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OPTIMAL RESOURCE ALLOCATION IN A LARGE SCALE SYSTEM UNDER SOFT CONSTRAINTS[1]

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In the paper there is discussed a problem of the resource allocation in a large scale system in the presence of the resource shortages. The control task is devided into two levels, with the coordinator on the upper level and local controllers on the lower one.

It is assumed that they have different information. The coordinator has an information on mean values of users demands, an inflow forecast and an estimation of the resource amount in a storage reservoir. On the basis on this information it determines (by a numerical way) values of a coordinating variable transmitted to the local controllers. The ith local controller receives the measurement of the ith user demand and the value of the coordinating variable from the coordinator. On the basis on this information it calculates the decision on the resource allocation.

For a coordination an isoperimetric constraint is proposed. Due to this, the lower level optimization problem consists in independent local tasks which depend on the coordinating variable.

In the paper two strategies of the coordinator are proposed. The first algorithm is based on the open-loop feedback strategy, while the second one takes into account probabilistic constraints on the aggregate variable and on the amount of the resource in a storage reservoir.

For static, scalar subsystems and a quadratic performance index some properties of an obtained solution are discussed.

1. INTRODUCTION

Control and optimization for large scale systems are usually based on a decomposition of a global system into subsystems so as to decrease computational requirements and decrease an amount of information to be transmitted to and processed by decision makers. A conflict between local controllers is softened by the coordinator on the upper level, which performs some supervisory tasks.

Decomposition and coordination methods have been developed for large scale systems. Studies on decomposition methods can be found e.g. in [3, 7, 8, 10, 11, 13, 15]. A lot of these methods are applied to steady-state deterministic systems.

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Problems with different controllers and different available information are studied in the team decision theory, as well as in the hierarchical control [2, 6].

Control problems with decentralized measurement information become more complicated. In [16] it is shown that the Linear Quadratic Gaussian case is nontrivial when the information pattern is nonclassical. Further results can be found in [1, 2, 14, 17].

In the present paper there is discussed a problem of the resource allocation in a large scale system, in the presence of the resource shortages. The control task is devided into two levels, with the coordinator on the upper level and local controllers on the lower one.

It is assumed that the coordinator has information on mean values of users demands, an inflow forecast and an estimation of the resource amount in a storage reservoir. On the basis of this information it determines (by a numerical way) values of an aggregate variable and then, (by an analytical way), values of a coordinating variable transmitted to the local controllers.

The ith local controller receives the measurement of the ith user demand and the value of the coordinating variable from the coordinator. On the basis of this information it calculates the decision on the resource allocation.

For a coordination an elastic constraint is proposed [4]. Due to this, the lower level optimization problem consists in independent local tasks which depend on the coordinating variable.

The upper level numerical problem is the one of the coordinator, which chooses the values of the aggregate variables.

In the paper two strategies of the coordinator are considered.

The first algorithm is based on the open-loop feedback (OLF) control strategy and it is most closely related to [4]. From a problem statement it results, that some reserve capacity is necessary in a storage reservoir, which depends on a control variance. Sometimes it can be found by an analytical way.

In the second algorithm the aggregate variable has a linear form and it is realized in a closed loop system. This strategy takes into account probabilistic constraints on the aggregate variable and on the amount of the resource in the storage reservoir.

2. MODEL OF A SYSTEM

Consider the system composed of M static subsystems (receivers of the resource), which derive the resources from the storage reservoir supplied by the inflow d_n. Users' demands z_n^i, $i = 1, 2, \ldots, M$, $n = 0, 1, \ldots, N$ as well as the inflow d_n, $n = 0, 1, \ldots, N$, where N denotes the stopping time, are random variables with given probability distribution functions.

In principle, the considerations concern the case of the shortages of the resource which means that sometimes users' demands cannot be fully satisfied.

The performance index defining the losses resulting from a deficit of the resource in some period of time has the form:

$$I = E \sum_{n=0}^{N} \sum_{i=1}^{M} (z_n^i - u_n^i)^2 \tag{1}$$

where E denotes the mean operation, z_n^i is the demand of the ith subsystem in the period of time $[n,(n+1)]$ and u_n^i represents an amount of the resource assigned to the ith subsystem in the period of time $[n,(n+1)]$.

The problem is to allocate the resource from the storage reservoir into M subsystems so as to minimize the performance index (1) under the constraint on the storage capacity:

$$h_{\min} \le h_n + d_n - \sum_{i=1}^{M} u_n^i \le h_{\max}, \quad n = 0,1,\dots,N \tag{2}$$

where $h_n, h_{\min}, h_{\max}$ are real, minimum and maximum admissible amount of the resource in the storage reservoir; d_n is the inflow.

Of course, one can introduce costs of the control u_n^i and consider the performance index in the general form:

$$I = E\sum_{n=0}^{N}\sum_{i=1}^{M}(Q_n^i z_n^{i2} + G_n^i u_n^i z_n^i + H_n^i u_n^{i2}). \tag{3}$$

3. PROBLEM FORMULATION

The complexity of the solution depends on information and control structures.

In the present paper it is assumed the two-level hierarchical control structure with the coordinator on the upper level and the local controllers on the lower level. Proposed structure is justified for large scale distributed systems large (M), in which transmission of the demands z_n^i, $i = 1,2,\dots,M$, $n = 0,1,\dots,N$ to one central controller is difficult to realize.

It is assumed that the coordinator has the information on the mean values of the demands $\bar{z}_n^i = Ez_n^i$, $i = 1,2,\dots,M$, $n = 0,1,\dots,N$, the inflow forecast and the estimation of the resource amount in the storage reservoir. On the basis of this information it determines (at time n) the amount of the resource e_n to be *preleminary* allocated to the receivers and then it calculates a value of the coordinating variable λ_n, which is transmitted (e.g. by radio) to the local controllers.

The ith local controller receives at time n the value of z_n^i and the value of the coordinating variable λ_n from the coordinator. On the basis of this information it calculates the decision u_n^i.

From assumed information and control structures it results that admissible control law of the ith local controller and the coordinator have the forms: $u_n^i = a_n^i(z_n^i, \lambda_n)$ and $e_n = c_n(\mathbf{m}_n)$, $\lambda_n = b_n(e_n, \bar{z}_n^1, \dots, \bar{z}_n^M)$, respectively, where $\mathbf{m}_n$ denotes the information available for the coordinator at time n.

Additionally it is assumed that the functions a_n^i and c_n fulfil the elastic constraint [4]:

$$E_{|\mathbf{m}_n} \sum_{i=1}^{M} a_n^i(.) = e_n \tag{4}$$

where $E_{|\mathbf{m}_n}(.)$ denotes the condition mean, given $\mathbf{m}_n$.

The information and control structures imply that the constraint on the amount of the resource in the storage reservoir in the form (2) can not be taken into account by the coordinator (u_n^i and d_n are random variables).

In the sequel are discussed two modifications of the constraint (2).
The first one relies on replacement (2) by:

$$h_{\min} + \Delta h_n \le \hat{h}_n + \hat{d}_n - e_n \le h_{\max}, \quad e_n \ge 0 \tag{5}$$

where Δh_n is a reserve capacity, $\hat{h}_n, \hat{d}_n$ are estimates of h_n and d_n, respectively.

Notice that for chosen realization it may happen that:

$$\sum_{i=1}^{M} u_n^i > e_n. \tag{6}$$

It suggests that some reserve capacity Δh_n taken into account in (5) is necessary in the storage reservoir.

The second strategy of the coordinator takes into account the constraints [5]:

$$Prob(e_{\min} \le e_n \le e_{\max}) \ge 2\alpha - 1, \quad \alpha \in (0.5, 1) \tag{7}$$

$$Prob(h_{\min} \le h_n \le h_{\max}) \ge 2\beta - 1, \quad \beta \in (0.5, 1) \tag{8}$$

where α, β are given numbers.

4. SYNTHESIS OF THE LOCAL CONTROL LAWS

From assumed information and control structures it results that the local optimal control laws a_n^{io}, $i = 1, 2, \ldots, M$, $n = 0, 1, \ldots, N$ can be found by the minimization of the performance index:

$$I_n = E \sum_{i=1}^{M} (z_n^i - a_n^i)^2 \tag{9}$$

under constraint (4).

Using the method of Lagrange multipliers, we can take into account the constraint (4) in (9) and perform independently, for each subsystem, the minimization of the Lagrange function:

$$L_n^i = E \min_{u_n^i} E_{|z_n^i, \lambda_n} [(u_n^i - z_n^i)^2 + 2\lambda_n u_n^i] \tag{10}$$

where $E_{|z_n^i, \lambda_n}$ denotes the mean operation given z_n^i, λ_n; λ_n is Lagrange multiplier.

Differentiating the expression in the bracket $[\cdot]$ in (10) with respect to u_n^i and equating to zero, the optimal control law takes the form:

$$u_n^i = z_n^i + \lambda_n \tag{11}$$

The values of λ_n, $n = 0, 1, \ldots, N$ are determined by the coordinator and transmitted to the subsystems.

Notice that for $\lambda_n = 0$ users' demands can be fully satisfied.

5. SYNTHESIS OF THE COORDINATOR CONTROL LAWS

The task of the coordinator at time n is to determine the values of the variables e_n and λ_n.

Substituting (11) into (4) we obtain λ_n in the form:

$$\lambda_n = \frac{1}{M}(e_n - \bar{z}_n) \tag{12}$$

where $\bar{z}_n = \sum_{i=1}^{M} E z_n^i = \sum_{i=1}^{M} \bar{z}_n^i$.

Substituting (11) into (1) and, resulting from (12), the performance index for the whole system takes the form:

$$I = \frac{1}{M} E \sum_{n=0}^{N} (e_n - \bar{z}_n)^2. \tag{13}$$

The problem of the coordinator is a numerical minimization of the performance index (13) with respect to e_n, $n = 0, 1, \ldots, N$ under the constraint (5) or (7)–(8) and then the determination of λ_n according to (12).

Notice that for $e_n = \sum_{i=1}^{M} \bar{z}_n^i = \bar{z}_n$, $\lambda_n = 0$, which gives $u_n^i = z_n^i$.

Determination of the optimal e_n by the minimization (13) under the constraint (5) may be difficult, even numerically. Thus it is proposed to solve a suboptimal problem based on the open-loop feedback (OLF) control strategy.

In accordance with this idea, the coordinator determines at time n values of variables $e^*_{n+N'|n} = \{e_{k|n}\}$, $k = n, n+1, \ldots, n+N'$, which minimize the performance index:

$$I_n = \sum_{k=n}^{n+N'} (e_{k|n} - \bar{z}_k)^2 \tag{14}$$

under constraints:

$$h_{\min} + \Delta h_{n+j-1} \le \hat{h}_{n+j|n}, \quad j = 1, 2, \ldots, N' + 1 \tag{15}$$

$$e_{n+j|n} \ge 0, \quad j = 0, 1, \ldots, N', \tag{16}$$

where $\hat{h}_{n+j|n}$ is the estimate of the variable h_{n+j} given information $\mathbf{m}_n$, N' is a moving horizon of the control.

For realization, at time n, only $e_n = e_{n|n}$ is applied.

The estimate $\hat{h}_{n+j|n}$ may be determined from the equation:

$$\hat{h}_{n+j|n} = \hat{h}_{n|n} + \sum_{k=n}^{n+j-1} \hat{d}_{k|n} - \sum_{k=n}^{n+j-1} e_{k|n}, \tag{17}$$

where $\hat{h}_{n|n}$ and $\hat{d}_{k|n}$ are the estimates of the h_n and d_k, given information $\mathbf{m}_n$. Determination of random variable estimate is known in the literature [9] and it is not discussed in this paper.

As it was mentioned earlier, the surplus of the resource over e_n is necessary to satisfy randomly increased demands and the reserve capacity Δh_n should be in the storage reservoir.

From (11) and (12) it results that:

$$\sum_{i=1}^{M} u_n^i = e_n + \sum_{i=1}^{M}(z_n^i - \bar{z}_n^i). \tag{18}$$

Let u_n^* be the minimal value of the resource for which the probability that $\sum_{i=1}^{M} u_n^i \leq u_n^*$ given e_n is equal to γ, i. e.:

$$P\left(\sum_{i=1}^{M} u_n^i \leq u_n^* | e_n\right) = \gamma \tag{19}$$

After substituting (18) into (19) we have:

$$P\left[\sum_{i=1}^{M}(z_n^i - \bar{z}_n^i) \leq u_n^* - e_n | e_n\right] = \gamma \tag{20}$$

From (20) it results that:

$$\Delta h_n = F_{z_n}^{-1}(\gamma) - \sum_{i=1}^{M} \bar{z}_n^i \tag{21}$$

where $\Delta h_n = u_n^* - e_n$ and $F_{z_n}^{-1}(\gamma)$ is the value of the inverse of a distribution function of $z_n = \sum_{i=1}^{M} z_n^i$ for given γ.

If the random variables $z_n^i, i = 1, 2, \ldots, M$ are gaussian $\rightarrow N(\bar{z}_n^i, \sigma_n^i)$, then the random variable z_n is gaussian $\rightarrow N(\sum_{i=1}^{M} \bar{z}_n^i, \sqrt{\sum_{i=1}^{M} \sigma_n^{i2}})$. For given γ, the value of $F_{z_n}^{-1}(\gamma)$ can be found with using e. g. a toolbox Stats in Matlab.

Example 1. Consider the system composed of M subsystems with gaussian demands $z_n^i \rightarrow N(3, 1)$.

In Table 1 are presented the values of Δh for different M and γ.

Notice that for given γ, the ratio $\frac{\Delta h}{\sum_{i=1}^{M} \bar{z}_n^i}$ decreases when number of the subsystems increases.

5.1. Synthesis of the aggregate variable e_n under the constraints (7) – (8)

The strategy of the coordinator presented above requires the determination of the reserve capacity Δh_n and a good inflow-forecast.

Table 1. The influence of M and γ on the reserve capacity Δh_n.

	γ	Δh
$M = 10$, $\sum_{i=1}^{M} \bar{z}_n^i = 30$	0.98	6.5
	0.90	4.05
	0.85	3.30
	0.80	2.65
$M = 50$, $\sum_{i=1}^{M} \bar{z}_n^i = 150$	0.98	14.52
	0.90	9.06
	0.85	7.33
	0.80	5.95
$M = 500$, $\sum_{i=1}^{M} \bar{z}_n^i = 1500$	0.98	45.90
	0.90	28.65
	0.85	23.10
	0.80	18.80

Now, it will be presented a strategy of the coordinator, which minimizes (13) under constraints (7)−(8). In this algorithm it is assumed that:

$$e_n = \bar{e}_n + G(h_n - \bar{h}_n) \tag{22}$$

where $\bar{e}_n, \bar{h}_n$ are mean values of the variables e_n, h_n, respectively, and h_n is described by the equation:

$$h_{n+1} = h_n + d_n - \sum_{i=1}^{M} u_n^i. \tag{23}$$

Substituting (18) into (23) we have:

$$h_{n+1} = h_n + d_n^* - e_n \tag{24}$$

where $d_n^* = d_n - \sum_{i=1}^{M}(z_n^i - \bar{z}_n^i)$.

In the sequel it is assumed that the random variables d_n^* and h_0 are gaussian $\rightarrow N(\bar{d}_n^*, \sigma_{d_n^*})$, $N(\bar{h}_0, \sigma_{h_0})$, respectively.

Substituting (22) into (13) it is obtained:

$$I = \frac{1}{M} E \sum_{n=0}^{N} [(\bar{e}_n - \bar{z}_n)^2 + G^2(h_n - \bar{h}_n)^2]. \tag{25}$$

After performing the mean operation of the both sides in (24) and subtracting from (24) we have:

$$\tilde{h}_{n+1} = (1 - G)\tilde{h}_n + d_n^* - \bar{d}_n^* \tag{26}$$

where $\tilde{h}_{n+1} = h_{n+1} - \bar{h}_{n+1}$.

Notice that the stochastic process $\tilde{h}_n$, $n = 0, 1, \ldots$ does not depend on $\bar{e}_n$. Then, the task of the coordinator is to minimize the performance index:

$$I^* = E \sum_{n=0}^{N} (\bar{e}_n - \bar{z}_n)^2 \tag{27}$$

with respect to $\bar{e}_n$, under the constraint:

$$\bar{h}_{n+1} = \bar{h}_n + \bar{d}_n - \bar{e}_n. \tag{28}$$

Further constraints on $\bar{e}_n$ result from (7) – (8) and will be discussed in the sequel.

5.2. Analysis of the constraints (7) – (8)

Write the constraint (7) in the form:

$$Prob(e_{\min} - \bar{e}_n \leq \tilde{e}_n \leq e_{\max} - \bar{e}_n) \geq 2\alpha - 1 \tag{29}$$

where $\tilde{e}_n = e_n - \bar{e}_n$. From (22) it results that

$$\tilde{e}_n = e_n - \bar{e}_n = G\tilde{h}_n. \tag{30}$$

Notice that the random variable $\tilde{e}_n$ is gaussian $\rightarrow N(0, \sigma_{\tilde{e}_n})$, where $\sigma^2_{\tilde{e}_n} = G^2 E\tilde{h}^2_n = G^2\sigma^2_{\tilde{h}_n}$.

It is seen that the inequalities:

$$Prob(\tilde{e}_n \leq e_{\max} - \bar{e}_n) \geq \alpha \tag{31}$$
$$Prob(\tilde{e}_n \leq e_{\min} - \bar{e}_n) \leq 1 - \alpha \tag{32}$$

guarantee a fulfilment of the constraint (29).

The inequalities (31), (32) can be written in the form:

$$F_{\tilde{e}_n}(e_{\max} - \bar{e}_n) \geq \alpha \tag{33}$$
$$F_{\tilde{e}_n}(e_{\min} - \bar{e}_n) \leq 1 - \alpha \tag{34}$$

where $F_{\tilde{e}_n}$ is the distribution function of the variable $\tilde{e}_n$.

From (33) it results that:

$$\bar{e}_n \leq e_{\max} - F^{-1}_{\tilde{e}_n}(\alpha) \tag{35}$$

For a gaussian distribution function with a zero mean value it is true that $F(z) = 1 - F(-z)$. Then (34) can be written in the form:

$$\bar{e}_n \geq e_{\min} + F^{-1}_{\tilde{e}_n}(\alpha). \tag{36}$$

Finally we have:

$$e_{\min} + F^{-1}_{\tilde{e}_n}(\alpha) \leq \bar{e}_n \leq e_{\max} - F_{\tilde{e}_n}(\alpha). \tag{37}$$

After similar analysis we can write (8) in the form:

$$h_{\min} + F_{\tilde{h}_n}^{-1}(\beta) \leq \bar{h}_n \leq h_{\max} - F_{\tilde{h}_n}^{-1}(\beta). \tag{38}$$

Notice that the constraints (37) or (38) are unfeasible in the case, when:

$$F_{\tilde{e}_n}^{-1}(\alpha) > \frac{e_{\max} - e_{\min}}{2} = \frac{\Delta e}{2} \tag{39}$$

or

$$F_{\tilde{h}_n}^{-1}(\beta) > \frac{h_{\max} - h_{\min}}{2} = \frac{\Delta h}{2}. \tag{40}$$

The inequalities (39), (40) can be written in the form:

$$\alpha > F_{\tilde{e}_n}\left(\frac{\Delta e}{2}\right) \tag{41}$$

$$\beta > F_{\tilde{h}_n}\left(\frac{\Delta h}{2}\right). \tag{42}$$

Remember that the random variables $\tilde{h}_n$ oraz $\tilde{e}_n$ are gaussians $\rightarrow$ $N(0, \sigma_{\tilde{h}_n})$, $N(0, G\sigma_{\tilde{e}_n})$, respectively. Then the distribution functions $F_{\tilde{e}_n}(\cdot)$ and $F_{\tilde{h}_n}(\cdot)$ in (41) and (42) depend on the distribution function of the random variable $\tilde{h}_n$.

From (26) it results that:

$$\sigma_{\tilde{h}_{n+1}}^2 = (1-G)^2 \sigma_{\tilde{h}_n}^2 + \sigma_{\tilde{d}_n^*}^2 \tag{43}$$

where $\sigma_{\tilde{d}_n^*}^2 = E(d_n^* - \bar{d}_n^*)^2$.

Let for given G and some $n = n^*$ the variance of the variable $\sigma_{\tilde{h}_{n^*}}^2$ is maximal. Then the variance of the variable $\tilde{e}_{n^*}$ takes the maximum, too. Then for $n = n^*$ the distribution functions $F_{\tilde{h}_{n^*}}(\frac{\Delta h}{2})$ and $F_{\tilde{e}_{n^*}}(\frac{\Delta e}{2})$ take minimal values, which depend on chosen G.

The constraints (37) and (38) are feasible if:

$$\alpha \leq F_{\tilde{e}_{n^*}}\left(\frac{\Delta e}{2}\right) \tag{44}$$

$$\beta \leq F_{\tilde{h}_{n^*}}\left(\frac{\Delta h}{2}\right). \tag{45}$$

The maximization of the expression:

$$\max_G \rightarrow (F_{\tilde{e}_{n^*}} + F_{\tilde{h}_{n^*}}) \tag{46}$$

provides the feedback gain G in (22) and gives the possibility to choose the values of α and β as good as possible.

Example 2. Consider a system composed of M subsystems with gaussian demands $z_n^i \to N(1,3)$. Assume that $e_{\min} = 0$, $e_{\max} = \sum_{i=1}^M \bar{z}_n^i$, $h_{\min} = 0$, $h_{\max} = 2\sum_{i=1}^M \bar{z}_n^i$.
The values of G for given M, N and $\sigma_{d_n}^2 = E(d_n - \bar{d}_n)^2$ are presented in Table 2.

Table 2. The influence of M, N and σ_{d_n} on G.

M	N	$\sigma_{d_n}^2$	G
10	2	0	0.415
50	2	0	0
500	2	0	0
10	3	0	0.480
50	3	0	0
500	3	0	0
10	2	10	0.485
50	2	50	0
500	2	500	0
10	3	10	0.485
50	3	50	0
500	3	500	0

From Table 2 it is seen that for a large number of the subsystems the coordinator determines e_n in the open-loop system.

From numerical investigations it results that for data from Table 2, the values of α and β may be less then one.

In the Tables 3 and 4 are presented the admissible values of $\bar{e}_n$ and $\bar{h}_n$ resulting from (37) and (38) for $M = 50, N = 2, \sigma_{d_n}^2 = 0, G = 0$ and $M = 50, N = 2, \sigma_{d_n}^2 = 50, G = 0$, respectively, given α and β.

Table 3. Admissible $\bar{e}_n$ and $\bar{h}_n$ for $M = 50, N = 2, \sigma_{d_n}^2 = 0, G = 0$ and chosen α, β.

α	β	Range of $\bar{e}_n$	Range of $\bar{h}_n$
0.98	0.98	$0 \le \bar{e}_0 \le 150$ $0 \le \bar{e}_1 \le 150$ $0 \le \bar{e}_2 \le 150$	$14.5 \le \bar{h}_1 \le 285$ $20.5 \le \bar{h}_2 \le 279$
0.90	0.90	$0 \le \bar{e}_0 \le 150$ $0 \le \bar{e}_1 \le 150$ $\le \bar{e}_2 \le 150$	$9.06 \le \bar{h}_1 \le 291$ $12.8 \le \bar{h}_2 \le 287$

Table 4. Admissible $\bar{e}_n$ and $\bar{h}_n$ for $M = 50, N = 2, \sigma^2_{d_n} = 50, G = 0$ and chosen α, β.

α	β	Range of $\bar{e}_n$	Range of $\bar{h}_n$
0.98	0.98	$0 \le \bar{e}_0 \le 150$ $0 \le \bar{e}_1 \le 150$ $0 \le \bar{e}_2 \le 150$	$20.53 \le \bar{h}_1 \le 279.5$ $29.04 \le \bar{h}_2 \le 271.2$
0.90	0.90	$0 \le \bar{e}_0 \le 150$ $0 \le \bar{e}_1 \le 150$ $0 \le \bar{e}_2 \le 150$	$12.8 \le \bar{h}_1 \le 287.1$ $18.1 \le \bar{h}_2 \le 281.9$

Notice that for $G = 0$, the variance of $\sigma^2_{\bar{h}_n}$ increases when n increases, while the variance of $\sigma^2_{\bar{e}_n} = 0$. Then the range of $\bar{e}_n$ is $e_{\min} \le \bar{e}_n \le e_{\max}$, while the range of $\bar{h}_n$ decreases for increased n.

6. CONCLUSIONS

In the paper the control of the resource allocation in the large scale system is considered. An interesting point of the considerations is the assumption that particular decision-makers have different information.

For assumed information and control structures it is possible to partially decompose the calculations and to realize the partially decentralized control.

The problem stated in the paper and the proposed method of the solution make it possible to obtain the analytical optimal control laws of the local controllers.

Two numerical algorithms for the coordinator are proposed. In the first strategy it is necessary to determine the value of the reserve capacity in the storage reservoir. For some distribution functions it can be done by an analytical way.
In the second algorithm are introduced the probabilistic constraints on the control and state variables. The control law is assumed to have the linear form. Then the deterministic problem is solved during the realization of control.

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