller does not work satisfactorily in the presence of time-varying disturbances. Then, we proposed an effective decentralized control scheme which is a combination of PI controller plus an adaptive controller. It is implemented on a hybrid computer. Hybrid computer control techniques combine the decision, storage and branching capabilities of the digital machine with the high speed solution of dynamic system equation on the analog machine. Simulation results present the advantages of the procedure on the two-area power system control design.

1. INTRODUCTION

Control of large inter-connected power system has drawn wide attention in the literature [1–4]. The primary function of an electric power system is to provide the real and reactive powers demanded by the various loads connected to the system. The load changes affects the frequency and magnitude of the bus voltage. The problem of load-frequency control (LFC) arises in multi-area power system in view of the dependence of the frequency of the generated voltages on the active power (or Load) demand on the system. A proportional integral PI feedback control law is generally required for ensuring that the actual frequency of the generator output will settle down to the desired value in the presence of small variation in the load demand on the given system. An application of the linear quadratic (LQ) optimal control theory for the design of a PI control law for multiarea power systems has been first proposed by Fosha and Elgerd [1]. These authors have assumed the load variations to be purely deterministic while in practice it is more logical to treat these variations as slow time-varying in nature. An application of the linear quadratic Gaussian (LQG) optimal control theory for designing PI control law in such cases has been discussed in [3] and [5]. However, the increasing complexity of modern power system has led to a greater dependence on automatic control at all levels of operation. A large number of investigators have pointed out that the implementation of centralized controller possesses certain difficulties when the complexity of interconnected areas increases. Specially, these difficulties can be traced to the need of enormous instrumentation and telementering of the required data to a central processing unit, which for the large-scale power system preclude any direct approach to centralized control due to the geographical distribution of power plants.

A class of research works deals with decentralized load frequency control schemes where the composite system is decomposed into several sub-systems, each of which is controlled separately on the basis of local information only. The first attempts in this sense were made by Sanders et al. [6], Calovic [7], Venkateswarlu and Mahalanabis [8], Park and Lee [9], Davison and Tripathi [10] and Bakule [11]. These works have faced some difficulties in order to implement a good quality of controller while the 'operating point' of a power system changes due to slow variation in system parameters and also for time-varying load disrurbances. Fortunately, these types of problems have been studied recently by employing adaptive control that adapts to changing system characteristics and has so much potential to improve power system performance [12–14].

In this paper, our aim is to implement a linear quadratic decentralized PI control scheme with prescribed close-loop eigenvalues for load frequency control problem while the power system is subjected to known constant plus time-varying load disturbances. Subsequently, we have been interested to examine the feasibility of developing a decentralized adaptive load frequency control scheme for the same system under the similar environmental conditions. This paper is organised as follows:

Section 2 deals with precise mathematical statement of the problem. In Section 3, we consider the development of decentralized PI controller based on decentralized state estimation with prescribed eigenvalues while the power system is under known constant plus time-varying disturbances. Section 4 deals with the development of an effective adaptive decentralized controller which is based on decentralized state estimation while the considered system subjected to the same environmental conditions. In Section 5, some simulation results of the proposed control algorithms are presented by considering load-frequency control problem of two-area power system model. Finally, some concluding remarks will be included in Section 6.

2. PROBLEM STATEMENT

Consider a multi-input-multi-output linear time-invariant continuous time interconnected power system, whose model is described by the following pair of equations:

$$\dot{X}(t) = F\,X(t) + G\,U(t) + \Gamma(D + E_D(t)) \qquad (1)$$

$$Y(t) = H\,X(t) \qquad (2)$$

where $X(t)$, $U(t)$ and $Y(t)$ are n, m and p dimensional states, inputs and outputs respectively and the constant matrices F, G, Γ and H have appropriate dimensions. The term $(D + E_D(t))$ represents known constant disturbance vector D and time-varying disturbance vector $E_D(t)$ of the same dimension as $U(t)$. The effect of parameter variation of system matrices from nominal values can be treated as time-varying disturbances to the state equation. The composite system is an N-area interconnected power system, where the ith area is described by the following pair of equations:

$$\dot{X}_i(t) = F_{ii} X_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} F_{ij} X_j(t) + G_i U_i(t) + \Gamma_i(D_i + E_{Di}(t)) \tag{3}$$

$$Y_i(t) = H_i X_i(t) \tag{4}$$

where $X_i(t) \in \mathbb{R}^{n_i}$ is the state vector, $U_i(t) \in \mathbb{R}^{m_i}$ is the control vector, $(D_i + E_{Di}(t)) \in \in \mathbb{R}^{m_i}$ is the constant known plus time-varying disturbances vector and $Y_i(t) \in \mathbb{R}^{p_i}$ is the output vector of the ith subsystem. It is assumed that the pairs (F_{ii}, G_i) and (F_{ii}, H_i) are completely controllable and observable.

The aim of this paper is to perform decentralized design of feedback control for the system (3), (4) so that the global closed-loop system should be stable and the closed-loop subsystems should satisfy given dynamical requirements. The closed-loop subsystems are designed according to the following two different specific local problem formulations. The philosophy of the decentralized design is described for instance in [15].

Problem 1. Let us consider the ith area power system model given by the equations (3)−(4) is subjected to a known constant plus time-varying disturbances. Such of N-subsystem comprises a composite system whose dynamic equations are given by (1)−(2). It is desired to implement a decentralized controller based on decentralized state estimator so that the control law $U_i(t)$, $t \geqq 0$ minimizes the performance index (note that $Z_i(t) = \int Y_i(t)\,\mathrm{d}t$)

$$J_{ai}(t_0, X_{ai}(t), U_i(t)) = \int_0^\infty \{[X_i'(t) \mid Z_i'(t)]\, Q_{ai} [X_i'(t) \mid Z_i'(t)]' + U_i'(t)\, R_i U_i(t)\}\, \mathrm{d}t$$
$$\text{for} \quad i = 1, 2, \ldots, N \tag{5}$$

of the interaction free ith augmented subsystem and moreover, ith subsystem controller/estimator eigenvalues are placed at prescribed locations. The performance index of the global system will be discussed in the next section.

Problem 2. The second problem studied by us corresponds to a decentralized adaptive and PI controller for the ith subsystem which is subjected to known constant plus time varying disturbances. Under such environmental condition, it is desirable to implement a decentralized adaptive PI controller which is a combination of two linear parallel controllers that are designed by minimizing two different performance indices to ensure some important states and outputs are as close as possible to the desired values (say; 0 to 0·4% of desired values).

3. DECENTRALIZED PI LOAD-FREQUENCY CONTROL SCHEME BASED ON PRESCRIBED EIGENVALUES

A two-level controller for multi area power systems was proposed by Bakule [11] to prescrible the controller eigenvalues at desired locations. But we will adopt here, a different approach to implement a decentralized controller for the ith subsystems with prescribed eigenvalues in presence of constant and time varying disturbances.

To implement such a decentralized control law $U_i(t)$ for the ith sub-system (3)–(4), the augmented state and output equation for the ith subsystem can be written as

$$\dot{X}_{ai}(t) = F_{aii} X_{ai}(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} F_{aij} X_{aj}(t) + G_{ai} U_i(t) + \Gamma_{ai}(D_i + E_{D_i}(t)) \tag{6}$$

$$Y_i(t) = H_{ai} X_{ai}(t) \tag{7}$$

with a new additional state vector

$$Z_i(t) = \int Y_i(t)\,\mathrm{d}t\,, \quad \text{for} \quad i = 1, 2, \ldots, N \tag{8}$$

and

$$X_{ai}(t) = [X_i'(t) \mid Z_i'(t)]'$$

The matrices F_{aii}, F_{aij}, G_{ai}, Γ_{ai} and H_{ai} have the following form:

$$F_{aii} = \begin{bmatrix} F_{ii} & 0 \\ H_i & 0 \end{bmatrix}; \quad F_{aij} = \begin{bmatrix} F_{ij} & 0 \\ 0 & 0 \end{bmatrix}$$

$$G_{ai} = \begin{bmatrix} G_i \\ 0 \end{bmatrix}; \quad \Gamma_{ai} = \begin{bmatrix} \Gamma_i \\ 0 \end{bmatrix} \quad \text{and} \quad H_{ai} = [H_i \mid 0]$$

The solution of decentralized control problem based on decentralized state estimation can be obtained by using the following decoupled model of the ith augmented system

$$\dot{X}_{ai}(t) = F_{aii} X_{ai}(t) + G_{ai} U_i(t) + \Gamma_{ai}(D_i + E_{D_i}(t)) \tag{9}$$

$$Y_i(t) = H_{ai} X_{ai}(t) \tag{10}$$

A unique solution $U_i(t)$ for the above problem exists and it can be obtained in the following manner by minimizing the quadratic performance index

$$J_{ai}(t_0, X_{ai}(t), U_i(t)) = \int_0^\infty [X_{ai}'(t)\, Q_{ai}\, X_{ai}(t) + U_i'(t)\, R_i\, U_i(t)]\,\mathrm{d}t \tag{11}$$

where $R_i > 0$ and $Q_{ai} \geqq 0$.

Decentralized control for the global system can be developed by minimizing such N decoupled quadratic performance indices. The overall performance index for the global system can be put in the following form

$$J_{as}(t_0, X_a(t), U(t)) = \sum_{i=1}^{N} (J_{ai}(t_0, X_{ai}(t), U_i(t))) \tag{12}$$

The problem is now to find out the desired state weighting matrix Q_{ai} for the ith augmented subsystem and subsequently to calculate the control law $U_i(t)$ in such a way that the performance index (11) for each augmented subsystem (9)–(10) is minimized and moreover close-loop eigenvalues for each subsystem are placed at prespecified locations. The desired control law $U_i(t)$ for the system (9)–(10) and (11) can be written as [16]

$$\begin{aligned} U_i(t) &= -K_{ai}\,\hat{X}_{ai}(t)\,; \quad \text{for} \quad i = 1, 2, \ldots, N \\ &= -K_{ai1}\,\hat{X}_i(t) - K_{ai2}\int_0^t Y_i(t)\,\mathrm{d}t \end{aligned} \tag{13}$$

where, the ith subsystem controller gain K_{ai} can be obtained using the procedure of Solheim [17] to assign controller eigenvalues at any desired locations.

Once the matrix K_{ai} is known, one can easily implement the decentralized control law (13) by estimating the states $X_i(t)$ of the ith sub-system and also placing the eigenvalues of the estimator at desired locations. It is a well-known fact that the state estimation problem is a dual of control problem. So, it is necessary to introduce some fictitious process noises in the state and output equations with noise covariances W_i and V_i respectively, in order to develop the state estimation algorithm. Without dealing with details, one can directly use the procedure by Solheim [17] to estimate the states $X_i(t)$ of the ith sub-system while the following basic symbols are made changes in the control algorithm:

$$\begin{aligned} &Q_{ai} \to W_i\,; \quad K_{ai} \to L_i' \\ &R_i \to V_i\,; \quad F_{aii} \to F_{ii}'\,; \quad G_{ai} \to H_i' \end{aligned}$$

If these changes are made, we obtain the final value of the estimator gain L_i which will place estimator eigenvalues at desired locations. The estimated state $\hat{X}_i(t)$ can be obtained using the following equation:

$$\begin{aligned} \dot{\hat{X}}_i(t) = {} & [F_{ii} - L_i H_i]\,\hat{X}_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} F_{ij}\,\hat{X}_j(t) + G_i\,U_i(t) + \\ & + \Gamma_i(D_i + E_{D_i}(t)) + L_i\,Y_i(t) \end{aligned} \tag{14}$$

where L_i is the estimator gain of the ith subsystem. Now by using (3)–(4) and (14), it is possible to find an expression for the estimator error equation

$$\begin{aligned} &\dot{e}_i(t) = X_i(t) - \hat{X}_i(t) = [F_{ii} - L_i H_i]\,e_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} F_{ij}\,e_j(t) \\ &\text{for} \quad i = 1, 2, \ldots, N \end{aligned} \tag{15}$$

This expression for $i = 1, 2, \ldots, N$ can be equivalently expressed for the global system as

$$\dot{e}(t) = \begin{bmatrix} F_{11} - L_1 H_1 & 0 & \ldots & 0 \\ 0 & F_{22} - L_2 H_2 & \ldots & 0 \\ 0 & & & 0 \\ 0 & & & F_{NN} - L_N H_N \end{bmatrix} e(t) + \begin{bmatrix} 0 & F_{12} & F_{13} & \ldots & F_{1N} \\ F_{21} & 0 & F_{23} & \ldots & F_{2N} \\ \vdots & & & & \\ F_{N1} & \ldots & & F_{N,N-1}, & 0 \end{bmatrix} e(t) \tag{16}$$

The convergence of the estimator error equation (16) is discussed in detail in [18]. Although the decentralized control scheme is attractive from computational point of view, one should not overlook the drawback of the scheme, where the information exchange between the sub-estimator is necessary in order to obtain the ith subsystem estimated state $\hat{X}_i(t)$. Moreover, controller does not act to the system effectively to maintain some of important states and outputs within the pre-specified limits. The performance of the controller can be improved considerably by introducing a decentralized adaptive controller in parallel with the PI controller.

Stability analysis for the global system

Substituting equation (13) into the equations (14) and (7), the following equations are obtained:

$$\dot{\hat{X}}_i(t) = [F_{ii} - L_iH_i - G_iK_{ai1}]\,\hat{X}_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} F_{ij}\,\hat{X}_j(t) - G_i K_{ai2}\,\hat{Z}_i(t) + \Gamma_i(D_i + E_{D_i}(t)) + L_i\,Y_i(t) \tag{17}$$

$$\dot{\hat{Z}}_i(t) = \hat{Y}_i(t) = H_i\,\hat{X}_i(t) \tag{18}$$

Augmenting the equations (17) and (18), we obtain

$$\begin{bmatrix} \dot{\hat{X}}_i(t) \\ \dot{\hat{Z}}_i(t) \end{bmatrix} = \dot{\hat{X}}_{ai}(t) = \left[\begin{array}{c|c} F_{ii} - L_iH_i - G_iK_{ai1} & -G_iK_{ai2} \\ \hline H_i & 0 \end{array}\right] \hat{X}_{ai}(t) + \left[\begin{array}{cc} L_iH_i & 0 \\ \hline 0 & 0 \end{array}\right] X_{ai}(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} \left[\begin{array}{c|c} F_{ii} & 0 \\ \hline 0 & 0 \end{array}\right] \hat{X}_{aj}(t) + \left[\begin{array}{c} \Gamma_i \\ \hline 0 \end{array}\right](D_i + E_{D_i}(t))$$

$$\dot{\hat{X}}_{ai}(t) = F^*_{aii}\,\hat{X}_{ai}(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} F_{aij}\,\hat{X}_{aj}(t) + \Gamma_{ai}(D_i + E_{D_i}(t)) + L^*_{ai}\,X_{ai}(t) \tag{19}$$

One can write combinely equations (19) and (6) in the following form:

$$\begin{bmatrix} \dot{X}_{a1}(t) \\ \dot{X}_{a2}(t) \\ \vdots \\ \dot{X}_{aN}(t) \\ \hline \dot{\hat{X}}_{a1}(t) \\ \dot{\hat{X}}_{a2}(t) \\ \vdots \\ \dot{\hat{X}}_{aN}(t) \end{bmatrix} = \left[\begin{array}{cccc|ccccc} F_{a11} & F_{a12} & \dots & F_{a1N} & -G_{a1}K_{a1} & 0 & 0 & \dots & 0 \\ F_{a21} & F_{a22} & \dots & F_{a2N} & 0 & -G_{a2}K_{a2} & 0 & \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & & \\ F_{aN1} & F_{aN2} & & F_{aNN} & 0 & 0 & & & -G_{aN}K_{aN} \\ \hline L^*_{a1} & 0 & \dots & 0 & F^*_{a11} & F_{a12} & F_{a13} & \dots & F_{a1N} \\ 0 & L^*_{a2} & \dots & 0 & F_{a21} & F^*_{a22} & F_{a23} & \dots & F_{a2N} \\ \vdots & \vdots & & & \vdots & \vdots & & & \\ 0 & 0 & \dots & L^*_{aN} & F_{aN2} & F_{aN2} & \dots & \dots & F^*_{aNN} \end{array}\right].$$

$$\cdot \begin{bmatrix} X_{a1}(t) \\ X_{a2}(t) \\ \vdots \\ X_{aN}(t) \\ \hline \hat{X}_{a1}(t) \\ \hat{X}_{a2}(t) \\ \vdots \\ \hat{X}_{aN}(t) \end{bmatrix} + \begin{bmatrix} \Gamma_{a1}(D_1 + E_{D1}(t)) \\ \Gamma_{a2}(D_2 + E_{D2}(t)) \\ \vdots \\ \Gamma_{aN}(D_N + E_{DN}(t)) \\ \hline \Gamma_{a1}(D_1 + E_{D1}(t)) \\ \Gamma_{a1}(D_1 + E_{D2}(t)) \\ \vdots \\ \Gamma_{aN}(D_N + E_{DN}(t)) \end{bmatrix} \tag{20}$$

The combined state and error equation of the augmented composite system can be obtained from equation (20) as follows:

$$\begin{bmatrix} \dot{X}_{a1}(t) \\ \dot{X}_{a2}(t) \\ \vdots \\ \dot{X}_{aN}(t) \\ \hline \dot{e}_{a1}(t) \\ \dot{e}_{a2}(t) \\ \vdots \\ \dot{e}_{aN}(t) \end{bmatrix} = \left[\begin{array}{cccc|cccc} F_{a11}-G_{a1}K_{a1} & F_{a12} & \dots & F_{a1N} & G_{a1}K_{a1} & 0 & 0 \dots & 0 \\ F_{a21} & F_{a22}-G_{a2}K_{a2} & \dots & F_{a2N} & 0 & G_{a2}K_{a2} & 0 \dots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \\ F_{aN1} & F_{aN2} & & F_{aNN}-G_{aN}K_{aN} & 0 & 0 & \dots & G_{aN}K_{aN} \\ \hline 0 & 0 & & 0 & F_{a11}-L^*_{a1} & F_{a12} & \dots & F_{a1N} \\ 0 & 0 & & 0 & F_{a21} & F_{a22}-L^*_{a2} & & F_{a2N} \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \\ 0 & 0 & & 0 & F_{aN1} & F_{aN2} & & F_{aNN}-L^*_{aN} \end{array}\right] \begin{bmatrix} X_{a1}(t) \\ X_{a2}(t) \\ \vdots \\ X_{aN}(t) \\ \hline e_{a1N} \\ e_{a2}(t) \\ \vdots \\ e_{aN}(t) \end{bmatrix} + \begin{bmatrix} \Gamma_{a1}(D_1 + E_{D1}(t)) \\ \Gamma_{a2}(D_1 + E_{D2}(t)) \\ \vdots \\ \Gamma_{aN}(D_N + E_{DN}(t)) \\ \hline 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{21}$$

The system (21) will behave as stable one, if all the eigenvalues of the upper and lower diagonal blocks lie in the left half of the s-plane. Using the equation (21), the ith subsystem estimator error equations is written in simplified form:

$$\dot{e}_i(t) = [F_{ii} - L_iH_i]\, e_i(t) + \sum_{\substack{j=1 \\ j \neq i}}^{N} F_{ij}\, e_j(t) = [F_{ii} - L_iH_i]\, e_i(t) + g_i(t, e(t)) \tag{22}$$

where, $e_i(t) = X_i(t) - \hat{X}_i(t)$, $\quad L_i = N_iH_i'V_i^{-1}$
and $g_i(t, e(t))$ = interaction error terms from other subsystems. The estimator gain matrix L_i can be obtained by solving a matrix Riccati equation as given below:

$$N_iF_{ii}' + F_{ii}N_i - N_iH_i'V_i^{-1}H_iN_i + W_i = [0] \tag{23}$$

Expression (22) and (23) are equivalently expressed (for $i = 1, 2, \dots, N$) as

$$\dot{e}(t) = [F_a - LH]\, e(t) + g(t, e(t)) \tag{24}$$

where

$$g(t, e(t)) = [g'_1(t, e(t)), g'_2(t, e(t)), \ldots, g'_N(t, e(t))]'$$

is a linear vector,

$$F_d = \text{Block-diag}\,[F_{11}, F_{22}, \ldots, F_{NN}]$$
$$N = \text{Block-diag}\,[N_1, N_2, \ldots, N_N]$$
$$H = \text{Block-diag}\,[H_1, H_2, \ldots, H_N]$$
$$V = \text{Block-diag}\,[V_1, V_2, \ldots, V_N]$$

and $L = NHV^{-1}$.

The stability condition for the system (24) employing subsystem interconnection structure is given by [18]

$$\underset{i}{\text{Min}}\,\{\chi_{\min}(S_i)\} \geqq 2\,\partial\,\underset{i}{\text{Max}}\,\{\chi_{\max}(N_i)\}$$

where ∂ = some nonnegative number and

$$S_i = W_i + L_i H_i N_i\,; \quad \text{for} \quad i = 1, 2, \ldots, N\,.$$

In order to establish the stability condition of the global system, the augmented global system (21) (with disturbance term is zero) is reproduced below for our convenience

$$\begin{bmatrix} \dot{X}_{a1}(t) \\ \dot{X}_{a2}(t) \\ \vdots \\ \dot{X}_{aN}(t) \end{bmatrix} = \begin{bmatrix} F_{a11} - G_{a1}K_{a1} & F_{a12} & \ldots F_{a1N} \\ F_{a21} & F_{a22} - G_{a2}K_{a2} & \ldots F_{a2N} \\ \vdots & & \\ F_{aN1} & F_{aN2} & \ldots F_{aNN} - G_{aN}K_{aN} \end{bmatrix} \begin{bmatrix} X_{a1}(t) \\ X_{a2}(t) \\ \vdots \\ X_{aN}(t) \end{bmatrix}$$
$$+ \begin{bmatrix} G_{a1}K_{a1} & 0 & 0 \ldots & 0 \\ 0 & G_{a2}K_{a2} & 0 \ldots & 0 \\ 0 & 0 & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & & G_{aN}K_{aN} \end{bmatrix} \begin{bmatrix} e_{a1}(t) \\ e_{a2}(t) \\ \cdot \\ \cdot \\ e_{aN}(t) \end{bmatrix} \tag{25}$$

It can be noted that the stability of the system (25) entirely depends on the eigenvalues of the matrix

$$\begin{bmatrix} F_{a11} - G_{a1}K_{a1} & F_{a12} & \ldots & F_{a1N} \\ F_{a21} & F_{a22} - G_{a2}K_{a2} & \ldots & F_{a2N} \\ \vdots & \vdots & & \vdots \\ F_{aN1} & F_{aN2} & & F_{aNN} - G_{aN}K_{aN} \end{bmatrix}$$

and moreover, it was shown earlier that the contribution of second term in the state equation (25) tends to zero while time is sufficiently large. However, it clearly indicates that the stability of equation (25) equivalently can be considered as the stability of the autonomous system and its ith augmented subsystem is described by (from equation (25)

$$\dot{X}_{ai}(t) = (F_{aii} - G_{ai}K_{ai})\,X_{ai}(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} F_{aij}\,X_{aj}(t)$$

$$= (F_{aii} - G_{ai}K_{ai})\,X_{ai}(t) + h_i(t, X_a(t)) \tag{26}$$

where

$$h_i(t, X_a(t)) = \text{interaction terms}$$

$$= \sum_{\substack{j=1\\ j\neq i}}^{N} F_{aij}\,X_{aj}(t)$$

Controller gain $K_{ai} = -R_i^{-1}G_{ai}M_i$ is obtained by solving the following algebraic Riccati matrix equation [17].

$$M_iF_{aii} + F'_{aii}M_i - M_iG_{ai}R_i^{-1}G'_{ai}M_i + Q_{ai} = 0 \tag{27}$$

Equation (27) is expressed equivalently for $i = 1, 2, \ldots, N$, as

$$\dot{X}_a(t) = [F_{ad} - G_aK_a]\,X_a(t) + h(t, X_a(t)) \tag{28}$$

where

$$F_{ad} = \text{Block diag}\,[F_{a11}, F_{a22}, \ldots, F_{aNN}]$$

$$G_a = \text{Block diag}\,[G_{a1}, G_{a2}, \ldots, G_{aN}]$$

$$K_a = \text{Block diag}\,[K_{a1}, K_{a2}, \ldots, K_{aN}]$$

The asymptotic stable controlled response of the system (25) can be obtained while the condition

$$\operatorname*{Min}_i\,\{\chi_{\min}(S_{ci})\} \geqq 2\alpha \operatorname*{Max}_i\,\{\chi_{\max}(M_i)\}$$

is satisfied [18]; where α = some nonnegative number and $S_{ci} = Q_{ai} + M_iG_{ai}K_{ai}$.

4. PROPOSED DECENTRALIZED ADAPTIVE AND PI STRATEGIES FOR LOAD FREQUENCY CONTROL

If the system parameters vary slowly in time or the state equation is disturbed with some time-varying disturbances then PI control algorithm alone will not work satisfactorily to reject the system disturbance. So, naturally one should look for an alternative control scheme that will ensure the system response as-close-as possible with the desired system response in the presence of constant plus time-varying disturbances. We shall discuss briefly the proposed decentralized adaptive control scheme below:

Let us rewrite the *i*th subsystem of an interconnected power system having its linearized model for our ready reference.

$$\dot{X}_i(t) = F_{ii}\,X_i(t) + \sum_{\substack{j=1\\ j\neq i}}^{N} F_{ij}\,X_j(t) + G_i\,U_i(t) + \Gamma_i(D_i + E_{Di}(t)) \tag{29}$$

$$Y_i(t) = H_i\,X_i(t), \quad \text{for} \quad i = 1, 2, \ldots, N \tag{30}$$

The term $E_{D_i}(t)$ represents the time-varying disturbances to the ith subsystem state equation. Now, we define the control strategy $U_i(t)$ as

$$U_i(t) = U_{i1}(t) + U_{i2}^a(t) \tag{31}$$

where

$U_{i1}(t)$ = Local PI decentralized control law based on the procedure ensuring prescribed eigenvalues and simultaneously minimizing L−Q performance index (11), and
$U_{i2}^a(t)$ = Local adaptive control low based on the minimum prediction error principle.

The solution of the local PI control law $U_{i1}(t)$, for $i = 1, 2, \ldots, N$, can be obtained by employing the procedure by Solheim [17]. The first part $(U_{i1}(t))$ of the control strategy is mainly responsible to reject the known constant disturbance effects while the second part $(U_{i2}^a(t))$ of the control strategy will act immediately on the system to reject the time-varying disturbance effects or is there is any deviation of system output from the desired outputs response. The digital version of desired adaptive control law $U_{i2}^a(k)$ (where k is the time index) can be obtained using the following procedure.

Let us consider that the ith subsystem input and output are processed in discrete form through analog to digital conversion. A digital computer will act as an intelligent controller in the plant and it offers the possibility to predict the plant output over a certain period in the future. The predicted system output can then be used as a guide in developing a suitable control input so as to minimize the variations of the system output around the desired value. The philosophy behind this control concept is very close to the heuristical but allows for more general control "strategies". The basic principle of this adaptive control consists of three steps, namely, controller parameter estimation, prediction and finally control, as discussed below:

The putput of the ith subsystem (3)−(4) at the time index $(k + d)$ can be put into the form of d-step ahead predictor form (cf. the Appendix)

$$Y_i(k + d) = \theta_i'(k)\,\phi_i(k) \tag{32}$$

where

$$\theta_i(k) = [A_{i,0}, A_{i,l}, \ldots, A_{i,l_i-1}, B_{i,0}, B_{i,l}, \ldots, B_{i,r_i+d-1}]$$

and,

$$\phi_i(k) = [Y_i'(k), Y_i'(k-1), \ldots, Y_i'(k - l_i + 1), U_i'(k), U_i'(k-1), \ldots$$
$$\ldots U_i'(k - r_i - d + 1)]$$

Note that $\{A_{i,j}\}$ and $\{B_{i,j}\}$ are the parameters of the model (32) in matrix form and it is assumed that the integers l_i, r_i and d are known while the parameters are unknown. Due to the inherent structures of inputs and outputs decentralization of the composite system, one can write immediately a d-step ahead predictor model for the global system as

$$Y(k + d) = \text{Block diag}\,[\theta_1'(k), \theta_2'(k), \ldots, \theta_N'(k)].$$
$$\text{Block diag}\,[\phi_1(k), \phi_2(k), \ldots, \phi_N(k)] \tag{33}$$

Define the output or tracking error as

$$\Delta Y_i(k + d) = Y_i(k + d) - Y_i^*(k + d) = \theta_i'(k)\,\phi_i(k) - Y_i^*(k + d) \qquad (34)$$

where $Y_i^*(k + d)$ = desired output of ith subsystem. If $\theta_1'(k)$ were known, then the tracking error $\Delta Y_i(k + d)$ could be made identically zero by setting:

$$\begin{aligned} U_{i2}^a(k) = & -\theta_{i,l_i+1}^{-1}(k)\,[\hat{\theta}_{i,1}(k)\,Y_i(k) + \hat{\theta}_{i,2}(k)\,Y_i(k-1) + \ldots \\ & \ldots + \hat{\theta}_{i,l_i}(k)\,Y_i(k - l_i + 1) + \hat{\theta}_{i,l_i+2}(k)\,U_{i2}^a(k-1) + \ldots \\ & \ldots + \hat{\theta}_{i,il+r_i+d}(k)\,U_{i2}^a(k - r_i - d + 1) - Y_i^*(k + d)\,, \\ & \text{for} \quad i = 1, 2, \ldots, N \end{aligned} \qquad (35)$$

where $\hat{\theta}_{i,j}(k)$ is the jth block matrix of the estimated parameter matrix $\hat{\theta}_i(k)$ for the ith subsystem at the time index k. The adaptive control law (35) is based on the minimum prediction error principle [19–20] with the desired output response of the ith subsystem $Y_i^*(k + d)$ and d representing a pure time delay. The sequence $\{\theta_i(k)\}$ is then computed recursively by the following algorithm [19]

$$\begin{aligned} \hat{\theta}_i(k) = \hat{\theta}_i(k-1) + & \frac{\text{ALFA}\,.\,\phi_i(k-d)}{\text{BETA} + \phi_i'(k-d)\,\phi_i(k-d)}\,. \\ & .\,[Y_i'(k) - \phi_i'(k-d)\,\hat{\theta}_i(k-1)] \end{aligned} \qquad (36)$$

where $0 < \text{ALFA} < 2$ and $\text{BETA} > 0$.

The convergence properties of the d-step prediction control lagorithm for a multi-input system are discussed in Goodwin and Sin [19]. However, the final form of the decentralized control law $U_i(t)$ can be written as

$$U_i(t) = U_{i1}(t) + U_{i2}^a(t) = -K_{i1}\,\hat{X}_i(t) - K_{i2}\int_0^t Y_i(t)\,dt + U_{i2}^a(t) \qquad (37)$$

while the discrete version of adaptive control signals $U_{i2}^a(k)$ are then converted into continuous adaptive signal simply by the D/A converter.

For implementation of control law (31) on a hybrid computer, we solved equations (17) and (29) on a analog computer. The computer law $U_i(t)$ was also computed on the analog computer using the equation (37). The digital computer is employed for parameters estimation (using equation (36)) and also for implementation of adaptive part of the control law (using equation (35)). In principle, any adaptive control law can be implemented on a hybrid computer configuration which combines the advantages of decision, storage and branching capabilities of the digital computer with the high speed solution of dynamic differential equation on the analog computer.

5. SIMULATION STUDY OF TWO-AREA POWER SYSTEM

We have considered the following eigth-order explicit model of a two-area power system having tie-line flow capacity of 0·1 p. u. [21]

$$\dot{X}(t) = \left[\begin{array}{cccc|cccc} -0{\cdot}05 & 6{\cdot}0 & 0{\cdot}0 & -6{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & -3{\cdot}333 & 3{\cdot}333 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ -5{\cdot}21 & 0{\cdot}0 & -12{\cdot}5 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}545 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & -0{\cdot}545 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ \hline 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & -0{\cdot}05 & 6{\cdot}0 & 0{\cdot}0 & -6{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & -3{\cdot}333 & 3{\cdot}333 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & -5{\cdot}21 & 0{\cdot}0 & -12{\cdot}15 & 0{\cdot}0 \\ -0{\cdot}545 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}545 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \end{array}\right] X(t)$$

$$+ \left[\begin{array}{cccc|cccc} 0{\cdot}0 & 0{\cdot}0 & 12{\cdot}5 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 12{\cdot}5 & 0{\cdot}0 \end{array}\right]' U(t)$$

$$+ \left[\begin{array}{cccc|cccc} -6{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & -6{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \end{array}\right]' (D + E_D(t)) \tag{38}$$

$$Y(t) = \left[\begin{array}{cccc|cccc} 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 1{\cdot}0 \end{array}\right] X(t) \tag{39}$$

The states represent the perturbations in the frequency, the turbine-generator output, and the governor position of the generators of the two-area along with the perturbations in the tie-line power flow. The outputs of the two areas are taken to be area control errors which are a linear combination of the perturbations in the frequency and the tie-line power flow. It should be mentioned here that the 8th state $(X_8(t))$ of the state vector $X(t) = [X_1(t), X_2(t), \ldots, X_8(t)]'$ is redundant (Since $X_4(t) = -X_8(t)$) and this state $X_8(t)$ is introduced intentionally into the system equation in order to make the composite system input and output decentralization.

Data used for decentralized PI control algorithm with prescribed eigenvalues

The augmented ith subsystem matrices have the following numerical values:

$$F_{aii} = \left[\begin{array}{cccc|c} -0{\cdot}05 & 6{\cdot}0 & 0{\cdot}0 & -60 & 0{\cdot}0 \\ 0{\cdot}0 & -3{\cdot}333 & 3{\cdot}333 & 0 & 0{\cdot}0 \\ -5{\cdot}21 & 0{\cdot}0 & -12{\cdot}5 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}545 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ \hline 1{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \end{array}\right]$$

$$F_{aij} = \left[\begin{array}{cccc|c} 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ -0{\cdot}545 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \\ \hline 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 & 0{\cdot}0 \end{array}\right]$$

$$G_{ai} = [0{\cdot}0 \;\; 0{\cdot}0 \;\; 12{\cdot}5 \;\; 0{\cdot}0 \,\vdots\, 0{\cdot}0]'$$

$$\Gamma_{ai} = [-6{\cdot}0 \;\; 0{\cdot}0 \;\; 0{\cdot}0 \;\; 0{\cdot}0 \,\vdots\, 0{\cdot}0]'$$

$$H_{ai} = [1{\cdot}0 \;\; 0{\cdot}0 \;\; 0{\cdot}0 \;\; 1{\cdot}0 \,\vdots\, 0{\cdot}0]$$

The initial choice for the ith subsystem's controller and estimator gains is chosen as follows:

$$K_{ai} = [0{\cdot}1557 \;\; 0{\cdot}3364 \;\; 0{\cdot}1302 \;\; -0{\cdot}4137 \,\vdots\, 0{\cdot}2314]$$

$$L_i = [1{\cdot}989 \;\; -0{\cdot}3 \;\; -0{\cdot}7662 \;\; -0{\cdot}2503]'$$

$$Q_{ai} = 0{\cdot}1I_{5\times 5}\,; \quad W_i = I_{4\times 4}\,; \quad R_i = I_{1\times 1}$$

$$D_1 + E_{D_1}(t) = [{\cdot}1 + {\cdot}1\text{s in}\,({\cdot}8t), 0]', \quad D_2 + E_{D_2}(t) = [0 \; 0]'$$

$$X_i(0) = [0, 0, 0, 0]'$$

For subsystem 1:

Controller and estimator eigenvalues are specified as $(-16{\cdot}0;\ -2 \pm j^*;\ -2{\cdot}5;\ -1{\cdot}0)$ and $(-2{\cdot}5 \pm j^*;\ -1{\cdot}6;\ -14{\cdot}5)$ respectively.

Employing Solheim's procedure [17], we obtain the following final values of Q_{a1}, W_1, K_{a1} and L_1 respectively.

$$Q_{a1} = \begin{bmatrix} 1{\cdot}256 & 1{\cdot}2262 & 0{\cdot}4462 & -1{\cdot}206 & 2{\cdot}340 \\ 1{\cdot}2262 & 1{\cdot}8529 & 0;3977 & -1{\cdot}279 & 2{\cdot}3109 \\ 0{\cdot}4422 & 0{\cdot}3977 & 0{\cdot}5977 & -0{\cdot}3083 & 0{\cdot}5016 \\ -1{\cdot}206 & -1{\cdot}279 & -0{\cdot}3083 & 21{\cdot}933 & -15{\cdot}19 \\ 2{\cdot}340 & 2{\cdot}3109 & 0{\cdot}5016 & -15{\cdot}19 & 13{\cdot}918 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 61{\cdot}54 & -\ 72{\cdot}46 & 182{\cdot}5 & -10{\cdot}74 \\ -72{\cdot}46 & 110{\cdot}02 & -229{\cdot}4 & 9{\cdot}853 \\ 182{\cdot}5 & -294{\cdot}5 & 834{\cdot}8 & -19{\cdot}5 \\ 10{\cdot}74 & 9{\cdot}853 & -\ 19{\cdot}5 & 4{\cdot}92 \end{bmatrix}$$

$$K_{a1} = [1{\cdot}333 \quad 1{\cdot}865 \;\; 0{\cdot}6094 \;\; -3{\cdot}687 \;\; 5{\cdot}724]$$

$$L_1 = [6{\cdot}474 \;\; -3{\cdot}24 \;\; 4{\cdot}028 \;\; -1{\cdot}257]'$$

With the above controller and estimator gains, we obtained following eigenvalues

* Indicate that the imaginary part of desired eigenvalues cannot be prespecified.

for subsystem 1.

Controller eigenvalues: -16; $-2 \pm j\,2{\cdot}95$; $-2{\cdot}5$; $-1{\cdot}0$

Estimator eigenvalues: $-2{\cdot}5 \pm j\,3{\cdot}15$; $-14{\cdot}5$; $-1{\cdot}6$

For subsystem 2:

Desired eigenvalues for controller and estimator are assumed as ($-17{\cdot}0$; $-2{\cdot}5 \pm$ $\pm$ j*; $-2{\cdot}0$; $-0{\cdot}5$) and ($-2 \pm$ j*; $-15{\cdot}0$; $-1{\cdot}5$) respectively. We then obtained the following final results of controller and estimator:

$$Q_{a2} = \begin{bmatrix} 1{\cdot}15 & 1{\cdot}41 & 0{\cdot}559 & 0{\cdot}445 & 1{\cdot}12 \\ 1{\cdot}41 & 2{\cdot}34 & 0{\cdot}501 & -0{\cdot}494 & 1{\cdot}015 \\ 0{\cdot}559 & 0{\cdot}501 & 0{\cdot}823 & -0{\cdot}249 & 0{\cdot}298 \\ 0{\cdot}445 & -0{\cdot}494 & -0{\cdot}259 & 6{\cdot}948 & -2{\cdot}245 \\ 1{\cdot}12 & 1{\cdot}015 & 0{\cdot}298 & -2{\cdot}245 & 3{\cdot}37 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 33{\cdot}505 & -44{\cdot}77 & 115{\cdot}31 & -5{\cdot}583 \\ -44{\cdot}77 & 75{\cdot}59 & -205{\cdot}90 & 5{\cdot}423 \\ 115{\cdot}31 & -205{\cdot}90 & 593{\cdot}69 & -9{\cdot}893 \\ -5{\cdot}582 & 4{\cdot}423 & -9{\cdot}893 & 3{\cdot}089 \end{bmatrix}$$

$$K_{a2} = [1{\cdot}311 \quad 1{\cdot}938 \quad 0{\cdot}6894 \quad -1{\cdot}683 \quad 1{\cdot}8237]$$

$$L_2 = [4{\cdot}473 \quad -2{\cdot}233 \quad 2{\cdot}487 \quad -0{\cdot}8557]'$$

Using these gains k_{a2} and L_2, we then obtained the following eigenvalues for the subsystem 2.

Controller eigenvalues: $-17{\cdot}0$; $-2{\cdot}5 \pm j\,2{\cdot}89$; $-2{\cdot}0$; $-0{\cdot}5$

Estimator eigenvalues: $-2{\cdot}0 \pm j\,3{\cdot}18$; $-14{\cdot}0$; $-1{\cdot}5$

With the above controller and estimator gains, we obtained global closed-loop system (25) eigenvalues as follows:

$$-16{\cdot}999\,; \quad -15{\cdot}77\,; \quad -5{\cdot}77\,; \quad -1{\cdot}244 \pm j\,3{\cdot}705\,; \quad -1{\cdot}972 \pm j\,3{\cdot}22$$
$$-3{\cdot}017\,; \quad -7{\cdot}0\,; \quad -0{\cdot}0$$

The composite system has been simulated with above control action and the results are presented in Figs. 1–6.

Data used for adaptive control scheme

We have considered the following initial data:

Sampling period $T = 0{\cdot}1$ sec.

Order of DARMA model (eq. 32) = (3, 2)

Initial choice of parameters $\theta_i(0) = [0\ 0\ 0\ 2{\cdot}0\ 0]'$

ALFA = 1, BETA = 1

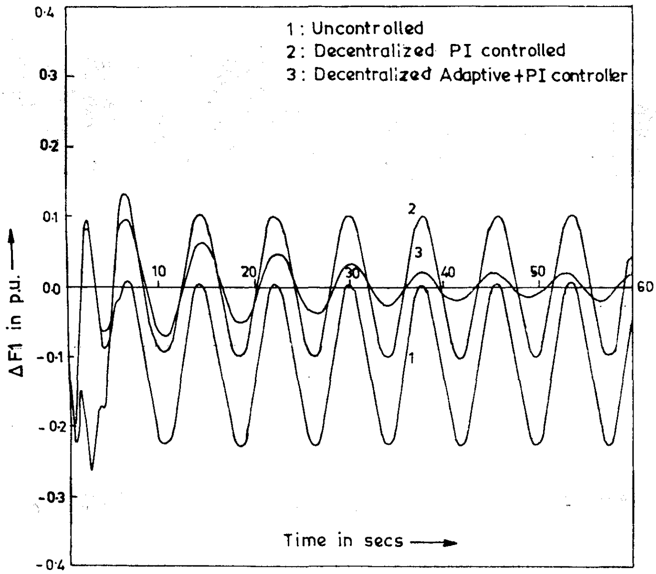

Fig. 1. Frequency deviation in area 1.

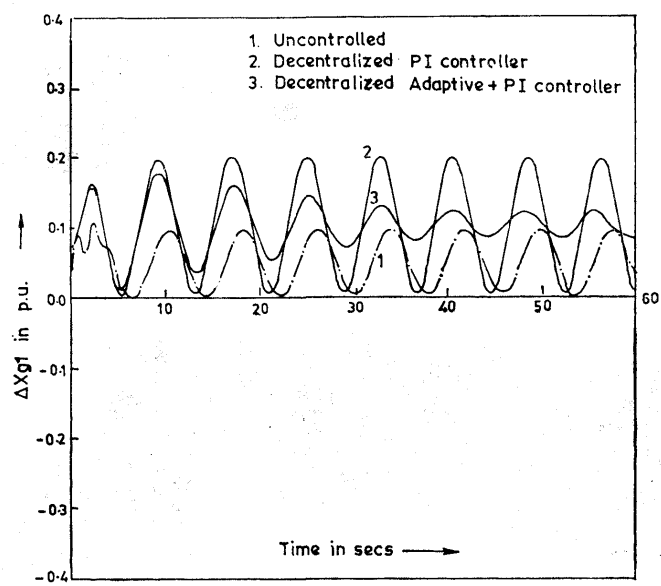

Fig. 2. Deviation in governer position in area 1.

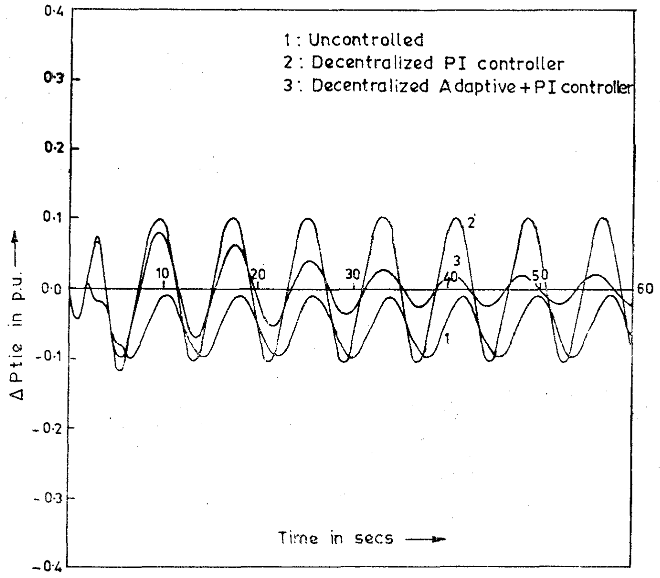

Fig. 3. Tie-line power variation.

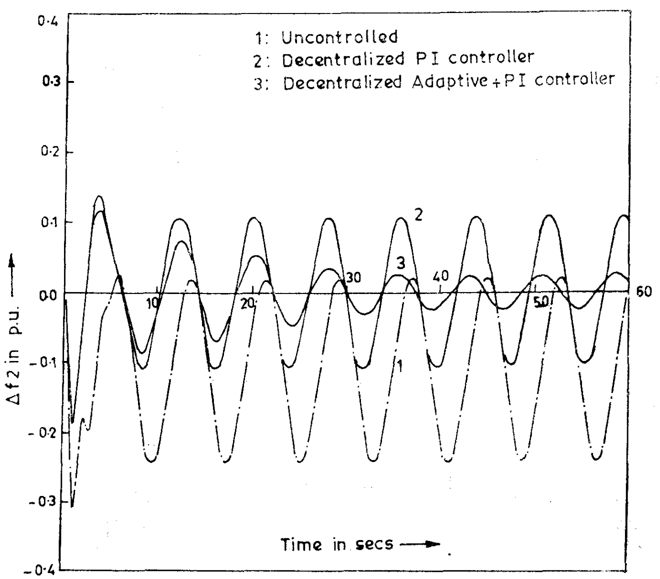

Fig. 4. Frequency deviation in area 2.

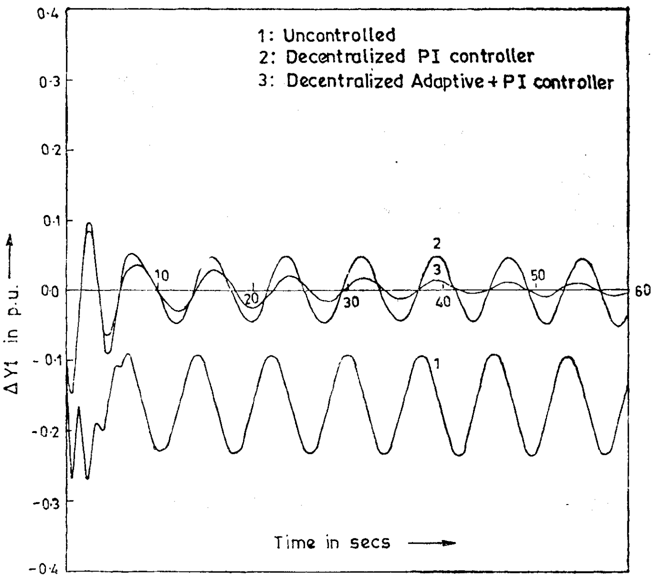

Fig. 5. Area control error in area 1.

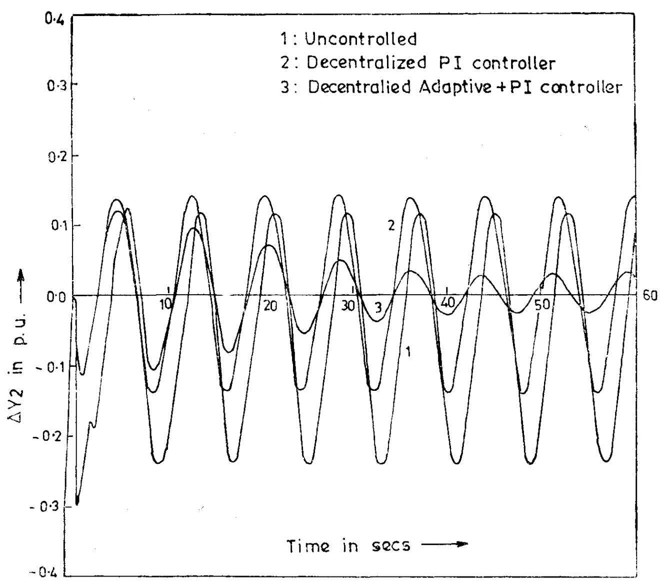

Fig. 6. Area control in area 2.

In order to implement decentralized adaptive and proportional plus integral control law (37), we have assumed that the proportional plus integral controller parameters for the first part of the control component $U_{i1}(t)$ have chosen exactly the same numerical values as we have calculated for decentralized PI control law (13). The other part of the control component $U^a_{i2}(k)$ can be easily obtained by using the equations (35) and (36). We then use the D/A converter to obtain continuous mode

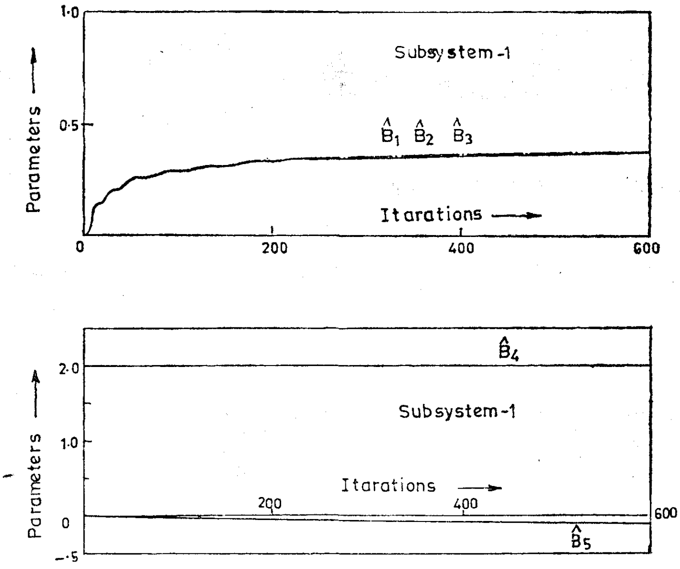

Fig. 7. Convergence of parameters of area 1.

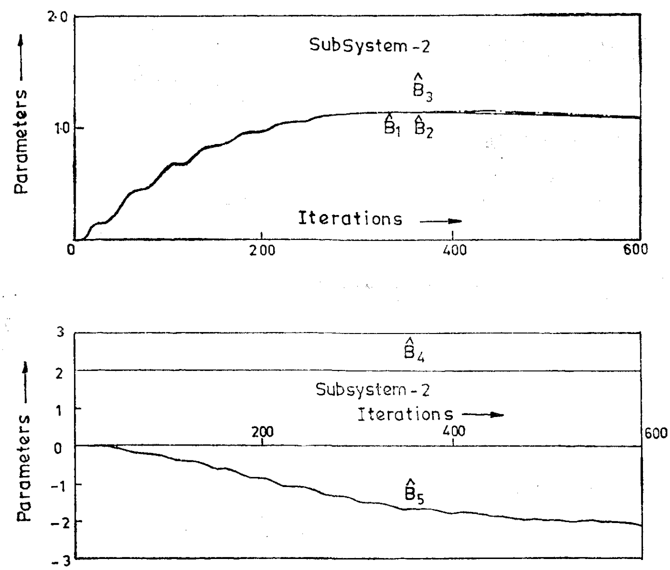

Fig. 8. Convergence of parameters of area 2.

of control signals. The convergence of parameters of DARMA model is then obtained recursively after 600 iterations as

Subsystem 1:

$$\theta_1 = [0{\cdot}3665\ \ 0{\cdot}3725\ \ 0{\cdot}3687\ \ 2{\cdot}0\ \ -0{\cdot}098]'$$

Subsystem 2:

$$\theta_2 = [1{\cdot}056\ \ 1{\cdot}0785\ \ 1{\cdot}0907\ \ 2{\cdot}0\ \ -2{\cdot}005]'$$

The system has been simulated using the control law (37) and its response is compared with decentralized controlled system response based on PI control law only (eq. 13). Uncontrolled and controlled system responses (in the presence of known constant plus time-varying disturbances) are shown in Figs. 1 – 6. The convergence of DARMA model parameters are also shown in Figs. 7 – 8.

6. CONCLUDING REMARKS

A study of different decentralized controllers for the load frequency control of interconnected power system subjected to known constant plus time-varying disturbances has been made in this paper. We have noticed that the decentralized proportional plus integral controller based on the decentralized state estimation with prescribed eigenvalues helps to reduce the computational burden and instrumentation costs. But on the other hand, this scheme does not work satisfactorily to keep the tie-line power system frequency and area control error within the acceptable limits from the set-points. However, it can be observed from Figs. 1 – 6 that the proposed decentralized (adaptive and proportional plus integral) controller keeps some important states and outputs as close as possible to the desired set points. The PI control acts immediately to the system to reject the effect of constant disturbances and it is implemented by analog computer. On the other hand, the adaptive control part which is corrective in nature implemented through digital computer. All digital control sequences are then converted into continuous signals form using the D/A converter. This control part acts to the system to reject the time-varying disturbances. It can be mentioned that the hybrid computation techniques can be adopted if the high speed implementation of adaptive control scheme is required.

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APPENDIX

Let us consider that the process to be controlled is a discrete-time multi-input, multi-output system model and its current output vector is expressed as a linear combination of past outputs $Y(k)$ and past inputs $U(k)$ [19].

$$A_{do} Y(k) = -\sum_{j=1}^{l} A_{dj} Y(k-j) + \sum_{j=0}^{r} B_{dj} U(k-j-d) \tag{A.1}$$

where A_{do} is a square and nonsingular matrix and d represents time delay. As mentioned in Section 2. the dimension of $Y(k)$ and $U(k)$ are p and m respectively. After rearranging, the equation (A.1) can be written as:

$$[A_{d0} Y(k) + \sum_{j=1}^{l} A_{dj} Y(k-j)]\, q^{d} = \sum_{j=0}^{r} B_{dj} U(k-j)$$

$$A(q^{-1})\, Y(k+d) = B(q^{-1})\, U(k) \tag{A.2}$$

where

$$A(q^{-1}) = A_{d0} + A_{d1}q^{-1} + \dots + A_{d1}q^{-l}$$

and

$$B(q^{-1}) = B_{d0} + B_{d1}q^{-1} + \dots + B_{dr}q^{-r}$$

By successive substitution of the following expressions

$$A(q^{-1})\, Y(k+d-1) = B(q^{-1})\, U(k-1)$$
$$A(q^{-1})\, Y(k+d-2) = B(q^{-1})\, U(k-2)$$
$$\vdots$$
$$A(q^{-1})\, Y(k+1) \quad = B(q^{-1})\, U(k-d+1)$$

in equation (A.2). we can express the above model (A.2) in d-step ahead predictor form as

$$Y(k+d) = A_d(q^{-1})\, Y(k) + B_d(q^{-1})\, U(k) \tag{A.3}$$

with

$$A_d(q^{-1}) = A_0 + A_1q^{-1} + A_2q^{-2} + \dots + A_{1-1}q^{-(l-1)}$$
$$B_d(q^{-1}) = B_0 + B_1q^{-1} + B_2q^{-2} + \dots + B_{r+d-1}q^{-(r+d-1)}$$

where $\{A_i\}$ and $\{B_i\}$ are function of the unknown $\{A_{di}\}$ and $\{B_{di}\}$. The equation (A.3) is known as d-step-ahead predictor model of multi-input muti-output discrete system.

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