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TWO-SIDED SYMMETRIC CONDITION IN THE ANALYSIS OF MAGNETIC FIELDS WITH PERMANENT MAGNETS*

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Abstract. Mathematical treatment of a planar magnetic field excited by permanent magnets is presented. A special two-sided condition for differential magnetic reluctivity is introduced to prove the unique existence of both the weak and the approximate solutions and also a certain error estimate. Notes to numerical algorithm and practical applications are given.

Keywords: magnetic field with permanent magnets, variational formulation, two-sided unique existence condition, finite element method

MSC 2000: 65N30

1. Introduction

A special two-sided symmetric condition for the differential (incremental) magnetic reluctivity was introduced in [14] to prove the unique existence of both the exact and the approximate solutions of an isotropic planar nonlinear magnetic field generated by electromagnets only. At the same time a certain estimate of the error between these two solutions was given. However, permanent magnets, for their good properties, are also used in modern electrical devices to excite the magnetic field. Efficient algorithms for numerical computation of magnetic fields with permanent magnets, based mostly on the finite element technique, have appeared very often in literature (see, e.g., [1]–[11], [16], [17]) but the corresponding papers do not usually contain any mathematical treatment. Nevertheless, an attempt at solving the problem theoretically is given in [12].

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† Professor F. Melkes passed away before the end of the referee process. The Editorial Board of Application of Mathematics, therefore, decided to publish his last scientific paper without any changes proposed by reviewers.

Situation with permanent magnets does not differ very much from the case without permanent magnets and therefore it is possible to analyse both cases in a similar way. In the present paper we will generalize the method described in [14] to the case when both sources of the magnetic field, electromagnets and permanent magnets, are accepted.

2. Preliminaries

Let $\Omega \subset R^2$ be a bounded domain with Lipschitz boundary. In the sequel we will use the obvious notation from the functional analysis such as the Lebesgue space $L_2(\Omega)$, the Sobolev spaces $H^1(\Omega)$, $W^{1,p}(\Omega)$ $(p > 2)$, the space of all square summable infinite sequences l_2 and the trace $\operatorname{tr} u$ of a function u. In the two-dimensional case we use, in agreement with [14], the notation

$$\operatorname{grad} u = \Big(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\Big), \quad \operatorname{curl} u = \Big(\frac{\partial u}{\partial y}, -\frac{\partial u}{\partial x}\Big), \quad \operatorname{grad}_1 u = \frac{\operatorname{grad} u}{|\operatorname{grad} u|}, \quad \operatorname{curl}_1 u = \frac{\operatorname{curl} u}{|\operatorname{curl} u|}.$$

The main result of this paper is based on an application of the following theorem and lemma from [14]. First let us recall them.

Theorem 1. *Let a potential operator $F(x) = \operatorname{grad} f(x)$ be defined on a Banach space E. Let $\langle\cdot,\cdot\rangle$ represents the duality between E and the dual space E^*. Let the operator fulfil*

$$(1) \qquad \alpha(\|x_2 - x_1\|) \leqslant \langle F(x_2) - F(x_1), x_2 - x_1\rangle \leqslant \beta(\|x_2 - x_1\|) \quad \forall\, x_1, x_2 \in E,$$

$\alpha(t)$, $\beta(t)$ being non-negative functions of a non-negative argument such that the functions

$$\overline{\alpha}(t) = \int_0^t \frac{\alpha(\tau)}{\tau}\,\mathrm{d}\tau, \quad \overline{\beta}(t) = \int_0^t \frac{\beta(\tau)}{\tau}\,\mathrm{d}\tau$$

are continuous, increasing and $\lim\limits_{t\to\infty} \overline{\alpha}(t)/t = \infty$. Let $M \subset E$ be a nonempty closed convex set. Then there exist unique $x^* \in E$, $f(x^*) = \min\limits_{x\in E} f(x)$ and $\bar{x} \in M$ with $f(\bar{x}) = \min\limits_{x\in M} f(x)$, and for their difference in the norm of E we have

$$(2) \qquad \|\bar{x} - x^*\| \leqslant \gamma(\|x - x^*\|) \quad \forall\, x \in M,$$

where γ is an increasing non-negative function of a non-negative argument, $\gamma(0) = 0$.

Lemma 1. *Let* $\mathbf{a},\mathbf{b}\in l_2$, $\mathbf{a}\neq\mathbf{0}$, $a=\|\mathbf{a}\|$. *Suppose* $\mathbf{a}$ *depends on a real parameter* t *so that all its components* $a_i=a_i(t)$, $i=1,2,\dots$ *are differentiable. Let* $\nu=\nu(a)$ *be a function fulfilling* (7), *where* $\nu_d(a)=(\mathrm{d}/\mathrm{d}a)(a\nu(a))$. *Then there exists* $\mathbf{c}\neq\mathbf{0}$ *orthogonal to* $\mathbf{a}$ *such that*

$$\frac{\mathrm{d}}{\mathrm{d}t}[\nu(a)\mathbf{ab}]=\nu(a)\frac{(\mathbf{cb})(\mathbf{c}\,\mathrm{d}\mathbf{a}/\mathrm{d}t)}{\mathbf{cc}}+\nu_d(a)\frac{(\mathbf{ab})(\mathbf{a}\,\mathrm{d}\mathbf{a}/\mathrm{d}t)}{\mathbf{aa}}. \tag{3}$$

It follows from [14] that we can choose $\mathbf{c}=\lambda a\big(a\,\mathrm{d}\mathbf{a}/\mathrm{d}t-\mathbf{a}\,\mathrm{d}a/\mathrm{d}t\big)$, $\lambda\neq0$ being a real constant.

3. Problem formulation

Let us apply the idea of [14] for the nonlinear planar magnetic field computation to the case with permanent magnets. That is why we suppose that besides standard media (ferromagnetic materials, electromagnets, air subregions) there are also permanent magnets in the domain of investigation. Each standard medium is determined by the z-component $J=J(x,y)$ of the current density and by the magnetic reluctivity $\nu=\nu(x,y,B)$. Each permanent magnet is characterized by the magnetization $\mathbf{M}$ which occurs in a preferred direction. Denoting the angle of this direction with the x-axis by ω and the corresponding unit vector by $\mathbf{m}=(\cos\omega,\sin\omega)$, we have $\mathbf{M}=M\mathbf{m}$, where $M=M(x,y,B_m)$ is the magnitude of $\mathbf{M}$. Notice that the magnetic reluctivity and the magnetization can depend, in addition to the space coordinates, also on the magnitude B of the flux density and on the projection B_m of the flux density into the direction of the magnetization, respectively. The relations between these two quantities and the sought solution u (the z-component of the vector magnetic potential) are given by

$$B=|\operatorname{grad}u|,\quad B_m=\mathbf{m}\operatorname{curl}u=\mathbf{k}\operatorname{grad}u, \tag{4}$$

where $\mathbf{k}=(-\sin\omega,\cos\omega)$. Evidently $\mathbf{mk}=0$.

The typical graph of the B-H characteristic (obtained experimentally) of a permanent magnet and the course of the corresponding magnitude of $\mathbf{M}$ are illustrated in Fig. 1 on the left, $\mu_0=4\pi\cdot10^{-7}\,\mathrm{H/m}$ being the permeability of vacuum. On the right-hand side of the same figure the B-H characteristic of iron including the behaviour of the corresponding relative reluctivity $\nu_r=\mu_0\nu$ is shown.

In order to formulate the problem in a general way we shall suppose that the physical medium is described by all three quantities mentioned above, namely by ν, J and $\mathbf{M}$. In particular, $\mathbf{M}\equiv0$ in standard media and $J\equiv0$, $\nu\equiv\nu_0$ in permanent magnets, $\nu_0=1/\mu_0$ meaning the reluctivity of vacuum.

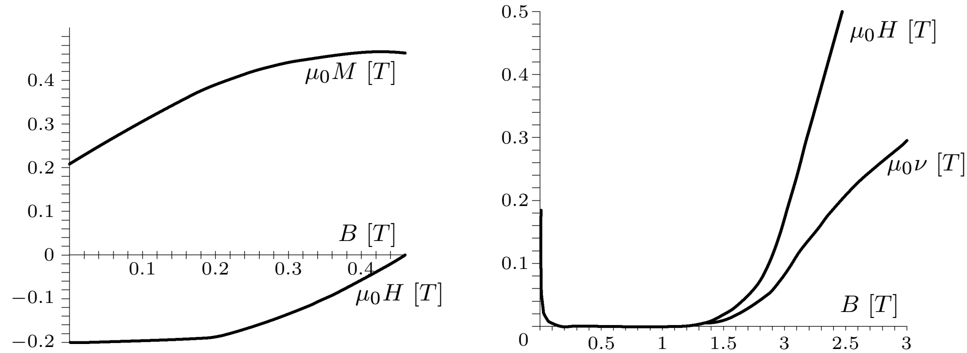

Figure 1. Magnetization characteristic of a permanent magnet and iron.

Problem 1. A bounded planar domain Ω with Lipschitz boundary $\partial\Omega$ is given. Suppose Ω to be divided into a finite number of mutually disjoint subregions Ω_i with Lipschitz boundaries $\partial\Omega_i$, $i = 1, \ldots, N$. Assume the physical medium in Ω to be described by the z-component $J = J(x, y)$ of the current density, by a positive magnetic reluctivity $\nu = \nu(x, y, B)$ and by a magnetization $\mathbf{M}$ given by its magnitude $M = M(x, y, B_m)$ and its angle $\omega = \omega(x, y)$ with the x-axis; each of these quantities can be discontinuous across the boundary $\Gamma = \bigcup_{i=1}^{N} \partial\Omega_i - \partial\Omega$ between different media. We look for a function $u = u(x, y)$ satisfying the partial differential equation

$$\operatorname{div}(\nu \operatorname{grad} u - M\mathbf{k}) = -J \quad \text{in } \Omega - \Gamma \tag{5}$$

which is continuous together with $\nu\, \partial u/\partial n$ across Γ, n being the unit normal to Γ oriented in a unique way, and which satisfies the boundary conditions

$$u = g_1 \text{ on } \Gamma_1, \qquad \nu\frac{\partial u}{\partial n} = g_2 \text{ on } \Gamma_2 = \partial\Omega - \Gamma_1, \tag{6}$$

n being the outer unit normal to $\partial\Omega$, $\Gamma_1 \subset \partial\Omega$ denoting a nonempty measurable set.

We assume that $\omega, J \in L_2(\Omega)$, $g_2 \in L_2(\Gamma_2)$ and g_1 is the trace of a function from $W^{1,p}(\Omega)$. Furthermore, we suppose the magnetic reluctivity $\nu = \nu(x, y, B)$ and the magnetization magnitude $M = M(x, y, B_m)$ to be measurable on Ω as functions of x, y and the differential reluctivities $\nu_d \equiv \nu_d(x, y, B) = (\partial/\partial B)(B\nu)$ and $\nu_m \equiv \nu_m(x, y, B_m) = \partial M/\partial B_m$ to exist for almost all $(x, y) \in \Omega$ and satisfy the inequalities

$$0 < \alpha_d \leqslant \nu_d \leqslant \beta_d, \quad -\infty < \alpha_m \leqslant \nu_m \leqslant \beta_m < \nu_0, \tag{7}$$

α_d, β_d, α_m, β_m being constant.

In real-life problems the functions ν and M are usually mediumwise constant, i.e., they do not depend on x and y explicitly within the same medium, and as functions of the third argument they are continuous. The left-hand side inequalities in (7) mean that the differential reluctivity must be bounded from both sides by positive constants. Integrating these inequalities from 0 to B, we find that also ν is bounded by the same constants (i.e. $0 < \alpha_d \leqslant \nu \leqslant \beta_d$), its graph being parallel with neither axis. The right-hand side inequalities in (7) admit the magnetization magnitude to be theoretically decreasing, since α_m could be negative.

To equation (5) let us assign the Dirichlet form

$$\begin{aligned} a(u,v) = \int_\Omega [&\nu(x,y,B) \operatorname{grad} u \operatorname{grad} v \\ &- M(x,y,B_m)\mathbf{k} \operatorname{grad} v - Jv] \,\mathrm{d}\Omega - \int_{\Gamma_2} g_2 v \,\mathrm{d}\Gamma, \end{aligned}$$

having sense for all $u, v \in H^1(\Omega)$ and being generally nonlinear with respect to the first argument and linear with respect to the second, the nonlinearity being caused by (4). The Dirichlet form allows us to determine the residual magnetic energy

$$\begin{aligned} f(u) &= \int_0^1 a(tu,u)\,\mathrm{d}t \\ &= \int_\Omega \left[\int_0^B \nu(x,y,b)b\,\mathrm{d}b - \int_0^{B_m} M(x,y,b)\,\mathrm{d}b - Ju\right] \mathrm{d}\Omega - \int_{\Gamma_2} g_2 u\,\mathrm{d}\Gamma. \end{aligned} \tag{8}$$

Then Problem 1 can be reformulated in the variational form:

P r o b l e m 2. Let $V = \{v \in H^1(\Omega)\colon\ v = 0 \text{ on } \Gamma_1\}$ be the space of all trial functions. Find $u_0 \in W^{1,p}(\Omega)$, $p > 2$ such that $\operatorname{tr} u_0 = g_1$ on Γ_1. We look for a function u, $u - u_0 \in V$ satisfying

$$a(u,v) = 0 \quad \forall\, v \in V.$$

4. Problem solution

Although the magnetic field with permanent magnets is described by a slightly different differential equation than that without permanent magnets, we can formulate the same assertion as in [14].

Theorem 2. *Let all the above assumptions be fulfilled and let $V_n \subset V$ be a nonempty finite-dimensional subspace. Then Problem 2 has a unique weak solution $u^* \in V$ minimizing the residual magnetic energy in V and a unique approximate solution $u \in V_n$ minimizing the same energy only in V_n. The error between these two solutions satisfies in the norm of $H^1(\Omega)$ the estimate*

$$\|u - u^*\| \leqslant \kappa \|v - u^*\| \quad \forall\, v \in V_n, \ \ \kappa > 1.$$

P r o o f. In order to simplify our further considerations let us determine the Gâteaux derivative $a'_u(v,w) = (\mathrm{d}/\mathrm{d}t)a(u+tv,w)|_{t=0}$ of the Dirichlet form. Put $\mathbf{a} = \operatorname{grad}(u+tv)$, $\mathbf{b} = \operatorname{grad} w$ and consequently, $\mathrm{d}\mathbf{a}/\mathrm{d}t = \operatorname{grad} v$. The remark after Lemma 1 implies

$$\begin{aligned} \mathbf{c} &= \lambda\Big(a^2 \frac{\mathrm{d}\mathbf{a}}{\mathrm{d}t} - \Big(a\frac{\mathrm{d}\mathbf{a}}{\mathrm{d}t}\Big)\mathbf{a}\Big)\Big|_{t=0} \\ &= \lambda(\operatorname{curl}(u+tv)\operatorname{grad} v)\operatorname{curl}(u+tv)|_{t=0} \\ &= \lambda(\operatorname{curl} u \operatorname{grad} v)\operatorname{curl} u. \end{aligned}$$

As λ is a constant, we choose it so that $\lambda \operatorname{curl} u \operatorname{grad} v = 1$. Then $\mathbf{c} = \operatorname{curl} u$. After using (3) and making some rearrangements, we obtain

$$\begin{aligned} a'_u(v,w) = \int_\Omega &[-\nu_m(\mathbf{k}\operatorname{grad} v)(\mathbf{k}\operatorname{grad} w) \\ &+ \nu(\operatorname{curl}_1 u \operatorname{grad} v)(\operatorname{curl}_1 u \operatorname{grad} w) \\ &+ \nu_d(\operatorname{grad}_1 u \operatorname{grad} v)(\operatorname{grad}_1 u \operatorname{grad} w)]\,\mathrm{d}\Omega \end{aligned} \tag{9}$$

valid for all $u, v, w \in H^1(\Omega)$. This derivative is symmetric with respect to v and w.

As the Dirichlet form represents a bounded linear functional with respect to the second argument, the Riesz Theorem implies that for any $u \in V$ there exists $F(u) \in V$ such that

$$\langle F(u), v\rangle = a(u,v) \quad \forall\, v \in V.$$

We verify that the operator F satisfies the assumptions of Theorem 1, i.e., it is potential, uniformly monotone and uniformly continuous. The potential of F is evidently given by (8). Now, choose $u, v \in V$ arbitrarily and calculate

$$\langle F(v) - F(u), v-u\rangle = a(v, v-u) - a(u, v-u) = \int_0^1 a'_{u+t(v-u)}(v-u, v-u)\,\mathrm{d}t.$$

Denoting $\mathbf{a} = \operatorname{grad}_1(u + t(v-u))$, $\mathbf{b} = \operatorname{curl}_1(u+t(v-u))$, $\mathbf{c} = \operatorname{grad}(v-u)$ and using (9), we get

$$\langle F(v) - F(u), v-u\rangle = \int_0^1\int_\Omega [-\nu_m(\mathbf{kc})^2 + \nu(\mathbf{bc})^2 + \nu_d(\mathbf{ac})^2]\,\mathrm{d}\Omega\,\mathrm{d}t.$$

Making use of (7), applying the fact that $(\mathbf{ac})^2+(\mathbf{bc})^2 = \mathbf{cc}$ for each t and integrating the intermediate results with regard to t, we arrive at

$$\alpha \int_\Omega \operatorname{grad}^2(v-u)\,\mathrm{d}\Omega \leqslant \langle F(v)-F(u), v-u\rangle \leqslant \beta \int_\Omega \operatorname{grad}^2(v-u)\,\mathrm{d}\Omega,$$

where the minimum and maximum $\alpha = \min(\alpha_d, \nu_0 - \beta_m)$ and $\beta = \max(\beta_d, \nu_0 - \alpha_m)$, both constructed from positive constants, must be taken all over Ω. Notice that $\alpha = \nu_0 - \beta_m$ for permanent magnets and $\alpha = \alpha_d$ for the other materials, while $\beta = \nu_0 - \alpha_m$ for permanent magnets with $\alpha_m < 0$ only and $\beta = \beta_d$ for permanent magnets with $\alpha_m \geqslant 0$ and other materials. Using the Friedrichs' inequality with a constant $c > 0$, we estimate the above seminorm in $H^1(\Omega)$ by the corresponding norm. Thus,

$$\frac{\alpha}{1+c}\,\|v-u\|^2 \leqslant \langle F(v)-F(u), v-u\rangle \leqslant \beta\|v-u\|^2. \tag{10}$$

The operator F is potential and satisfies (1) with quadratic bounded functions. The assertion is a consequence of Theorem 1 with $\kappa = \sqrt{(1+c)\beta/\alpha}$. □

It follows from Theorem 2 that the error between the weak and the approximate solution is, at worst, equal to the error of the best approximation of the weak solution over V_n.

5. Notes to the algorithm

For a polygonal domain and linear splines constructed on a triangular network (the case often used in engineering practice), Theorem 2 claims that the error between the weak and the approximate solution is of the first order. The corresponding computational algorithm based on the finite element method represents an iterative process of the first order of accuracy. Regarding the nonlinear characteristics of both the ferromagnetic materials and the permanent magnets, the discretization system forms a system of generally nonlinear equations. In order to solve it the Newton method has been applied, each Newton iterate being inverted by the Gaussian elimination exploiting both the symmetry and the band structure of the system matrix.

When deriving the algorithm, we restrict ourselves to the case $g_2 \equiv 0$ which appears most frequently in the heavy current electrical engineering. Additivity of the magnetic energy with respect to the integration region enables us to proceed on element-by-element basis. Without loss of generality we restrict ourselves to the local discrete system describing the situation in one triangle only. Denote the double area of this triangle by D, its side lengths by l_1, l_2, l_3 and its interior angles by α_1, α_2, α_3. In this case

$$u = \sum_{k=1}^{3} u_k \omega_k(x, y),$$

$\omega_k(x, y)$ being a linear polynomial satisfying $\omega_k(x_i, y_i) = \delta_{ik}$ (Kronecker's delta), $i = 1, 2, 3$, i.e.,

$$\omega_k(x, y) = \frac{1}{D} \begin{vmatrix} x & y & 1 \\ x_{k+1} & y_{k+1} & 1 \\ x_{k+2} & y_{k+2} & 1 \end{vmatrix}, \qquad D = \begin{vmatrix} x_k & y_k & 1 \\ x_{k+1} & y_{k+1} & 1 \\ x_{k+2} & y_{k+2} & 1 \end{vmatrix},$$

where the subscripts are understood modulo 3. In this case B and B_m are constant within each triangle and therefore all integrals over each triangle can be calculated exactly.

An important role in the construction of the local discretization equation system is played by the symmetric dimensionless matrix

$$\mathbf{S} = \sum_{k=1}^{3} \mathbf{Q}_k \operatorname{cotg} \alpha_k = \frac{1}{D} \sum_{k=1}^{3} \mathbf{R}_k l_k^2. \tag{11}$$

This matrix, being expressed in two ways, depends on the triangle shape only, since

$$\mathbf{Q}_1 = \frac{1}{2} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}, \quad \mathbf{Q}_2 = \frac{1}{2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad \mathbf{Q}_3 = \frac{1}{2} \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$\mathbf{R}_i = \mathbf{Q}_1 + \mathbf{Q}_2 + \mathbf{Q}_3 - 2\mathbf{Q}_i$, $i = 1, 2, 3$, are fixed matrices and the coefficients in each sum in (11) are mutually dependent due to the identities $2l_1^2 l_2^2 + 2l_2^2 l_3^2 + 2l_3^2 l_1^2 - l_1^4 - l_2^4 - l_3^4 = 4D^2$ and $\alpha_1 + \alpha_2 + \alpha_3 = \pi$. In order to simplify the resulting discretization system let us introduce some auxiliary quantities. First put $\mathbf{u} = (u_1, u_2, u_3)^\top$, $\mathbf{w} = \mathbf{S}\mathbf{u}$ and denote by $\mathbf{v}$ the column vector whose entries are projections of the counterclockwise-oriented triangle sides into the direction of $\mathbf{M}$, namely $v_i = D\mathbf{k} \operatorname{grad} \omega_i$. These three vectors enable us to introduce additional four quantities

$$B^2 = \frac{1}{D} \mathbf{u}^\top \mathbf{w}, \quad B_m = \frac{1}{D} \mathbf{u}^\top \mathbf{v}, \quad \mathbf{T} = \frac{1}{D} \mathbf{w}\mathbf{w}^\top, \quad \mathbf{P} = \frac{1}{D} \mathbf{v}\mathbf{v}^\top,$$

$\mathbf{T}$ and $\mathbf{P}$ being symmetric matrices. By means of these quantities the local system of discretization equations reduces to the simple form

$$\nu(B)\mathbf{w} - M(B_m)\mathbf{v} = \frac{1}{3} J D \mathbf{\Lambda},$$

$\mathbf{\Lambda}$ being the column unit vector. Each Newton iterate for the increment $\mathbf{z} = \mathbf{u}^{n+1} - \mathbf{u}^n$ can be written in the matrix form

$$\mathbf{A}\mathbf{z} = \mathbf{b},$$

where

$$\begin{aligned} \mathbf{A} &= \nu(B)\mathbf{S} - \nu_m(B_m)\mathbf{P} + \frac{1}{B}\frac{\mathrm{d}\nu(B)}{\mathrm{d}B}\mathbf{T}, \\ \mathbf{b} &= -\nu(B)\mathbf{w} + M(B_m)\mathbf{v} + \frac{1}{3}DJ\mathbf{\Lambda}. \end{aligned}$$

The first terms in both $\mathbf{A}$ and $\mathbf{b}$ are present in all media, the second terms arise in permanent magnets, the third term in $\mathbf{A}$ appears in nonlinear ferromagnetics only and that in $\mathbf{b}$ is related to current sources. Assembling the local contributions of all finite elements the global system of discretization equations is built up, its system matrix being symmetric and of a band structure.

6. Some applications

The algorithm has been verified on a transversally magnetized rod permanent magnet situated in an external homogeneous magnetic field which contains the angle φ with the permanent magnet magnetization. Though the magnetization characteristic of the magnet is nonlinear, the magnetic potential can be calculated exactly (see [13]) and therefore, it is possible to determine the actual error. The maximum relative error in the potential u is much less than 1 per mille. The corresponding field maps for some φ's are in Fig. 2, where the thick arrow represents the direction of the external magnetic field, while the direction of permanent magnet magnetization coincides with the positive direction of the x-axis. We can observe how the left-hand rotation of the external field weakens the field in the permanent magnet.

With help of the algorithm, various electrical machines have been numerically analyzed (see [10]). As an illustration the eight-pole (Fig. 3) and the six-pole (Fig. 4) synchronous machines, both in on-load state, can serve. In these cases the magnetic field is generated by both the permanent magnets situated in the rotor part and the electromagnets the excitation windings of which are located in the stator slots. In the figures the influence of the permanent magnet on the resulting field distribution and the strongly saturated machine parts can be easily determined.

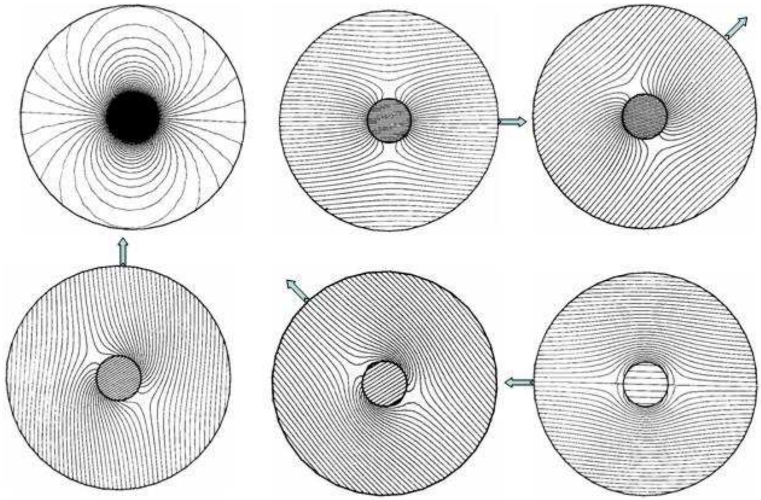

Figure 2. Rod permanent magnet.

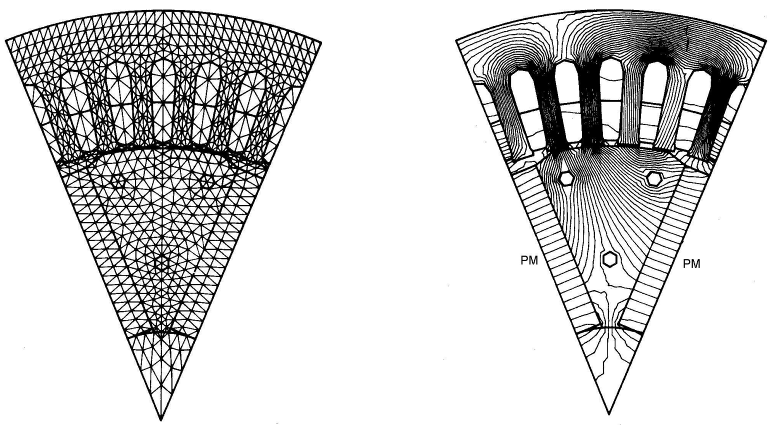

Figure 3. Eight-pole synchronous machine.

7. Conclusion

Although the above described algorithm represents an iterative process of the first order of accuracy only and in spite of strong nonlinearity of ferromagnetics and permanent magnets, discontinuity of physical properties and geometrical complexity of all materials occurring in the investigated domains, many practical problems from the heavy current electrical engineering point to a sufficiently high level of both the convergence of the algorithm and the accuracy of the results.

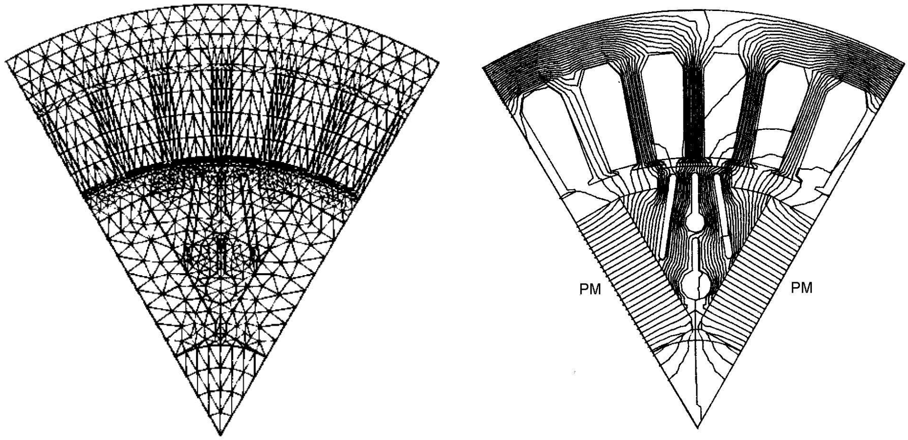

Figure 4. Six-pole synchronous machine.

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