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INTEGRAL CRITERIA OF OSCILLATION FOR A THIRD ORDER LINEAR DIFFERENTIAL EQUATION

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ABSTRACT. A new oscillation criterion for the equation $y''' + p(t)y' + q(t)y = 0$ with a nonpositive coefficient p and a positive coefficient q is established. This result extends and improves some oscillation criteria for third order linear differential equations in this case.

1. Introduction

Consider the differential equation

$$y''' + p(t)y' + q(t)y = 0\,, \tag{L}$$

where $p, q, p' \colon I \to \mathbb{R}$, $I = (a, \infty) \subset (0, \infty)$, $\mathbb{R} = (-\infty, \infty)$, are continuous.

We shall investigate two cases:

$$p(t) \le 0\,, \quad q(t) > 0\,, \qquad t \in I \tag{P}$$

and

$$p(t) \le 0\,, \quad p'(t) - q(t) > 0\,, \qquad t \in I\,. \tag{PA}$$

We consider only nontrivial solutions of (L). Such a solution is called *oscillatory* on I if it has arbitrarily large zeros, otherwise, it is called *nonoscillatory* on I. Equation (L) is said to be oscillatory on I if it has at least one oscillatory solution. Furthermore, equation (L) is said to be of *Class I* (*Class II*) on I if and only if every solution y of (L) with $y(c) = y'(c) = 0$, $y''(c) > 0$, $c \in (a, \infty)$, has the property that $y(t) > 0$ in (a, c) (in (c, ∞)).

In the particular case, when $p(t) \equiv 0$, $q(t) > 0$, $t \in I$, there is the well-known oscillation criterion for (L) of the form

$$\int^{\infty} t^{2-\varepsilon} q(t)\, \mathrm{d}t = \infty \qquad \text{for some} \quad \varepsilon > 0\,, \tag{1.1}$$

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see, e.g., [1] and [11].

The condition (1.1) has been improved several times. We present a result of Chanturiya. For analogous results, the reader is referred to [3] and [13].

THEOREM A. ([2; Theorem 2.1]) *Let* $p(t) \equiv 0$ *and* $q(t) > 0$ *for* $t \in I$. *If*

$$\liminf t^2 \int_t^\infty q(s)\, \mathrm{d}s > \frac{1}{3\sqrt{3}}, \tag{1.2}$$

then equation (L) *is oscillatory.*

In general, we assume that $p(t) \not\equiv 0$ on I. For equation (L) there are oscillation criteria due to Lazer and Erbe.

THEOREM B. ([10; Theorem 1.3]) *Let condition* (P) *hold. If*

$$\int^\infty \left(q(t) - \frac{2}{3\sqrt{3}} \big(-p(t)\big)^{3/2} \right) \mathrm{d}t = \infty, \tag{1.3}$$

then equation (L) *is oscillatory.*

THEOREM C. ([5; Theorem 2.4–2.6]) *Let condition* (P) *hold and* $2q(t) - p'(t) \geq 0$ *for* $t \in I$. *Assume further that for each* $\lambda > 0$ *there exists* $t_\lambda \geq a$ *such that* $q(t) + \lambda p(t) \geq 0$ *for every* $t \geq t_\lambda$ *and such that the equation* $y''' + \big[q(t) + \lambda p(t)\big] y = 0$ *is oscillatory. Finally assume that*

$$\int^\infty p(t)\, \mathrm{d}t > -\infty \quad \textit{or that} \quad |p(t)| < K \qquad \textit{for some} \quad K > 0\,.$$

If $\int^\infty t^2 \big(2q(t) - p'(t)\big)\, \mathrm{d}t = \infty$, *then equation* (L) *is oscillatory.*

The result of Lazer is applicable to the equation with constant coefficients $y''' + p_0 y' + q_0 y = 0$, where $p_0 < 0$, $q_0 > 0$ are some constants, Theorem C of Erbe is not applicable to the equation above. On the other hand, Erbe presented an example (see [5; p. 378, Remark]) when Theorem C is applicable, but Theorem B is not. Neither Theorem B nor Theorem C is applicable to the Euler equation $t^3 y''' + p_0 t y' + q_0 y = 0$, where $p_0 < 0$, $q_0 > 0$ are some constants.

The aim of this paper is to establish some new criteria for equation (L) which extend and improve Theorem B and Theorem C. Our results are applicable to the Euler equation and equations with constant coefficients. Also they are verified easier than Theorem C. Even in the case when $p(t) \equiv 0$, $t \in I$, our result is not worse than condition (1.2).

Remark. For Kneser-type oscillation criteria, the reader is referred to [7], [9] and [14].

2. Some helpful assertions

The following assertions describe the structure of solutions of equation (L). The proofs of these assertions may be omitted since they are similar to proofs in the references.

Let us note that, if y is a solution of equation (L), then also $-y$ is a solution of this equation. Thus, concerning nonoscillatory solutions of (L) we can restrict our attention only to positive ones.

The following lemmas are satisfied even for some third order nonlinear differential equations, see [4], [6], [12], [15], and [16].

LEMMA 2.1. *Let* (P) *hold and y be a nontrivial nonoscillatory solution of* (L). *Then there exists $b \geq a$ such that*

$$y(t)y'(t) < 0 \tag{2.1a}$$

or

$$y(t)y'(t) \geq 0, \quad y(t) \neq 0 \tag{2.1b}$$

for every $t \geq b$.

Furthermore, some positive solution y of type (2.1a) *satisfies*

$$\begin{gathered} y(t) > 0, \quad y'(t) < 0, \quad y''(t) > 0, \quad y'''(t) < 0 \qquad \textit{for all} \quad t \geq a \\ \textit{and} \\ \lim_{t\to\infty} y''(t) = \lim_{t\to\infty} y'(t) = 0, \quad \lim_{t\to\infty} y(t) = L < \infty . \end{gathered} \tag{2.2}$$

Proof. See [10; Lemma 1.1, Lemma 1.3, Theorem 1.1], [5; Lemma 2.2].

LEMMA 2.2. *Let* (P) *hold. Then there exists a positive solution y of* (L) *with property* (2.1a).

Proof. See [10; Theorem 1.1].

THEOREM 2.3. *Let* (P) *hold. A necessary and sufficient condition for* (L) *to be oscillatory is that for any nontrivial nonoscillatory solution y the condition* (2.1a) *hold.*

Proof. See [10; Theorem 1.2].

THEOREM 2.4. *Let* (P) *hold and equation* (L) *be oscillatory. Then any nonoscillatory solution y satisfies*

$$\lim_{t\to\infty} y(t) = 0 .$$

Proof. See [8].

DEFINITION 2.5. Equation (L) is said to have *property A* if each solution y of this equation is either oscillatory, or satisfies condition (2.2) with $L = 0$.

Remark 2.6. From Theorem 2.3 and Theorem 2.4, it follows that equation (L) is oscillatory if and only if it has the property A.

Remark 2.7. From the above results, it follows that, in order to prove oscillatoriness of equation (L), it is sufficient to prove that (L) does not have any nonoscillatory positive solution of type (2.1b).

3. Oscillation criteria

The following lemma is elementary but quite useful in the sequel.

LEMMA 3.1. *Let* (P) *hold. Let Q be the polynomial in the variable z,*

$$Q(z) = \frac{1}{t^4}z^3 - \frac{3}{t^3}z^2 + \left(\frac{2}{t^2} + p(t)\right)z + t^2q(t), \qquad t > 0.$$

Then

$$Q(z) \geq t^2q(t) + tp(t) - \frac{2}{3\sqrt{3}t}\left(1 - t^2p(t)\right)^{3/2} = Q(z_0) \tag{3.1}$$

for all $z \geq 0$.

Remark 3.2. The right-hand side of (3.1) is the local minimum of Q in the point $z_0 = t\Big(1 + 3^{-1/2}\big(1 - t^2p(t)\big)^{1/2}\Big)$.

The following theorem generalizes, improves and extends Theorem B and Theorem C.

THEOREM 3.3. *Let* (P) *hold. If*

$$\int^{\infty} \left[t^2q(t) + tp(t) - \frac{2}{3\sqrt{3}t}\left(1 - t^2p(t)\right)^{3/2}\right] \mathrm{d}t = \infty, \tag{3.2}$$

then equation (L) *is oscillatory.*

Proof. Let y be a nonoscillatory solution of (L). Suppose without loss of generality that y is positive. We prove that y cannot have property (2.1b). To prove this, we assume the contrary, i.e., $y(t) > 0$, $y'(t) \geq 0$, $t \geq b \geq a$. Now, we denote

$$z(t) = \frac{t^2y'(t)}{y(t)}, \qquad t \geq b.$$

So $z(t) \geq 0$, and it is easy to verify that z satisfies the second-order Riccati equation

$$\left[z' + \frac{3}{2}t^{-2}z^2 - 4t^{-1}z\right]' + t^{-4}z^3 - 3t^{-3}z^2 + \left[2t^{-2} + p(t)\right]z + t^2q(t) = 0, \qquad t \geq b. \tag{3.3}$$

Substituting the estimate (3.1) to (3.3) we have

$$\Big[z' + \frac{3}{2}t^{-2}z^2 - 4t^{-1}z\Big]' \le -\Big[t^2q(t) + tp(t) - \frac{2}{3\sqrt{3}t}\big(1 - t^2p(t)\big)^{3/2}\Big] = -Q\big(z_0(t)\big)$$
$$\text{for all} \quad t \ge b\,.$$

Integrating the above inequality from b to $t \ge b$ we get

$$z'(t) + \frac{3}{2}t^{-2}z^2(t) - 4t^{-1}z(t) \le K_0 - \int\limits_b^t Q\big(z_0(s)\big)\,\mathrm{d}s\,,$$

where K_0 is a constant. Since $\frac{3}{2}t^{-2}z^2(t) - 4t^{-1}z(t) \ge -\frac{8}{3}$, integration of the above inequality from b to $t \ge b$ yields

$$z(t) \le K_1 + K_2 t - \int\limits_b^t \int\limits_b^s Q\big[z_0(u)\big]\,\mathrm{d}u\,\mathrm{d}s\,, \tag{3.4}$$

where $K_1 = z(b) + \frac{8}{3}b - K_0 b$, $K_2 = K_0 + \frac{8}{3}$. So it follows from (3.2) and (3.4) that $z < 0$ for sufficiently large t, which contradicts the nonnegativity of z. Therefore equation (L) cannot have any solution with property (2.1b), and, by Remark 2.7, we get the assertion of Theorem 3.3. □

The next result generalizes, improves, and extends [10; Theorem 2.6] and [5; Corollary 2.7–2.8].

COROLLARY 3.4. *Let* (PA) *hold. If*

$$\int^{\infty}\Big[t^2\big[p'(t) - q(t)\big] + tp(t) - \frac{2}{3\sqrt{3}t}\big(1 - t^2p(t)\big)^{3/2}\Big]\,\mathrm{d}t = \infty\,,$$

then equation (L) *is oscillatory.*

P r o o f . The adjoint equation to (L) is

$$y''' + p(t)y' + \big[p'(t) - q(t)\big]y = 0\,. \tag{LA}$$

By results of H a n a n [7, Theorem 3.3, Lemma 2.9], equation (LA) is of Class I, and so equation (L) is of Class II. Hence, by [7; Theorem 4.7], equation (L) is oscillatory if and only if equation (LA) is oscillatory. So, applying Theorem 3.3 to equation (LA) we obtain the assertion of Corollary 3.4.

4. Final remarks and comparisons

In this section, we compare our results with previous results obtained for equation (L).

In the special case when $p(t) \equiv 0$ on I, there is the well-known Kneser-type condition of oscillation for the equation

$$y''' + q(t)y = 0. \tag{4.1}$$

Let $q(t) > 0$ on I. Then (4.1) is oscillatory if

$$\liminf_{t\to\infty} t^3 q(t) > \frac{2}{3\sqrt{3}}, \tag{4.2}$$

see [7; Theorem 5.7]. From (4.2), it follows that there exist $\varepsilon > 0$ and $T \geq a$ such that

$$q(t) - \frac{2}{3\sqrt{3}t^3} \geq \varepsilon t^{-3}, \tag{4.3}$$

and

$$t^2\left[q(t) - \frac{2}{3\sqrt{3}t^3}\right] \geq \varepsilon/t, \tag{4.4}$$

for all $t \geq T$.

For equation (4.1) conditions (1.2) and (3.2) may be rewritten as

$$\liminf_{t\to\infty} t^2 \int_t^\infty \left(q(s) - \frac{2}{3\sqrt{3}s^3}\right) \mathrm{d}s > 0, \tag{4.5}$$

and

$$\int^\infty s^2\left(q(s) - \frac{2}{3\sqrt{3}s^3}\right) \mathrm{d}s = \infty, \tag{4.6}$$

respectively.

Using inequalities (4.3) and (4.4) respectively, we get

Remark 4.1. Let $p(t) \equiv 0$, $q(t) > 0$ on I. Then (4.2) implies (4.5) and (4.2) implies (4.6).

To compare conditions (4.5) and (4.6), we suppose that

$$q(t) - \frac{2}{3\sqrt{3}t^3} \geq 0 \qquad \text{for sufficiently large } t. \tag{4.7}$$

ASSERTION 4.2. *Let* $p(t) \equiv 0$, *and* $q(t) > 0$ *on* I *and assume* (4.7) *satisfied. Then* (4.5) *implies* (4.6).

Proof. Let (4.5) and (4.7) hold. So there exist $\delta > 0$ and $T_1 \geq a$ such that

$$t^2 \int_t^\infty \left(q(s) - \frac{2}{3\sqrt{3}s^3} \right) \mathrm{d}s \geq \delta \qquad \text{for all} \quad t \geq T_1 .$$

If $\int^\infty s^2 \left(q(s) - \frac{2}{3\sqrt{3}s^3} \right) \mathrm{d}s < \infty$, then there exists $T_2 \geq T_1$ such that

$$\int_{T_2}^\infty s^2 \left(q(s) - \frac{2}{3\sqrt{3}s^3} \right) \mathrm{d}s \leq \delta/2 .$$

So we have

$$\begin{aligned}
\delta/2 &\geq \int_{T_2}^\infty s^2 \left(q(s) - \frac{2}{3\sqrt{3}s^3} \right) \mathrm{d}s \\
&= \liminf_{t\to\infty} \left[\int_{T_2}^t s^2 \left(q(s) - \frac{2}{3\sqrt{3}s^3} \right) \mathrm{d}s + \int_t^\infty s^2 \left(q(s) - \frac{2}{3\sqrt{3}s^3} \right) \mathrm{d}s \right] \\
&\geq \liminf_{t\to\infty} \int_t^\infty s^2 \left(q(s) - \frac{2}{3\sqrt{3}s^3} \right) \mathrm{d}s \geq \liminf_{t\to\infty} \int_t^\infty t^2 \left(q(s) - \frac{2}{3\sqrt{3}s^3} \right) \mathrm{d}s \\
&= \liminf_{t\to\infty} t^2 \int_t^\infty \left(q(s) - \frac{2}{3\sqrt{3}s^3} \right) \mathrm{d}s \geq \delta ,
\end{aligned}$$

a contradiction. Hence (4.6) is satisfied. □

Remark 4.3. Condition (1.1) cannot be applied to the Euler equation, while (4.5) and (4.6) may be applied.

Now we suppose that (P) holds, and $p(t) \not\equiv 0$ for $t \in I$. To compare our result with known oscillation criteria, we consider the equation

$$y''' + p_0 t^\beta y' + q_0 t^\delta y = 0 , \tag{4.8}$$

where $p_0 < 0$, $q_0 > 0$ are some constants, and $\delta \geq -3$, $2\delta \geq 3\beta$.

For $\beta = -2$, $\delta = -3$ equation (4.8) becomes the Euler equation. Neither Theorem B nor Theorem C is applicable. The necessary and sufficient condition for oscillation of Euler's equation (4.8) is

$$q_0 + p_0 - \frac{2}{3\sqrt{3}}(1 - p_0)^{3/2} > 0 . \tag{4.9}$$

It is easy to check that for oscillation of Euler's equation (4.8) condition (4.9) is equivalent to condition (3.2) of Theorem 3.3.

In the case $\delta = -3$, $2\delta > 3\beta$, Theorem B is not applicable and Theorem C is applicable only when $\beta < -3$, and $q_0 > 2/(3\sqrt{3})$, also see example of E r b e in [5; p. 378, Remark].

ASSERTION 4.4. *Let* $\delta = -3$, $\beta < -2$, *and* $q_0 > 2/(3\sqrt{3})$. *Then equation* (4.8) *is oscillatory.*

P r o o f. Since

$$(1+x)^{3/2} = 1 + \frac{3}{2}x + \frac{3}{8}x^2 + \frac{3x^2}{4}\sum_{k=1}^{\infty}\frac{(-1)^k(2k-1)!!}{2^k(k+2)!}x^k\,, \qquad |x| < 1\,, \quad (4.10)$$

where $(2k-1)!! = (1)(3)(5)\dots(2k-1)$, substituting the coefficients of equation (4.8) to the left-hand side of (3.2) for $t > a_0 \ge (-p_0)^{-1/(\beta+2)}$ we obtain

$$\begin{aligned}
&\int_{a_0}^{\infty}\frac{1}{t}\Big[q_0 + p_0t^{\beta+2} - \frac{2}{3\sqrt{3}}\big(1 - p_0t^{\beta+2}\big)^{3/2}\Big]\,\mathrm{d}t\\
&= \int_{a_0}^{\infty}\frac{1}{t}\Big[q_0 + p_0t^{\beta+2} - \frac{2}{3\sqrt{3}}\Big(1 - \frac{3}{2}p_0t^{\beta+2} + \frac{3}{8}p_0^2t^{2\beta+4} + \dots\Big)\Big]\,\mathrm{d}t\\
&= \int_{a_0}^{\infty}\frac{1}{t}\Big[q_0 - \frac{2}{3\sqrt{3}} + p_0t^{\beta+2} - \frac{2}{3\sqrt{3}}\Big(-\frac{3}{2}p_0t^{\beta+2} + \frac{3}{8}p_0^2t^{2\beta+4} + \dots\Big)\Big]\,\mathrm{d}t\,.
\end{aligned}$$

Since $q_0 > 2/(3\sqrt{3})$, $\beta + 2 < 0$, it is easy to see that condition (3.2) is fulfilled. So the assertion follows immediately by Theorem 3.3. □

In the case $\delta > -3$, $2\delta = 3\beta$, Theorem B may be applied only when $\delta \ge -1$, and $q_0 > 2(-p_0)^{3/2}/(3\sqrt{3})$. Theorem C is not at all applicable.

ASSERTION 4.5. *Let* $\delta > -3$, $2\delta = 3\beta$, *and* $q_0 > 2(-p_0)^{3/2}/(3\sqrt{3})$. *Then equation* (4.8) *is oscillatory.*

P r o o f. Since $\delta > -3$, $2\delta = 3\beta$, there exists $\varepsilon > 0$ such that $\delta = -3 + \varepsilon$, and $\beta = -2 + (2/3)\varepsilon$. Let $t > a_0 \ge (-p_0)^{-3/2\varepsilon}$. Substituting the coefficients of equation (4.8) to the left-hand side of (3.2) and using (4.10) we get

$$\int_{a_0}^{\infty}\left[q_0 t^{-1+\varepsilon}+p_0 t^{-1+2\varepsilon/3}-\frac{2}{3\sqrt{3}t}\left((-p_0t^{2\varepsilon/3})\left(1-\frac{t^{-2\varepsilon/3}}{p_0}\right)^{3/2}\right]dt$$

$$=\int_{a_0}^{\infty} t^{-1+\varepsilon}\left[q_0+p_0t^{-\varepsilon/3}-\frac{2}{3\sqrt{3}}(-p_0)^{3/2}\left(1-\frac{3t^{-2\varepsilon/3}}{2p_0}+\frac{3t^{-4\varepsilon/3}}{8p_0^2}+\dots\right)\right]dt$$

$$=\int_{a_0}^{\infty} t^{-1+\varepsilon}\left[q_0-\frac{2}{3\sqrt{3}}(-p_0)^{3/2}+p_0t^{-\varepsilon/3}\right.$$

$$\left.-\frac{2}{3\sqrt{3}}(-p_0)^{3/2}\left(-\frac{3t^{-2\varepsilon/3}}{2p_0}+\frac{3t^{-4\varepsilon/3}}{8p_0^2}+\dots\right)\right]dt=\infty$$

since $q_0-2(-p_0)^{3/2}/(3\sqrt{3})>0$. The proof is complete. □

In the last case $\delta>-3$, $2\delta>3\beta$, Theorem B may be applied again only when $\delta\ge -1$. Theorem C is applicable only when $\beta<\delta$, and $\beta<0$.

ASSERTION 4.6. *Let $\delta>-3$, $2\delta>3\beta$. Then equation* (4.8) *is oscillatory.*

Proof. Similarly as before, after substituting the coefficients of equation (4.8) to the left-hand side of (3.2) and using (4.10) for sufficiently large t, we obtain:

$$\int_{a_0}^{\infty}\left[q_0t^{\delta+2}+p_0t^{\beta+1}-\frac{2}{3\sqrt{3}t}(-p_0)^{3/2}t^{3(\beta+2)/2}\left(1-\frac{3t^{-\beta-2}}{2p_0}-\frac{3t^{-2\beta-4}}{8p_0^2}+\dots\right)\right]dt,$$

$$\text{for } \beta+2>0;$$

$$\int_{a_0}^{\infty}\left[q_0t^{\delta+2}+p_0t^{-1}-\frac{2}{3\sqrt{3}t}(1-p_0)^{3/2}\right]dt,\qquad \text{for } \beta+2=0;$$

$$\int_{a_0}^{\infty}\left[q_0t^{\delta+2}+p_0t^{\beta+1}-\frac{2}{3\sqrt{3}t}\left(1-\frac{3}{2}p_0t^{\beta+2}+\frac{3}{8}p_0^2t^{2\beta+4}+\dots\right)\right]dt,$$

$$\text{for } \beta+2<0.$$

It is easy to check that $\delta+2>\beta-1$, and since $\delta+2>-1$, all the integrals above satisfy (3.2). So, from Theorem 3.3, it follows that equation (4.8) is oscillatory. □

R e m a r k 4.7. If condition (PA) holds, then using Corollary 3.4 we can derive analogous assertions for equation (4.8).

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