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ON THE CLASSES OF HEREDITARILY ℓ_p BANACH SPACES

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Abstract. Let X denote a specific space of the class of $X_{\alpha,p}$ Banach sequence spaces which were constructed by Hagler and the first named author as classes of hereditarily ℓ_p Banach spaces. We show that for $p > 1$ the Banach space X contains asymptotically isometric copies of ℓ_p. It is known that any member of the class is a dual space. We show that the predual of X contains isometric copies of ℓ_q where $1/p+1/q=1$. For $p=1$ it is known that the predual of the Banach space X contains asymptotically isometric copies of c_0. Here we give a direct proof of the known result that X contains asymptotically isometric copies of ℓ_1.

Keywords: Banach spaces, asymptotically isometric copy of ℓ_p, hereditarily ℓ_p Banach spaces

MSC 2000: 46B04, 46B20

1. Introduction

J. Hagler and the first named author have introduced a class of Banach sequence spaces, the $X_{\alpha,p}$ spaces. For $p = 1$ each of the spaces is hereditarily complementably ℓ_1 and yet fails the Schur property [2]. For $p > 1$ each of the spaces is hereditarily complementably ℓ_p [1]. In this paper we show that $X_{\alpha,p}$ spaces for $p > 1$ contain asymptotically isometric copies of ℓ_p. Any $X_{\alpha,p}$ is a dual space. We show that the preduals of the spaces contain isometric copies of ℓ_q.

For $p=1$, Azimi showed that the preduals of $X_{\alpha,1}$ spaces contain asymptotically isometric copies of c_0 and by a result of S. Chen and B. L. Lin [3] deduced that $X_{\alpha,1}$ contains asymptotically isometric copies of ℓ_1. As an immediate consequence of the results of J. Dilworth, M. Girardi and J. Hagler [4], we observe that $C^*[a,b]$ is linearly isometric to a subspace of $X^*_{\alpha,1}$. Here we give a direct proof to show that any $X_{\alpha,1}$ contain asymptotically isometric copies of ℓ_1. A result of P. N. Dowling and C. J. Lennard [5] implies that $X_{\alpha,1}$ spaces fail to have the fixed point property, i.e., there exists a nonexpansive self-mapping on a bounded closed convex subset of $X_{\alpha,1}$ which has no fixed point.

Now we go through the construction of the $X_{\alpha,p}$ spaces.

A block F is an interval (finite or infinite) of integers. For any block F, and $x = (t_1, t_2, \ldots)$ a finitely non-zero sequence of scalars, we let $\langle x, F\rangle = \sum_{j\in F} t_j$. A sequence of blocks $F_1, F_2, \ldots$ is admissible if $\max F_i < \min F_{i+1}$ for each i. Finally, let $1 = \alpha_1 \geqslant a_2 \geqslant \alpha_3 \geqslant \ldots$ be a sequence of real numbers with $\lim_{i\to\infty} \alpha_i = 0$ and $\sum_{i=1}^{\infty} \alpha_i = \infty$.

We now define a norm which uses the α_i's and an admissible sequence of blocks in its definition. Let $1 \leqslant p < \infty$ and let $x = (t_1, t_2, \ldots)$ be a finitely non-zero sequence of reals. Define

$$\|x\| = \max\left[\sum_{i=1}^{n} \alpha_i |\langle x, F_i\rangle|^p\right]^{1/p}$$

where the max is taken over all n, and admissible sequences $F_1, F_2, \ldots$. The Banach space $X_{\alpha,p}$ is the completion of the finitely non-zero sequences of scalars in this norm.

2. Definitions and Notation

Definitions and notation are standard, but we give some of these here.

Let ℓ_1 be the space of absolutely summable sequences and c_0 the space of all null sequences $x = (t_1, t_2, \ldots)$ with $\|x\| = \max_n |t_n|$.

A Banach space X is hereditarily ℓ_1 if every infinite dimensional subspace of X contains a subspace isomorphic to ℓ_1.

Definition 2.1. We say that a Banach space X contains asymptotically isometric copies of ℓ_1 if for some sequence $\varepsilon_n \downarrow 0$ $(0 < \varepsilon_n \leqslant 1)$, there is a norm-one sequence (x_n) in X such that for all m and scalars $(t_n\colon\ 0 \leqslant n \leqslant m)$

$$\sum_{n=0}^{m} (1-\varepsilon_n)|t_n| \leqslant \left\|\sum_{n=0}^{m} t_n x_n\right\| \leqslant \sum_{n=0}^{m} |t_n|, \quad (t_n) \in \ell_1.$$

We say that a Banach space X contains an asymptotically isometric copy of ℓ_p $(1 < p < \infty)$ if for any $\varepsilon_n \downarrow 0$ $(0 < \varepsilon_n \leqslant 1)$ X contains a norm-one sequence (x_n) such that

$$\Bigl(\sum_n (1-\varepsilon_n)^p |\beta_n|^p\Bigr)^{1/p} \leqslant \Bigl\|\sum_n \beta_n x_n\Bigr\| \leqslant \Bigl(\sum_n (1+\varepsilon_n)^p |\beta_n|^p\Bigr)^{1/p}, \quad (\beta_n) \in l_p.$$

3. The results

The key to the analysis of the space X is the following result (Lemma 4 of [2]).

Lemma 3.1. *Let the sequence (α_i) be as above, let $N > 0$ be an integer and let $\varepsilon > 0$. Then there exist a $\delta > 0$ such that, if $b_1, b_2, \ldots, b_n$ are $\geqslant 0$, $b_i < \delta$ for all i, and $\sum\limits_{i=1}^{n} \alpha_i b_i = 1$, then $\sum\limits_{i=1}^{n} \alpha_{i+N} b_i \geqslant 1 - \varepsilon$.*

The following summarize the main result of [1]. Let (e_i) denote the sequence of the usual unit vectors in $X_{\alpha,p}$, $e_i(j) = \delta_{ij}$.

Theorem 3.2. *Let $X_{\alpha,p}$ denote a specific space of the class, then we have the following:*

1. *$X_{\alpha,p}$ is hereditarily complementably ℓ_p.*
2. *The sequence (e_i) is a normalized boundedly complete bases for $X_{\alpha,p}$. Thus, $X_{\alpha,p}$ is a dual space.*
3. *The predual of $X_{\alpha,p}$ contains complemented subspaces isomorphic to ℓ_q where $1/p + 1/q = 1$.*
   - (a) *Each complemented non weakly sequentially complete subspace of $X_{\alpha,p}$ contains a complemented isomorph of $X_{\alpha,p}$.*
   - (b) *$X_{\alpha,p}$ and $X_{\beta,p}$ are isomorphic if and only if they are equal as sets.*
   - (c) *The sequence (x_n) with $x_n = e_{2n-1} - e_{2n}$ is weakly null sequence in $X_{\alpha,p}$ but not in norm.*

     Since $X_{\alpha,p}$ contains ℓ_p hereditarily complementably, thus,
   - (d) *$X_{\alpha,p}$ spaces are not prime.*

     Since for $p > 1$, $X_{\alpha,p}$ does not contain ℓ_1 and is not reflexive,
   - (e) *$X_{\alpha,p}$ is a Banach space without unconditional basis.*

Theorem 3.3. *The Banach space $X_{\alpha,1}$ contains asymptotically isometric copies of l_1.*

P r o o f. Let (u_i) be a sequence of norm one vectors in $X_{\alpha,1}$ and (G_i) an admissible sequence of blocks such that $\{j\colon u_i(j) \neq 0\} \subset G_i$. For each i, put $s_i = s(u_i)$ where $s(u_i) = \max\limits_{G} |\langle u_i, G\rangle|$. If $\lim\limits_{i\to\infty} s_i = 0$, then a subsequence (v_j) of (u_j) satisfies

$$\Bigl\|\sum_{j=1}^{n} t_j v_j\Bigr\| \geqslant \sum_{j=1}^{n} (1-\varepsilon_j)|t_j|$$

where (ε_j) is a decreasing sequence, $\varepsilon_i < 1$ for all i and (t_j) is a sequence of scalars.

We select (v_j) by induction. Let $v_1 = u_1$. Pick n_1 and $F_1, F_2, \ldots, F_{n_1}$ satisfying $\max F_{n_1} = \max G_1$ and $\sum_{i=1}^{n_1} \alpha_i |\langle v_1, F_i\rangle| = \|v_1\| = 1$. Let δ_1 be any δ guaranteed by Lemma 3.1 for the integer n_1 and ε_1. We let $n_0 = 0$. Assume now that we have selected for $k = 1, \ldots, p-1$

1. an integer m_k $(> m_{k-1})$ so that $v_k = u_{m_k}$.
2. an integer n_k $(> n_{k-1})$, blocks $F_{n_{k-1}+1}, \ldots, F_{n_k}$ and $\delta_k > 0$ such that
   (a) $\max F_{n_k} = \max G_{m_k}$,
   (b) the sequence $F_1, F_2, \ldots, F_{n_1}, \ldots, F_{n_2}, \ldots, F_{n_k}$ is admissible,
   (c) $\sum_{i=1}^{n_k - n_{k-1}} \alpha_i |\langle v_k, F_i\rangle| = \|v_k\| = 1$,
   (d) δ_k is any δ guaranteed by Lemma 3.1 for the integer n_{k-1} and ε_k.

Now let $\delta_p > 0$ be any δ guaranteed by Lemma 3.1 for the integer n_{p-1} and ε_p. Pick m_p $(> m_{p-1})$ so that $s_{m_p} < \delta_p$ and $v_p = u_{m_p}$. Finally, pick blocks $F_{n_{p-1}}, \ldots, F_{n_p}$ such that (a), (b) and (c) above are satisfied for v_p and G_{m_p}. This completes the induction process.

Observe that $|\langle v_k, F_{i+n_{k-1}}\rangle| < s_{n_k} < \delta_k$ for $i = 1, \ldots, n_k - n_{k-1}$. By Lemma 3.1

$$\sum_{i=1}^{n_k - n_{k-1}} \alpha_{i+n_{k-1}} |\langle v_k, F_{i+n_{k-1}}\rangle| > 1 - \varepsilon_k.$$

This inequality can be rewritten as

$$\sum_{i=n_{k-1}+1}^{n_k} \alpha_i |\langle v_k, F_i\rangle| > 1 - \varepsilon_k.$$

Now, let scalars $t_1, t_2, \ldots, t_k$ be given. Since the sequence $F_1, \ldots, F_{n_k}$ is admissible, it follows from the observation above that

$$\begin{aligned}\left\|\sum_{j=1}^{n} t_j v_j\right\| &\geqslant \sum_{i=1}^{n_k} \alpha_i \left|\left\langle \sum_{j=1}^{n} t_j v_j, F_i \right\rangle\right| = \sum_{j=1}^{n} |t_j| \left(\sum_{i=1}^{n_k} \alpha_i |\langle v_j, F_i\rangle|\right) \\ &= \sum_{j=1}^{n} |t_j| \left(\sum_{i=n_{j-1}+1}^{n_j} \alpha_i |\langle v_j, F_i\rangle|\right) \geqslant \sum_{j=1}^{n} (1-\varepsilon_j)|t_j|.\end{aligned}$$

To complete the proof we need to establish the result for norm one vectors (u_i) and blocks (G_i) with $\max G_i < \min G_{i+1}$ such that $\{j\colon u_i(j) \neq 0\} \subset G_i$ if some subsequence of $(s_i) \to 0$, then we are done. If not we use an argument similar to the proof of Theorem 1 (1) of [2]. □

The following lemma shows that if for a sequence (u_i) in $X_{\alpha,p}$, $s(u_i) \nrightarrow 0$, then we can construct a sequence (x_i) from (u_i) such that $s(x_i) \to 0$. Proof of the lemma is analogous to those of the theorem 1 (1) of [2].

Lemma 3.4. *Let (u_i) be a sequence of norm one vectors in $X_{\alpha,p}$ and (G_i) an admissible sequence of blocks such that $\{j\colon u_i(j)\neq 0\}\subset G_i$. Then, a sequence (x_i) obtained from (u_i) such that $s(x_i)\to 0$.*

Lemma 3.5. *Let (v_i) be a sequence in $X_{\alpha,p}$, (G_i) an admissible sequence of blocks such that $\{j\colon v_i(j)\neq 0\}\subset G_i$ and*

1. $\|v_i\|=1$,
2. $\langle v_i, N\rangle = 0$,
3. $s(v_i)\to 0$.

Then

$$\Big\|\sum_{i=1}^{k} t_i v_i\Big\|^p \leqslant \sum_{i=1}^{k}|t_i|^p.$$

Proof. Let $u_i = 2v_i$. By induction, we show that for any n, and admissible blocks $F_1, F_2, \ldots, F_m$, we have

$$\text{(A)}\qquad \sum_{j=1}^{m}\alpha_j\Big|\Big\langle\sum_{i=1}^{n} t_i u_i, F_j\Big\rangle\Big|^p \leqslant 2K\sum_{i=1}^{n-1}|t_i|^p + K|t_n|^p$$

for $K=2^{p-1}$. Now we assume that (A) is true for all $k\leqslant n-1$, and note that it holds for $k=1$. Let l be the largest integer for which

$$\operatorname{support}(u_{n-1})\cap F_l \neq \emptyset$$

and suppose that for $i=k,\ldots,n-1$

$$\operatorname{support}(u_i)\cap F_l\neq\emptyset$$

yet

$$\operatorname{support}(u_{k-1})\cap F_l = \emptyset.$$

Thus $u_{k+1},\ldots,u_n$ are entirely supported in F_l.

Next

$$\text{(B)}\qquad \sum_{j=1}^{m}\alpha_j\Big|\Big\langle\sum_{i=1}^{n} t_i u_i, F_j\Big\rangle\Big|^p = \sum_{j=1}^{l-1}\alpha_j\Big|\Big\langle\sum_{i=1}^{k} t_i u_i, F_j\Big\rangle\Big|^p + \alpha_l\Big|\Big\langle\sum_{i=k}^{n} t_i u_i, F_l\Big\rangle\Big|^p + \sum_{j=l+1}^{m}\alpha_j|\langle t_n u_n, F_j\rangle|^p = \sum\nolimits_1 + \sum\nolimits_2 + \sum\nolimits_3.$$

We will use the induction hypothesis on $\sum_1$, we will leave $\sum_3$ basically as it is, and estimate the middle term in $\sum_2$:

$$\begin{aligned}\sum\nolimits_2 &= \alpha_l \left| t_k\langle u_k, F_l\rangle + \sum_{i=k+1}^{n-1} \langle t_i u_i, F_l\rangle + t_n\langle u_n, F_l\rangle \right|^p \\ &= \alpha_l |t_k\langle u_k, F_l\rangle + t_n\langle u_n, F_l\rangle|^p \\ &\leqslant \alpha_l 2^{p-1}[|t_k\langle u_k, F_l\rangle|^p + |t_n\langle u_n, F_l\rangle|^p].\end{aligned}$$

Returning to (B) we obtain

$$\begin{aligned}&\sum_{j=1}^{m} \alpha_j \left| \left\langle \sum_{i=1}^{n} t_i u_i, F_j \right\rangle \right|^p \\ &\leqslant \left[2K \sum_{i=1}^{k-1} |t_i|^p + K|t_k|^p \right] + [K|t_k\langle u_k, F_l\rangle|^p + K \sum_{i=k+1}^{n-1} |t_i|^p \\ &\quad + \alpha_l K |t_n\langle u_n, F_l\rangle|^p] + \sum_{j=l+1}^{m} \alpha_j |\langle t_n u_n, F_j\rangle|^p \\ &\leqslant 2K \sum_{i=1}^{n-1} |t_i|^p + K \sum_{j=l}^{m} \alpha_j |\langle t_n u_n, F_j\rangle|^p \leqslant 2K \sum_{i=1}^{n-1} |t_i|^p + K|t_n|^p,\end{aligned}$$

thus

$$\left\| \sum_{i=1}^{k} t_i u_i \right\|^p \leqslant 2^p \sum_{i=1}^{k} |t_i|^p.$$

□

Lemma 3.6. *Let* (v_i) *be as above and* (G_i) *an admissible sequence of blocks such that* $\{j\colon\ v_i(j) \neq 0\} \subset G_i$. *Then for a subsequence* (v_k) *(not renaming) of* (v_k)*and for a given sequence* $t_1, t_2, \ldots, t_k$ *of scalars we have*

$$\left\| \sum_{i=1}^{k} t_i v_i \right\|^p \geqslant \sum_{i=1}^{k} (1-\varepsilon_i)^p |t_i|^p$$

where $0 < \varepsilon_i \leqslant 1$ *is a decreasing sequence.*

P r o o f. An argument similar to the proof of Theorem 3.3 shows that we may assume the following.

There exists subsequence (v_i) (not renaming) of (v_i) and sequence (n_i) of integers and $\delta_i > 0$ satisfying:

1. $\|v_i\| = 1$ for all i.
2. For integer n_i ($> n_{i-1}$) put $N_i = n_1 + n_2 + \ldots + n_{i-1}$, $i > 1$ and $N_1 = 0$. Then δ_i satisfies Lemma 3.1 for $\varepsilon = \varepsilon_i$ and $N = N_i$.
3. For each block F and each i, $|\langle v_i, F\rangle|^p \leqslant \delta_i$.
4. For each i, there is a sequence of admissible blocks $F_{n_{i-1}+1}, F_{n_{i-1}+2}, \ldots, F_{n_i}$ with
   (a) $\max F_{n_i} < \min F_{n_i+1}$
   (b) $\sum_{j=1}^{n_i-n_{i-1}} \alpha_j |\langle v_i, F_{n_{i-1}+j}\rangle|^p = \|v_i\|^p = 1$
   (c) $\langle v_k, F_{n_{i-1}+j}\rangle = 0$ if $i \neq k$, and by Lemma 3.1, we have
   (d) $\sum_{j=n_{i-1}+1}^{n_i} \alpha_j |\langle v_i, F_j\rangle|^p > 1 - \varepsilon_i$.

Since the sequence $F_1, F_2, \ldots, F_{n_1}, \ldots, F_{n_2}, \ldots, F_{n_k}, \ldots$ is admissible, it follows from 1–4 above that for scalars $t_1, \ldots, t_k$ and admissible blocks $F_1, F_2, \ldots, F_{n_k}$,

$$\Big\|\sum_{i=1}^{k} t_i v_i\Big\|^p \geqslant \sum_{i=1}^{n_k} \alpha_i \Big|\Big\langle \sum_{j=1}^{k} t_j v_j, F_i\Big\rangle\Big|^p = \sum_{j=1}^{k} |t_j|^p \sum_{i=n_{j-1}+1}^{n_j} \alpha_i |\langle v_j, F_i\rangle|^p$$
$$\geqslant \sum_{j=1}^{k} (1-\varepsilon_j)|t_j|^p \geqslant \sum_{j=1}^{k} (1-\varepsilon_j)^p |t_j|^p.$$

□

Lemmas 3.4, 3.5 and 3.6 have the following consequence.

Theorem 3.7. *The Banach space $X_{\alpha,p}$ contains asymptotically isometric copies of l_p.*

The following corollary is an immediate consequence of Theorem 3.7 and a result of Chen and Lin [3] (Theorem 7).

Corollary 3.8. *For any sequence $\varepsilon_n \downarrow 0$ ($0 < \varepsilon_n < 1$), $X_{\alpha,p}$ contains a subspace X_0 such that X_0^* has a normalized basis (x_n^*) satisfying*

$$\Big(\sum_n (1-\varepsilon_n)^q |\beta_n|^q\Big)^{1/q} \leqslant \Big\|\sum_n \beta_n x_n^*\Big\|_{X_0^*} \leqslant \Big(\sum_n (1+\varepsilon_n)^q |\beta_n|^q\Big)^{1/q}, \quad (\beta_n) \in \ell_q$$

where $1/p + 1/q = 1$.

Remark 3.9. Let (f_i) in X^* be the biorthogonal sequence to the usual basis (e_i) in X, and let Y be the subspace of X^* generated by the sequence (f_i). Theorem 3.2 (2) and a well known result [6] (Proposition 1.b.4, page 9) show that $X = Y^*$. For $p > 1$, Theorem 3.2 (3) shows that Y contains complemented subspaces isomorphic to ℓ_q where $1/p + 1/q = 1$.

Now, we show that Y contains isometric copies of l_q, where $1/p+1/q=1$.

Theorem 3.10. *The predual of $X_{\alpha,p}$ spaces contains isometric copies of l_q where $1/p+1/q=1$.*

Proof. Let (v_i) be as above and

$$\varphi_i(x)=\sum_{j=1}^{n_i}\alpha_j|\langle v_j,F_j^i\rangle|^{p-1}\varepsilon_j^i\langle x,F_j^i\rangle$$

where v_i is normed by $F_1^i,\dots,F_{n_i}^i$ and $\varepsilon_j^i=\operatorname{sgn}\langle v_i,F_j^i\rangle$. Then $\varphi_i\in Y$ where $Y^*=X_{\alpha,p}$ (Remark 3.9) and $\|\varphi_i\|=1$ since $\varphi_i(v_i)=1$.

Now we go through the calculation of the norm. By Hölder's inequality and the fact that $q(p-1)=p$, we have

$$\begin{aligned}\Big|\sum_{i=1}^k s_i\varphi_i(x)\Big| &\leqslant \sum_{i=1}^k|s_i|\,|\varphi_i(x)|\leqslant\sum_{i=1}^k|s_i|\Big(\sum_{j=1}^{n_i}\alpha_j|\langle v_i,F_j^i\rangle|^{p-1}|\langle x,F_j^i\rangle|\Big)\\ &=\sum_{i=1}^k\sum_{j=1}^{n_i}|s_i|\alpha_j|\langle v_i,F_j^i\rangle|^{p-1}|\langle x,F_j^i\rangle|\\ &=\sum_{i=1}^k\Big(\sum_{j=1}^{n_i}|s_i|\alpha_j^{1/q}|\langle v_i,F_j^i\rangle|^{p-1}\alpha_j^{1/p}|\langle x,F_j^i\rangle|\Big)\\ &\leqslant\Big[\sum_{i=1}^k\Big(\sum_{j=1}^{n_i}|s_i|^q\alpha_j|\langle v_i,F_j^i\rangle|^{q(p-1)}\Big)\Big]^{1/q}\\ &\quad\times\Big[\sum_{i=1}^k\Big(\sum_{j=1}^{n_i}\alpha_j|\langle x,F_j^i\rangle|^p\Big)\Big]^{1/p}\leqslant\Big(\sum_{i=1}^k|s_i|^q\Big)^{1/q}\|x\|.\end{aligned}$$

Therefore

$$\Big\|\sum_{i=1}^k s_i\varphi_i\Big\|\leqslant\Big(\sum_{i=1}^k|s_i|^q\Big)^{1/q}.$$

Now we prove that

$$\Big\|\sum_{i=1}^n s_i\varphi_i\Big\|\geqslant\Big(\sum_{i=1}^n|s_i|^q\Big)^{1/q}.$$

Let $x=\sum\limits_{i=1}^n\varepsilon_i|s_i|^{q-1}v_i$, $\varepsilon_i=\operatorname{sgn}(s_i)$. By Lemma 3.5

$$\|x\|\leqslant\Big(\sum_{i=1}^n|s_i|^{p(q-1)}\Big)^{1/p}=\Big(\sum_{i=1}^n|s_i|^q\Big)^{1/p}.$$

This implies that

$$\Big\|\sum_{i=1}^n s_i\varphi_i\Big\| \geqslant \Big|\sum_{i=1}^n s_i\varphi_i\Big(\frac{x}{\|x\|}\Big)\Big| = \frac{1}{\|x\|}\Big|\sum_{i=1}^n s_i\varphi_i(\varepsilon_i|s_i|^{q-1}v_i)\Big|$$
$$= \frac{1}{\|x\|}\sum_{i=1}^n |s_i|^q \geqslant \frac{1}{\Big(\sum\limits_{i=1}^n |s_i|^q\Big)^{1/p}}\sum_{i=1}^n |s_i|^q = \Big(\sum_{i=1}^n |s_i|^q\Big)^{1/q}.$$

Thus

$$\Big\|\sum_{i=1}^n s_i\varphi_i\Big\| \geqslant \Big(\sum_{i=1}^n |s_i|^q\Big)^{1/q}.$$

Therefore

$$\Big\|\sum_{i=1}^n s_i\varphi_i\Big\| = \Big(\sum_{i=1}^n |s_i|^q\Big)^{1/q}.$$

□

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