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SUM-FUZZY IMPLEMENTATION OF A CHOICE FUNCTION USING ARTIFICIAL LEARNING PROCEDURE WITH FIXED FRACTION

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Abstract. In one if his paper Luo transformed the problem of sum-fuzzy rationality into artificial learning procedure and gave an algorithm which used the learning rule of perception. This paper extends the Luo method for finding a sum-fuzzy implementation of a choice function and offers an algorithm based on the artificial learning procedure with fixed fraction. We also present a concrete example which uses this algorithm.

Keywords: decision making, choice function, artificial learning

MSC 2000: 92B20

1. Introduction

The choice theory is of great interest in many problems of practice. Those who have contributed essentially to the development of the theory of choice functions were the economists. There is a large number of classical choice functions proposed in literature.

The concept of rationality of a choice function was introduced by Richter [7]. If we consider a set X and a set $P(X)$ of nonempty subsets of X, the choice function represents a mechanism which indicates choice sets for a given preference relation R over a set of alternatives.

Human thinking and preferences are inherently imprecise. Their vague character is modelled by fuzzy theory. The first who pointed out the importance of fuzzy preference relations was Orlovsky [6]. He was also the one who extended the concept of rationality to the concept of fuzzy rationality. Afterwards, the domain has been studied by many other researchers, see [1], [2], [6], [8].

In this paper we deal with the sum-fuzzy rational choice function which was introduced by Luo et al [4]. Sum-fuzzy implementation of a choice function consists in finding the matrix representation of the fuzzy binary relation which describes the preference relation.

Based on the artificial learning rule with fixed fraction, we give an algorithm for finding a sum-fuzzy implementation of a choice function and, in the conclusion, offer an example.

In the following section, we introduce some basic concepts and notation related to sum-fuzzy rational choice functions. Section 3 deals with relationships between sum-fuzzy rational choice functions and decision functions from the artificial learning procedures. Section 4 presents the learning rule with fixed fraction. Section 5 offers a new algorithm for finding sum-fuzzy implementation. The paper ends with some remarks.

2. Notation and definitions

Let X be a finite set of objects called "alternatives" with at least two elements

$$X = \{x_1, x_2, \dots x_n\}.$$

Let R be a binary relation on X, i.e. $R \subseteq X \times X$.

Definition 1. Let X be a set and $P(X)$ a set of nonempty subsets of X. A choice function C on X is a function from $P(X)$ to $P(X)$ such that $C(S) \subseteq S$ for all $S \in P(X)$ and it is a rational choice function if there exists a binary relation R on X such that

$$C(S) = f_R(S)$$

for all $S \in P(X)$, where

$$f_R(S) = \{x \in S\colon\ (x, y) \in R \text{ for all } y \in S\}.$$

Definition 2. A fuzzy binary relation on X is a function

$$r_R\colon\ X \times X \to [0, 1]$$

which associates each ordered pair of alternatives $(x, y) \in X^2$ with an element of $[0, 1]$ such that

$$r_R(x, y) \in (0, 1] \quad \text{if } (x, y) \in R$$

and

$$r_R = 0 \quad \text{if } (x, y) \notin R.$$

The matrix

$$\mathbf{M}(r_R) = \begin{pmatrix} r_R(x_1, x_1) & r_R(x_1, x_2) & \dots & r_R(x_1, x_n) \\ r_R(x_2, x_1) & r_R(x_2, x_2) & \dots & r_R(x_2, x_n) \\ \vdots & \vdots & \vdots & \vdots \\ r_R(x_n, x_1) & r_R(x_n, x_2) & \dots & r_R(x_n, x_n) \end{pmatrix}$$

is the matrix representation of the fuzzy binary relation r_R.

Definition 3. Let r_R be a fuzzy binary relation on X. Barret et al. [2] defined

$$SF(x) = \sum_{y \in S - \{x\}} r_R(x, y)$$

with the corresponding choice set

$$SF(S, R) = \{x \in S \colon\ SF(x) = \max_{x \in S} SF(x)\}.$$

They called this choice function max-SF.

Luo et al. [4] proposed a choice function called the sum-fuzzy choice function on r_R. Is is denoted by f_R.

Definition 4. The sum-fuzzy choice function on r_R is

$$f_R(S) = \Big\{x \in S \colon\ \sum_{z \in S} r_R(x, z) \geqslant \sum_{z \in S} r_R(y, z) \text{ for all } y \in S\Big\}.$$

Definition 5. If there exists a fuzzy binary relation r_R such that a choice function C on X is the sum-fuzzy choice function on r_R, i.e. $C = f_R$, then C is called sum-fuzzy rational.

The sum-fuzzy rational function is one of preference-based choice functions proposed by Barrett et al, because C is sum-fuzzy rational if, and only if, C is max-SF [5].

3. Artificial learning and the sum-fuzzy rational choice function

Let $l(S) = \{l\colon\ x_l \in S\}$ be the index set for all units which belong to S. Define a function

$$\delta_{ij}^{lk}(S) = \begin{cases} 1 & \text{if } i = l \text{ and } j \in l(S), \\ -1 & \text{if } i = k \text{ and } j \in l(S), \\ 0 & \text{otherwise} \end{cases}$$

for any $S \in P(X)$ and for any $l, k \in l(S)$, $l \neq k$ and $i, j = 1, 2, \ldots, n$. Let us consider a vector $z \in \mathbb{R}^{n^2}$,

$$z = (z_{11}, z_{12}, \ldots, z_{1n}; z_{21}, z_{22}, \ldots, z_{2n}; \ldots z_{n1}, z_{n2}, \ldots, z_{nn})$$

and a choice function C on X.

Luo [5] defined

$$U[C(S)] = \bigcup_{l \in l(C(S))} \bigcup_{k \in l(S)-l} \{z \in \mathbb{R}^{n^2} :\ z_{ij} = \delta_{ij}^{lk}(S) \text{ for } i, j = 1, 2, \ldots, n\}$$

and

$$U[C^+(S)] = \bigcup_{l \in l(C(S))} \bigcup_{k \in l(S)-l(C(S))} \{z \in \mathbb{R}^{n^2} :\ z_{ij} = \delta_{ij}^{lk}(S) \text{ for } i, j = 1, 2, \ldots, n\},$$

$$U[C] = \bigcup_{S \in P(X)} U[C(S)]$$

and

$$U^+[C] = \bigcup_{S \in P(X)} U^+[C(S)].$$

Theorem 1. *A choice function C on X is sum-fuzzy rational if and only if there exists $w \in \mathbb{R}^{n^2}$ such that $wz^T \geqslant 0$ for all $z \in U[C]$ and $wz^T > 0$ for all $z \in U^+[C]$.*

In the proof, Luo [5] defined the matrix

$$\mathbf{W} = \begin{pmatrix} w_{11}^* & w_{12}^* & \ldots & w_{1n}^* \\ w_{21}^* & w_{22}^* & \ldots & w_{2n}^* \\ \vdots & \vdots & \vdots & \vdots \\ w_{n1}^* & w_{n2}^* & \ldots & w_{nn}^* \end{pmatrix}$$

where $w_{ij}^* = (w_{ij} - \underline{w})/(\overline{w} - \underline{w})$ for $i, j = 1, 2, \ldots, n$ and $\overline{w}$ and $\underline{w}$ are the maximum and the minimum of components of w.

Theorem 1 makes it possible to transform the problem of sum-fuzzy rationality into the artificial learning procedure. In the forthcoming sections we use the procedure with fixed fraction.

4. Learning rule with fixed fraction

The procedure with fixed fraction is an artificial learning procedure for neural network. The two classes of methods presented by Dumitrescu [3] use the linear decision function

$$g\colon\ \mathbb{R}^n \to \mathbb{R},$$
$$g(z) = w^T z$$

where

$$w = (w_1, w_2, \ldots, w_n)^T$$

is a weight vector and

$$z = (z_1, z_2, \ldots, z_n)^T$$

is an input vector.

This procedure starts with an arbitrary weight vector and constructs a sequence which converges to a solution weight vector. This vector describes a decision function which classifies in a proper way any object z.

The method represents a training method based on error correction. Correction is achieved by modifying the weight vector in case of an incorrect classification.

So, the correction rule is: Modify w into $w - c(w^T z/\|z\|^2)z$ if $w^T z \leqslant 0$, where $c > 0$.

In the case of correct classification the weight vector remains unchanged.

5. An algorithm which uses the learning rule with fixed fraction

We present an algorithm for finding a sum-fuzzy implementation of a choice function. This algorithm finds the matrix representation using one of the artificial learning rules in neural network, namely the procedure with fixed fraction.

The algorithm is:

Step 1. Set the value of w_{ij} at random for all $i, j = 1, 2, \ldots n$.

Step 2. Calculate $U[C]$ and $w^T z = \sum w_{ij} z_{ij}$.

Step 3. If $w^T z = 0$ when $z \in U[C] - U^+[C]$ and $w^T z > 0$ when $z \in U^+[C]$ then go to Step 5; otherwise go to Step 4.

Step 4. Modify w_{ij} to $w_{ij} - c(w_T z/\|z\|^2)z$ with $0 < c < 1$.

Step 5. Go to Step 2 until w is stable.

Step 6. Let $w^*_{ij} = (w_{ij} - \underline{w})/(\overline{w} - \underline{w})$ with $\underline{w} = \min w_{ij}$ and $\overline{w} = \max w_{ij}$.

Step 7. Write W.

I made some calculations in Microsoft Excel for the case when $X = \{x_1, x_2, x_3\}$ and the choice function on X is given in Tab. 1.

$P(X)$	x_1	x_2	x_3	x_1x_2	x_2x_3	x_1x_3	$x_1x_2x_3$
C	x_1	x_2	x_3	x_1	x_3	x_1	x_1

Table 1. The choice function on $X = \{x_1, x_2, x_3\}$.

Note that a choice function such that $C(S)$ is a singleton for each $S \in P(X)$ is called single-valued.

With the above algorithm, the sum-fuzzy implementation is

$$\mathbf{W} = \begin{pmatrix} 1.00 & 0.21 & 0.47 \\ 0.11 & 0.40 & 0.47 \\ 0.39 & 0.80 & 0.40 \end{pmatrix}.$$

From the performance point of view, for this peculiar example, our algorithm seems to be similar to Luo's algorithm [5]. The number of the algorithm repetitions is approximately the same (27 for our algorithm and 24 for Luo's algorithm). However, our algorithm can be used as an alternative method in practical cases.

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