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Mathematica Slovaca, Vol. 35 (1985), No. 3, 243--249

Persistent URL: http://dml.cz/dmlcz/130472

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PROLONGATION OF NATURAL BUNDLES

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We discuss some special aspects of the theory of natural functions (see [3], [5], [6], [7]) in the case of fibre bundles. Our considerations are in the category C^∞.

1. Point fibre frames. Let $(N, \bar{y})$ be a manifold with a fixed point $y \in N$.

Definition 1. *Let $\pi: Y \to X$ be a fibre space with a fibre type $(N, \bar{y})$, $\dim X = m$. The set of r-jets $J^r_{(0,\bar{y})}\Phi$ of all local fibre isomorphisms Φ from $R^m \times N$ to Y will be called the space of point fibre frames and denoted $FH^r_{\bar{y}}Y$.*

Let $FL^r_m N_{\bar{y}}$ be the Lie group of all r-jets of local isomorphisms of the fibre space $p_1: R^m \times N \to R^m$ with source and target in $(0, \bar{y})$, the composition law of which is given by the jet composition.

Proposition 1. *Let β be the target jet projection. Then the space $\beta: FH^r_{\bar{y}}Y \to Y$ is a principal fibre bundle with the structure group $FL^r_m N_{\bar{y}}$.*

Proof is routine.

Remark 1. In the definition of the manifolds $FH^r_{\bar{y}}Y$, $FL^r_m N_{\bar{y}}$ the space $(R^n, 0)$, $n = \dim N$, can be used instead of $(N, \bar{y})$. In this case we use the notations $FH^r Y$, $FL^r_{m,n}$. It is easy to see that $FH^r Y$ is a reduction of the space $H^r Y$ of all r-frames on Y to the subgroup $FL^r_{m,n}$ of the group L^r_{m+n} of r-jets of all O-preserving local diffeomorphisms of R^{m+n}.

Let us describe some properties of the group $FL^r_m N_{\bar{y}}$. Let $DL(N)$ be the set of all local diffeomorphisms on N. Remember that a local map from M to $DL(N)$ is differentiable, i.e. $\varphi \in CL(M, DL(N))$, if the map $\bar{\varphi}$, $\bar{\varphi}(x, y) = \varphi(x)(y)$, from $M \times N$ to N is differentiable. We define $j^r_{(0,\bar{y})}\varphi := j^r_{(0,\bar{y})}\bar{\varphi}$. Let φ_1, $\varphi_2 \in CL(M, DL(N))$. Put $\varphi_1 \circ \varphi_2(x) = \varphi_1(x) \cdot \varphi_2(x)$ where in all our considerations the dot means the composition of maps or jets. Let $L^r_m N_{\bar{y}}$ be the set of r-jets $j^r_{(0,\bar{y})}\varphi$ of all maps $\varphi \in CL(R^m, DL(N))$ that $\varphi(0)(\bar{y}) = \bar{y}$. Let $a_1 = j^r_{(0,\bar{y})}\varphi_1$, $a_2 = j^r_{(0,\bar{y})}\varphi_2 \in L^r_m(N)_{\bar{y}}$. Then $a_1 \circ a_2 = j^r_{(0,\bar{y})}(\varphi_1 \circ \varphi_2)$ is the composition rule of the Lie group $L^r_m N_{\bar{y}}$. Denote by $L^r_m(N, id)_{\bar{y}}$ or $L^r_{\bar{y}}N$ the Lie group of such $j^r_{(0,\bar{y})}\varphi \in L^r_m N_{\bar{y}}$ for which $j^r_{\bar{y}}\varphi(0) = j^r_{\bar{y}} id_N$ or $\varphi(x) = \varphi(0)$, $x \in R^m$, respectively. Clearly the group $L^r_{\bar{y}}N$ can be identified with the group of r-jets $j^r_{\bar{y}}g$ of all local diffeomorphisms of N such that $g(\bar{y}) = \bar{y}$. It is easy to show that $L^r_m(N, id)_{\bar{y}}$ is a normal subgroup of $L^r_m N_{\bar{y}}$, $L^r_y \cap L^r_m(N, id)_{\bar{y}} = \{e\}$ and $L^r_{\bar{y}}(N) \circ L^r_m(N, id)_y = L^r_m N_{\bar{y}}$.

Remark 2. Let $A = j^r_{(0,\bar y)}\varphi \in L^r_m N_{\bar y}$. Then ε: $L^r_m N_y \to L^r_{\bar y} N \times T^r_m N_y$, $\varepsilon(A) = (j^r_{\bar y}\varphi(0),\ j^r_0\bar\varphi(x, \bar y))$ is a diffeomorphism iff $r = 1$. Identifying $T^1_m N_{\bar y}$ with $T_{\bar y}N \otimes (R^m)^*$ we get an Abelian group structure on $T^1_m N_y$. According to the left-hand action of $L^1_{\bar y}$ on $T^1_m N_{\bar y}$ given by the jet composition we construct the semi-direct product $L^1_{\bar y}N \bar\times R^1_m N_{\bar y}$ of the groups $L^1_y N$ and $T^1_m N_{\bar y}$ with the composition rule $(a_1, b_1)(a_2, b_2) = (a_1 \cdot a_2,\ b_1 + a_1 \cdot a_1 b_2)$. In this case ε is an isomorphism of groups.

Every local isomorphism Φ of trivial fibre space p_1: $R^m \times N \to R^m$ determines the local diffeomorphism $f = p_1 \Phi$ on R^m and $\varphi \in CL(R^m, DL(N))$, $\varphi(x) = \Phi|_{\{x\}\times N}$ so that $\Phi(x, y) = (f(x), \varphi(x)(y))$. In local coordinates (x^i) on R^m and (y^α) on N we have for Φ: $\hat x^i = f^i(x^j)$, $\hat y^\alpha = \varphi^\alpha(x^i, y^\beta)$. *This gives*

Lemma 1. *Let $\Phi_i = (f_i, \varphi_i)$, $i = 1, 2$ be two local isomorphi ms of $R^m \times N$. Then*

$$(j^r_{(0,\bar y)}\Phi_1 = j^r_{(0,\bar y)}\Phi_2) \Leftrightarrow (j^r_0 f_1 = j^r_0 f_2,\ j^r_{(0,y)}\varphi_1 = j^r_{(0,y)}\varphi_2)$$

The group L^r_m of r-jets $j^r_0 f$, where f is a local diffeomorphism of R^m such that $f(0) = 0$, acts on the right-hand side on $L^r_m N_y$ by the rule $j^r_{(0,y)}\varphi \Leftrightarrow j^r_{(0)}f = j^r_{(0,y)}(\varphi \cdot f)$. Let $L^R_m \bar x L^r_m N_{\bar y}$ be the semi-direct product with the group operation

$$(a_1, A_1)(a_2, A_2) = (a_1 \cdot a_2,\ (A_1 \cdot a_2) \circ A_2) \tag{1}$$

It is easy to prove

Lemma 2. *Let $a = j^r_{(0,\bar y)}\Phi \in FL^r_m N_y$, $\Phi(f, \varphi)$. Then the map i: $FL^r_m N_y \to L^r_m \bar\times L^r_m N_y$, $i(a) = (j^r_0 f,\ j^r_{(0,\bar y)}\varphi)$ is an isomorphism of groups.*

Remark 3. Let $c = j^r_{(0,\bar y)}\psi \in FH^r_{\bar y}Y$. Denote $c_1 = j^r_0(z \mapsto \pi \cdot \psi(z, \bar y) = g(z)) \in H^r X$, $c_2 = j^r_{\bar y}(\psi|_{\{0\}\times N}) \in FJ^r_{\bar y}(N, Y)$, $c_3 = j^r_{g(0)}(x \mapsto \psi(g^{-1}(x), \bar y)) \in J^r Y$, where $FJ^r_{\bar y}(N, Y)$ is a manifold of all r-jets $j^r_y\xi$ of all local diffeomorphisms from N to fibres of Y. Clearly the map π^r: $FH^r_y Y \to H^r X \times_X [FJ^r_{\bar y}(N, Y) \times_Y J^r Y]$, $\pi^r(c) - (c_1, c_2, c_3)$ is a submersion. If $r = 1$, then in the coordinates

$$(x^i, y^\alpha, A^i_j, A^\alpha_i, A^\beta_\alpha) \xrightarrow{\pi^1} (x^i, y^\alpha, A^i_j, A^\alpha_\beta, A^\alpha_i A^i_j)$$

where $A^k_i \bar A^i_j = \delta^k_j$. Every $A \in J^1_{x_0}Y$, $\beta A = y_0 \in Y$ determines a map A': $T_{x_0}X \to T_{y_0}Y$ such that $T\pi \cdot A' = id_{T_{x_0}X}$ and vice versa.

The group $L^1_m \bar x(L^r_y N \bar\times T^1_m N_{\bar y})$ acts on the right-hand side on $H^1 X \times_X [FJ^1_{\bar y}(N, Y) \times_Y J^1 Y]$ by the rule $(H, B, A)(h, b, a) = (H \cdot h,\ B \cdot b,\ A' + B' \cdot a' \cdot h'^{-1} \cdot H'^{-1})$ where the prime denotes the maps of the corresponding tangent spaces determined by 1-jets. Then (π^1, i) is an isomorphism of principal fibre bundles. It is directly seen that π_σ: $FH^r_y Y \to H^r X \times_X FJ^r_{\bar y}(N, Y)$ or π_H: $FH^r_{\bar y}Y \to H^r X \times_X Y$ is the principal fibre bundle with the structure group $L^r_m(N, \mathrm{id})_{\bar y}$ or $L^r_m N_y$, respectively.

2. Fibre base r-frames. Let us remember the notion of fibre r-jets, see [2].

Definition 2. *Let π_i: $Y_i \to X_i$, $i=1,2$, be two fibre spaces. Then the fibre morphisms ψ, $\bar{\psi}$: $Y_1 \to Y_2$ belong to the same fibre r-jet $j^r_{x_0|B}\psi$ with the source $x_0 \in X_1$ if $j^r_y\psi = j^r_y\bar{\psi}$ for any $y \in Y_1$, $\pi_1 y = x_0$. The point $\bar{x} = \pi_2\psi(y)$, $\pi_1 y = x_0$ is called the target of $j^r_{x_0|B}\psi$.*

Let π: $Y \to X$ be a fibre space with the type fibre N. By LB^r_mN we mean the set of all fibre r-jets $j^r_{0|B}\Phi$ of local isomorphisms $\Phi = (f, \varphi)$ of the space $R^m \times N$ such that $f(0)=0$, $\varphi \in CL(R^m, D(N, N))$, where $D(N, N)$ is the set of all diffeomorphisms of N. Let $J^r_{0|B}\Phi$, $j^r_{0|B}\Phi' \in LB^r_mN$. Then the group structure on LB^r_mN determined by the composition rule

$$(j^r_{0|B}\Phi)(j^r_{0|B}\Phi') = j^r_{0|B}(\Phi \cdot \Phi')$$

is not a Lie group structure in the classical sense.

Definition 3. *The set FH^r_BY of all fibre r-jets $j^r_{0\,B}\psi$ of local isomorphisms ψ: $R^m \times N \to Y$, whose domain is a set $p_1^{-1}(U)$, where U is an open set in X, is called the space of basic r-frames on Y.*

Let $\pi^r_B(a)$, $a \in FH^r_BY$, be the target of a. The set $(FH^r_BY)_x = \{a \in FH^r_BY, \pi^r_B(a) = x \in X\}$ is called the fibre over x. Let us recall that π^r_B: $FH^r_BY \to X$ is not a fibre manifold in the classical sence. Let $B = j^r_{0|B}\psi \in FH^r_BY$, $A = j^r_{0|B}\Phi \in LB^r_mN$. Denote $B \cdot A = j^r_{0|B}(\psi \cdot \Phi)$ and $\varkappa(B, A) = B \cdot A$. It is easy to prove

Proposition 2. *The map $\varkappa$: $FH^r_BY \times LB^r_mN \to FH^r_BY$, $\varkappa(B, A) = B \cdot A$ is a right-hand fibre preserving action of the group LB^r_BN on FH^r_BY and is free and transitive on fibres of FH^r_BY.*

Definition 4. *Let G be both a Lie group and an algebraic subgroup of LB^r_BN. Every subset P of FH^r_BY, which is a principal G-fibre bundle over X, will be called a reduction of FH^r_BY to group G and is said to be a space of G-basic r-frames on Y.*

3. G-frames. Let G be a Lie group, ε: $G \times N \to N$ be its left-hand action on N and let $\hat{g}$: $N \to N$, $\hat{g}(y) = \varepsilon(g, y)$ be the diffeomorphism determined by $g \in G$. We use $\widehat{g_1g_2}(y) = \hat{g}_2(\hat{g}_1(y))$.

Definition 5. *The action ε is said of order k (at $\bar{y} \in N$) if $j^k_y\hat{g}_1 = j^k_y\hat{g}_2 \Rightarrow g_1 = g_2$ for any $y \in N$ (for $y = \bar{y}$).*

Let $H^r_{\bar{y}} = \{g \in G, j^r_{\bar{y}}\hat{g} = j^r_{\bar{y}}\, \mathrm{id}_N\}$ be the isotropic group of order r at $\bar{y} \in N$.

Lemma 3. *If the action ε is effective, ($\hat{g} = \hat{h} \Rightarrow g = h$), then it is of order k at $\bar{y}$ iff $H^k_{\bar{y}} = \{e\}$ where e is the unit of G.*

Definition 6. *A local isomorphism $\Phi = (f, \varphi)$ of $R^m \times N$ is called a G-isomorphism if φ is such a local map from R^m to G that $\Phi(x, y) = (f(x), \varphi(x)(y))$. A G-isomorphism Φ is said to be trivial if $\Phi = (\mathrm{id}_{R^m}, \varphi) \equiv \Phi_\varphi$.*

Lemma 4. *Let* φ_1, φ_2: $R^m \to G$ *and let* $j_0^r\varphi_1 = j_0^r\varphi_2 \in T_m^r G$. *Then* $j_{(0,y)}^r \Phi_\varphi = j_{(0,y)}^r \Phi_{\varphi_2}$ *for any* $y \in N$.

Proof follows from identity

$$\Phi_\varphi = (\mathrm{id}_{R^m} \times \varepsilon)\cdot(\mathrm{id}_{R^m} \times \varphi_i \times \mathrm{id}_N)(\Delta \times \mathrm{id}\ )$$

where Δ: $R^m \to R^m \times R^m$, $\Delta(x) = (x, x)$ is the diagonal map

Corollary 1. *If* $j_0^r\varphi_1 = j_0^r\varphi_2 \in T_m^r G$, *then* Φ_{φ_1}, Φ_{φ_2} *belong to the same fibre r-jet with source and target* $0 \in R^m$. *This gives the map* ξ: $T_m^r G \to LB_m^r N$, $\xi(j_0^r\varphi) = j_{0|B}^r \Phi_\varphi$.

Let $GL_{my}^r N$ be the manifold of all jets $j_{(0,y)}^r \Phi_\varphi$ where Φ_φ is a trivial G-morphism. We will describe th set

$$\eta^{-1}(j_{(0,y)}^r \mathrm{id}_{R^m \times N}) \quad \text{for} \quad \eta\colon T_m^r G \to GL^r\ N, \quad \eta(j_0^r\ )\ j_{(0\ y)}^r \Phi_\varphi.$$

Let M, Q be differentiable m nifolds and $S \subset Q$ be a clo ed submanifold of Q. A mapping h: $M \to Q$ is sa'd to have the contact of order r with S at $x_0 \in M$ if there is such a map $\bar{h}$: $M \to S$ that $j_x^r \bar{h} = j_x^r h$. The action ε of G on N determines the mappings ε_y^k: $G \to J_y^k(N, N)$, $\varepsilon_y^k(h) - j_y^k \hat{h}$, $\varepsilon_y^0(h) - \hat{h}(y)$. The action ε is called r-normal if $\dim(\varepsilon_y^k(G)) = q - d_k$, $q - \dim G$, $d\ - \dim H^k$, for $k = 0, .., r$.

Lemma 5. *Let the action* ε *of G on N be r-normal. Then* $j_{(0,\ )}^r \Phi_\varphi - j_{(0,y)}^r \mathrm{id}_{R^m \times N}$ *iff* φ: $R^m \to G$ *has the contact of order k with* $H_y^{r\ k}$ *at* $0 \in R^m$ *for* $k = 0, \dots, r$.

Proof. There is a sequence $H_y^0 \supset H_y^1 \supset \dots \supset H_y$ of the clo ed i otropic subgroups of the point $\bar{y} \in N$. There is a local chart (z^p) on G such that $e = (0, \dots, 0)$ and $(z^1, \dots, z^{d_k}, 0, \dots, 0) \in H_y^k$, $k = 0, .., r$. In this chart let ε: $G \times N \to N$ be given by $\hat{y}^\alpha = F^\alpha(y^\beta, z^p)$. Then for $a = (z^1, \dots, z^{d_j}, 0, .., 0) \in H_y^j$

(2)
$$F^\alpha(y, a) - \bar{y}^\alpha, \quad \partial F^\alpha(\bar{y}, a)/\partial y^\beta - \delta^\alpha,$$
$$\partial^{s+k} F^\alpha(y, a)/\partial y^{\beta_1} \dots \partial y^{\beta_s} \partial z^{p_1} \dots \partial z^{p_k} = 0$$

where $s = 0, 1, \dots, j$, $p_i - 1, \dots, d_j$, $j - 0, \dots, r$. Let φ be given by $z^p = \varphi^p(x^i)$ and let $j_0^r\varphi = (a^p, a_i^p, .., a_{i_1\ i_r}^p$, $\varphi(0) = a$. Then the equations for Φ_φ are: $\hat{x}^i = x^i$, $\hat{y}^\alpha = F^\alpha(y^\beta, z^p = \varphi^p(x))$ and $j_{(0\ \bar{y})}^r \Phi_\varphi - (b^\alpha, b_i^\alpha, ., b^\alpha\ _i, .., b_{\beta_1\ \beta_{k i_1}\ i_u}^\alpha, b_\beta^\alpha, \dots, b_{\beta_1 . \beta_r}^\alpha)$, where

(3)
$$b_{i_1 . i_u}^\alpha - \sum_{s=1}^{u} \partial^s F^\alpha(y, a)/\partial z^{p_1} \dots \partial z^{p_s} \sum a_{\sigma 1}^{p_1} \dots a_{\sigma}^{p_s}, \quad u = 1, \dots, r$$

(4)
$$b_{\beta_1 . \beta_k i\ . i_u}^\alpha = \sum_{s\ 1}^{u} \partial^{s+k} F^\alpha(\bar{y}, a)/\partial y^\beta \dots \partial y^{\beta_k} \partial z^{p_1} .. \partial z^{p_s} \sum_\sigma a^{p_1}_1 \dots a_\sigma^{p_s},$$
$$k\ \ 1, .., r-1; \quad u = 1, .., r-k$$

(5)
$$b_\beta^\alpha\ \ \partial F^\alpha(y, a)/\partial y\beta, \ .., b_{\beta_1}^\alpha \quad \partial\ F^\alpha(\bar{y}, a)/\partial y^\beta ..\ y^{\beta_r}$$

where σ denotes the set of all σ-decompositions of the sequence $i_1 \dots i_u$ (an s-part decomposition π of the sequence $i_1 \dots i_u$ is called a σ-decomposition if $\pi(i_1 \dots i_u) = \sigma_1 \dots \sigma_s = (i_{c_1} \dots i_{h_1})(i_{c_2} \dots i_{h_2}) \dots (i_{c_s} \dots i_{h_s})$ and $c_1 < c_2 < \dots < c_s$, $c_j < \dots < h_j$. For instance $\pi(i_1 i_2 i_3 i_4) = (i_1 i_3)(i_2 i_4)$ or $= (i_1)(i_2 i_4) i_3$ are examples of σ-decompositions). Since $j^r_{(0,\bar{y})} \mathrm{id}_{R^m \times N} = (\bar{y}^\alpha, b^\alpha_\beta = \delta^\alpha_\beta, 0, \dots, 0)$ the assertion of our Lemma follows from (2), (3), (4), (5).

We supose that the action ε is r-normal. Then by Lemma 5 we can prove

Lemma 6. *The group homomorphism* $\xi\colon T^r_m G \to LB^r_m N$ *is injective iff the action* ε *is effective.*

Corollary. *If the action* ε *is effective, then the group of all fibre r-jets* $j^r_{0|B}\Phi_\varphi$ *of all trivial G-isomorphisms* Φ_φ *can be identified with the group* $T^r_m G$. *The homomorphism* ξ *can be extended on* $\xi\colon L^r_m \bar{\times} T^r_m G \to LB^r_m N$, $\xi(j^r_0 f, j^r_0 \varphi) = j^r_{0|B}(f, \varphi)$, *where* $L^r_m \bar{\times} T^r_m G$ *denotes the semi-direct product of the groups,* $(a, A)\cdot(b, B) = (a\cdot b, (A\cdot b)B)$. *Then the group of all fibre r-jets* $j^r_{0|B}\Phi$ *of all G-isomorphisms* Φ *can be identified with* $L^r_m \bar{\times} T^r_m G$ *iff the action* ε *is effective.*

Lemma 5 implies

Assertion. *The map* η *is injective iff* $H^0_y = \{e\}$, *i.e. iff the action* ε *is free at* $\bar{y} \in N$, *i.e. iff* $\hat{g}_1(\bar{y}) = \hat{g}_2(\bar{y}) \Rightarrow g_1 = g_2$.

Corollaries: 1. *If the action* ε *is free at* $\bar{y} \in N$, *then the group of r-jets* $j^r_{(0,\bar{y})}\Phi$ *of local G-isomorphisms* $\Phi = (f, \varphi)$ *such that* $\Phi(0, \bar{y}) = (0, \bar{y})$ *can be identified with* $L^r_m \bar{\times} T^r_m G_e$, *where* $j^r_0 \varphi \in T^r_m G_e \Leftrightarrow \varphi(0) = e$.

2. Let $LB^r_m N_G$ be the set of all fibre r-jets $j^r_{0|B}\Phi_\varphi$ of all local trivial G-isomorphisms of $R^m \times N$. The map $\zeta\colon LB^r_m N_G \to GL^r_{m\bar{y}} N$, $\zeta(j^r_{0|B}\Phi_\varphi) = j^r_{(0,\bar{y})}\Phi_\varphi$ is injective iff the action ε is free. In this case the manifold $GL^r_{m\bar{y}} N$ is the Lie group which can be identified with $T^r_m G$ and with $LB^r_m N_G$.

Let $\pi\colon P \to X$ be a principal fibre bundle. Its structure group G acts transitively and freely on itself by the right translation $\hat{a}(g) = ga$. Therefore the group $L^r_m BG$ of fibre r-jets $j^r_{0|B}\Phi$ of all local G-isomorphisms Φ of $R^m \times G$ is identified with $L^r_m \bar{\times} T^r_m G$. Let $F\colon R^m \times G \to P$ be a local isomorphism of fibre bundles. Then $f(z) = \pi \cdot F(z, e)$ or $\sigma_F(x) = F(f^{-1}(x), e)$ is a local isomorphism from R^m to X or a local cross-section of P, respectively, so that $F(z, g) = [\sigma(f(z))]\bar{g}$ and $j^r_{(0,g)}F = j^r_{\sigma_F f(0)}\bar{g} \cdot j^r_{f(0)}\sigma_F \cdot j^r_0 f$, where $\bar{g}$ denotes the diffeomorphism of P determined by $g \in G$. It yields

Lemma 7. *Local isomorphisms* $F_1 = (f_1, \sigma_{F_1})$, $F_2 = (f_2, \sigma_{F_2})$ *of principal fibre bundles* $R^m \times G$, P *belong to the seme fibre r-jet* $j^r_{0|B}F_1$ *iff* $j^r_0 f_1 = j^r_0 f_2$, $j^r_{f_1(0)}\sigma_{F_1} = j^r_{f_2(0)}\sigma_{F_2}$.

Corollaries: 1. *The space* $W^r P$ *of fibre r-jets* $j^r_{0|B}$ *of all local isomorphisms from* $R^m \times G$ *to* P *can be identified with the Whitney sum* $H^r X \times_X J^r P$, *which is the* *principal fibre bundle with the structure group $L^r_m \rtimes T^r_m G$, see [1]. Hence $W^r P$ is a reduction of the space $FH^r_B P$ of all basic r-frames on P to the group $L^r_m \rtimes T^r_m G$.*

2. Let $p: Q \to X$ be a fibre bundle associative to P with a fibre type N on which the group G acts effectively on the left-hand side. Quite analogously to the above it can be shown that $W^r P = H^r X \times_X J^r P$ is the reduction of the space $FH^r_B Q$ of all basic r-frames on Q to the group $L^r_m \rtimes T^r_m G$.

Remark 4. It is known (see [3]) that the space $H^r X \times J^r P \to P$ is a principal $L^r_m \rtimes T^r_m G_e$-bundle. It is clear that it is the reduction both of the space $FH^r_e P$ of all point r-frames on P and of the space $FH^r_e Q$ (where Q is a fibre space associated to P) to the group $L^r_m \rtimes T^r_m G_e$.

4. Natural fibre functor. Let FB be the category of fibre bundles, B_m be the category of manifolds M ($m = \dim M$) whose morphisms are diffeomorphisms, FB_m be the category whose objects (morphisms) are m-dimensional manifolds (fibre morphisms over diffeomorphisms of bases).

A natural functor F restricted to the category FB_m will be called fibre, i.e. if (π: $Y \to X) \in \mathrm{Obj}(FB_m)$ and (f: $Y_1 \to Y_2) \in \mathrm{Mor}(FB_m)$, then ($\pi_F$: $FY \to Y) \in \mathrm{obj}(FB)$ and the morphism Ef: $FY_1 \to FY_2$ is over f.

Remark 5. If F is a natural fibre functor, then the rule $F_m(Y) = (\pi \cdot \pi_F$: $FY \to X)$ determines a functor F_m from FB_m to FB_m.

Let us recall that a natural fibre functor F is of order r if $j^r_y f = j^r_y g$ implies $Ff|_{(FY)_y} = Fg|_{(FY)_y}$ for any fibre morphisms (f, g: $Y \to \bar{Y}) \in \mathrm{Mor}(FB_m)$.

Example. The prolongation functor J^r from FB_m to FB($J^r Y$ is the r-jet prolongation of Y) is a natural fibre functor of order r.

Remark 6. Let F be a natural fibre functor of order r. Then every jet $A = j^r_y f \in J^r_y(Y, \bar{Y})_{\bar{y}}$ defines a map $\bar{A}$: $(FY)_y \to (F\bar{Y})_{\bar{y}}$, $\bar{A} = Ff|_{FY_y}$.

A small modification of the well-known assertions in the theory of natural bundles gives

Proposition 3. *Let F be a natural fibre functor of order r. Let π: $Y \to X$ be a fibre bundle, $m = \dim X$, $n + m = \dim Y$. Let N_F be the fibre of $F(R^m \times R^n)$ over $(0, 0) \in R^m \times R^n$. Then π_F: $FY \to Y$ is associated to the principal fibre bundle $FH^r Y$ of all point r-frames on Y with the type fibre N_F.*

Let $P(K) \to X$ be a reduction of the space $FH^r_B Y$ of all basic r-frames on Y to a Lie group $K \subset LB^r_m R^n$ of fibre jets $j^r_{0|B}\Phi$ of all local isomorphisms of $R^m \times R^n$.

Proposition 4. *Let F be natural fibre functor of order r. Let N_{F_m} be the fibre of $F_m(R^m \times R^n) \to R^m$ over $O \in R^m$. Then the space $\pi \cdot \pi_F$: $FY \to X$ with fibre type N_{F_m} is associated to $P(K)$.*

Remark 7. Let $P \to X$ or $Q \to X$ be a principal fibre bundle with a structure group G or a space associated to P with a fibre type N on which the group G acts effectively. Then $W^r P = H^r X \times J^r P$ is the reduction of $FH^r_B Y$ to the group $L^r_m \rtimes T^r_m G$. It is well known that if ψ is a natural functor of order k, then the space

$\psi X \to X$ is associated to $H^k X$. Therefore $F\psi X \to X$ is associated to $W^r(H^k X) = H^r X \times_X J^r H^k X$. Since $F\psi$ is a natural functor of order $k+r$, then $F\psi X$ is associated to the principal fibre bundle $H^{k+r}X$, which is a reduction of $W^r(H^k X)$ to the group $L_m^{r+k} \subset L_m^r \bar{\times} T_m^r L_m^k$.

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Received February 24, 1983

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ПРОДОЛЖЕНИЕ НАТУРАЛЬНЫХ РАССЛОЕННЫХ ПРОСТРАНСТВ

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Резюме

В статье исследованы некоторые специальные аспекты теории натуральных функторов в случае расслоенных пространств.