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A NEW NYQUIST–BASED TECHNIQUE FOR TUNING ROBUST DECENTRALIZED CONTROLLERS

Alena Kozáková, Vojtech Veselý and Jakub Osuský

An original Nyquist-based frequency domain robust decentralized controller (DC) design technique for robust stability and guaranteed nominal performance is proposed, applicable for continuous-time uncertain systems described by a set of transfer function matrices. To provide nominal performance, interactions are included in individual design using one selected characteristic locus of the interaction matrix, used to reshape frequency responses of decoupled subsystems; such modified subsystems are termed "equivalent subsystems". Local controllers of equivalent subsystems independently tuned for stability and specified feasible performance constitute the decentralized controller guaranteeing specified performance of the full system. To guarantee robust stability, the $M-\Delta$ stability conditions are derived. Unlike standard robust approaches, the proposed technique considers full nominal model, thus reducing conservativeness of resulting robust stability conditions. The developed frequency domain design procedure is graphical, interactive and insightful. A case study providing a step-by-step robust DC design for the Quadruple Tank Process [8] is included.

Keywords: multivariable system, decentralized controller, frequency domain, independent design, robust stability, unstructured uncertainty

AMS Subject Classification: 93D15

1. INTRODUCTION

Most industrial processes are naturally multi-input multi-output (MIMO) plants arising as interconnection of a finite number of subsystems. Due to interactions among subsystems, MIMO systems are more difficult to control compared to the SISO ones. If strong interactions within the plant are to be compensated for then multivariable controllers are used. However, there may be practical reasons that make restrictions on controller structure necessary or reasonable. If the controller is split into several local feedbacks it becomes a decentralized controller. Compared with centralized full-controller systems the decentralized control (DC) structure brings about certain performance deterioration; however, this drawback is weighted against important benefits, e. g. hardware, operation and design simplicity as well as reliability improvement [19, 20]. Thus, decentralized controllers (DC) and DC design techniques remain popular among control engineers, in particular the frequency domain ones as they provide insightful solutions and link to the classical control theory.

Major important multivariable frequency-response Nyquist-type design techniques were developed in the late 60's and throughout the 70's: the Inverse-Nyquist Array (INA) and Direct Nyquist Array (DNA) methods by Rosenbrock and the Sequential design technique [15]. Almost simultaneously the non-interacting Characteristic Loci (CL) technique was developed [14]. With the come up of robust frequency domain approaches in the 80's, robust approach to the decentralized controller design has become very popular and many practice-oriented DC design techniques were developed.

The DC design proceeds in two main steps: 1) selection of a suitable control configuration (pairing inputs with outputs); 2) design of local controllers for individual subsystems. The main approaches applicable in Step 2 are the following: sequential (dependent) design, independent design, detuning methods and overlapping decompositions.

When using sequential design [1, 3, 15] local controllers are designed sequentially as a series controller. Usually, the controller corresponding to a fast loop is designed first and this loop is then closed before the design proceeds with the next controller. Thus, the information about "lower level" controllers is directly used as more loops are closed. If performance of the overall system is not satisfactory, the design procedure repeats with a corrective design. Main drawbacks are lack of failure tolerance when "lower level" controllers fail, strong dependence of performance on the loop closing order, and a "trial and error" design process. The method is recommended if bandwidths of individual loops are different.

If the independent design [2, 9, 19] is applied, interactions between loops are examined first and sufficient conditions are derived to guarantee robust stability and performance of the full system, and translated into bounds for individual loops. Local controllers designed in compliance with these bounds constitute the resulting decentralized controller. Main advantages with this approach are failure tolerance and direct design. Main limitation is conservatism of the derived stability and performance conditions since information on other controllers is not exploited in individual controllers design.

Using detuning methods [13, 17], local controllers are tuned first ignoring interactions from individual loops. Interactions are considered in the next step when each controller is detuned using an appropriate interaction measure until some stability criterion is met (typical example is the Ziegler–Nichols tuning formula plus a detuning factor evaluated from RGA). This method only provides reasonable preliminary controller settings with guaranteed closed-loop stability.

Use of overlapping decompositions to design robust decentralized controller for uncertain interconnected systems was introduced in [4]. Based on overlapping decompositions and expansions, a robustness bound in form of a single frequency-dependent scalar function accounting for neglected interactions between subsystems, uncertainties in interactions and subsystem models is developed, and further used to design local controllers. Satisfying a simple condition which involves this function guarantees closed-loop robust stability under decentralized controller. Recently this approach has also been extended for time delay systems [5]. Decentralized LQF/LTR controller design can be found in [6, 7].

In this paper an original Nyquist-based frequency domain robust decentralized controller (DC) design technique for robust stability and guaranteed nominal performance is proposed, applicable for continuous-time uncertain systems described by a set of transfer function matrices. To guarantee nominal performance the idea proposed in [10, 11] is developed. According to it, the effect of interactions on the full system is assessed first, using characteristic loci (CL's) of the matrix of interactions; the CL's are then used to reshape frequency responses of decoupled subsystems thus generating the so-called equivalent subsystems. For the equivalent subsystems, local controllers are designed according to the independent design approach using any frequency-domain design procedure. Resulting local controllers guarantee fulfilment of performance requirements imposed on the full nominal system. To guarantee robust stability over the specified operating range of the plant, the M-Δ stability conditions are used; their fulfilment is achieved by modification of control parameters. Unlike standard robust approaches, the proposed technique allows to consider full nominal model, thus reducing conservativeness of resulting robust stability conditions. The developed frequency domain design procedure is graphical, interactive and insightful. Theoretical conclusions are supported by simple examples.

The paper is organized as follows: Preliminaries for the development of the proposed design technique and Problem formulation are in Section 2. Main results including the proposed design procedure along with simple illustrative examples are presented in Section 3. A Case study providing a step-by-step robust DC design for the quadruple tank process [8] is in Section 4. Conclusions are drawn at the end of the paper.

2. PRELIMINARIES AND PROBLEM FORMULATION

Consider a MIMO system described by a transfer function matrix $G(s) \in \mathbb{R}^{m\times m}$ and a decentralized controller $R(s) \in \mathbb{R}^{m\times m}$ in the standard feedback configuration (Figure 1):

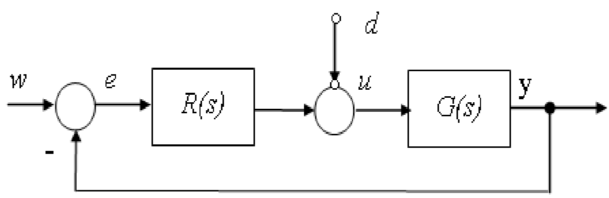

Fig. 1. Standard feedback configuration.

where w, u, y, e, d are respectively vectors of reference, control, output, control error and disturbance of compatible dimensions.

When designing a controller, the major source of difficulty is the plant model uncertainty brought about by identification/modelling errors; a control system is robust if it is insensitive to the differences between the actual plant and its model used to design the controller. To deal with an uncertain plant a suitable uncertainty model is to be selected and instead of a single model, the behavior of a whole class of models is to be considered. A simple uncertainty model is obtained in terms of unstructured uncertainty, i. e. a full complex perturbation matrix with the same dimensions as the plant. Four single perturbation forms are commonly used, [19, 20]:

- additive ('a'),
- multiplicative input ('i'),
- multiplicative output ('o') ones and
- their inverse counterparts are used for uncertainty associated with plant poles located in the closed right half-plane .

Denote $G(s)$ any member of a set of a possible plants Π, $G_0(s)$ the nominal model used to design the controller, $\sigma_M(\cdot)$ the maximum singular value of $(\cdot)$ and $l_k(s), k = a, i, o, ia$ the scalar weights on a normalized perturbation $\Delta(s) \in \mathbb{R}^{m\times m}, \sigma_M(\Delta) \leq 1$. Individual perturbation forms generate related families of plants $\Pi_k, k = a, i, o, ia$ as follows:

- Additive uncertainty

$$\Pi_a : \quad G(s) = G_0(s) + l_a(s)\Delta(s) \tag{1}$$

$$l_a(s) = \max_{G(s)\in\Pi_a} \sigma_M[G(s) - G_0(s)].$$

- Multiplicative input uncertainty

$$\Pi_i : \quad G(s) = G_0(s)[I + l_i(s)\Delta(s)] \tag{2}$$

$$l_i(s) = \max_{G(s)\in\Pi_i} \sigma_M\{G_0(s)^{-1}[G(s) - G_0(s)]\}.$$

- Multiplicative output uncertainty

$$\Pi_o : \quad G(s) = [I + l_o(s)\Delta(s)]G_0(s) \tag{3}$$

$$l_o(s) = \max_{G(s)\in\Pi_o} \sigma_M\{[G(s) - G_0(s)]G_0(s)^{-1}\}.$$

- Inverse-additive type uncertainty

$$\Pi_{ia} : \quad G(s) = (I + l_{ia}G_0(s)\Delta(s))^{-1}G_0(s) \tag{4}$$

$$l_{ia} = \max_{G(s)\in\Pi_{ia}} \sigma_M\{G^{-1}(s)(G(s) - G_0(s))G_0(s)^{-1}).$$

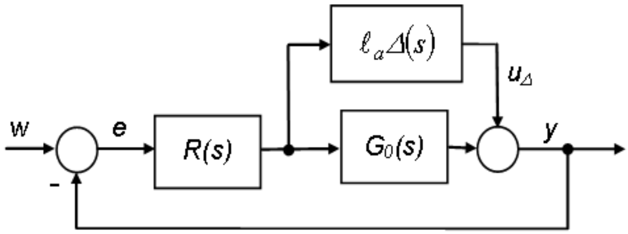

Fig. 2. Standard feedback configuration with additive uncertainty.

Closed-loop with uncertain plant represented by means of additive uncertainty (1) is in Figure 2.
Block diagram for other uncertainty types are derived similarly. Each such block diagram can be easily put into the $M-\Delta$ structure (Figure 3).

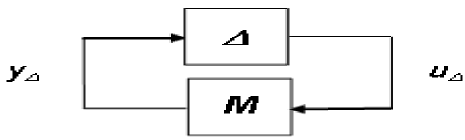

Fig. 3. $M-\Delta$ structure.

If the nominal closed-loop system is stable then M is stable and Δ is a stable uncertainty which can destabilize the system. The following theorem establishes conditions on M so that the $M-\Delta$ closed-loop system is stable [20].

Theorem 1. (Robust stability for unstructured uncertainty) Assume that the nominal closed-loop system $M(s)$ is stable and the uncertainty $\Delta(s)$ is stable. Then the $M-\Delta$ system in Figure 3 is stable for all uncertainty models $\Delta(s)$ satisfying $\sigma_M(\Delta) \leq 1$ if and only if

$$\sigma_M(M(s)) < 1, \quad \forall \omega. \tag{5}$$

For individual uncertainty forms the corresponding matrices $M_k, k = a, i, o, ia$ are as follows:

$$\begin{aligned} M_a(s) &= -l_a(s)[I + R(s)G_0(s)]^{-1}R(s) \\ M_i(s) &= -l_i(s)[I + R(s)G_0(s)]^{-1}R(s)G_0(s) \\ M_o(s) &= -l_o(s)G_0(s)R(s)[I + G_0(s)R(s)]^{-1} \\ M_{ia}(s) &= l_{ia}(s)(I + G_0(s)R(s))^{-1}G_0(s). \end{aligned} \tag{6}$$

Stability of the closed-loop with the nominal model (nominal stability) can be examined using the Generalized Nyquist stability theorem [14, 20]. The multivariable stability theory relies on the concept of the system return difference [3]

$$F(s) = [I + Q(s)] \tag{7}$$

where $F(s) \in \mathbb{R}^{m\times m}$ is the system return-difference matrix, $Q(s) = G(s)R(s) \in \mathbb{R}^{m\times m}$ is the open loop transfer function matrix for the system in Figure 1.

Following standard notation is used throughout the paper. The *Nyquist D-contour* in the complex plane consists of the imaginary axis $s = j\omega$ and an infinite semi-circle into the right-half plane avoiding locations where $Q(s)$ has $j\omega$ -axis poles by making small indentations to the right-half plane around these points; *Nyquist plot* of a complex function $g(s)$ is the image of the *Nyquist D-contour* under $g(s)$; $N[k, g(s)]$ denotes the number of anticlockwise encirclements of the point $(k, j0)$ by the Nyquist plot of $g(s)$. Characteristic functions of $Q(s)$ are the set of m algebraic functions $q_i(s),\ i = 1, 2, \ldots, m$ defined as [14]

$$\det[q_i(s)I - Q(s)] = 0 \quad i = 1, 2, \ldots, m. \tag{8}$$

Characteristic loci (CL) are the set of loci in the complex plane traced out by the characteristic functions of $Q(s), \forall\, s \in D$. The closed-loop characteristic polynomial expressed in terms of characteristic functions of $Q(s)$ reads as follows

$$\det F(s) = \det[I + Q(s)] = \prod_{i=1}^{m}[1 + q_i(s)]. \tag{9}$$

Theorem 2. (Generalized Nyquist Stability Theorem) The closed-loop system in Figure 1 is stable if and only if

$$\begin{gathered}\det F(s) \neq 0 \quad \forall\, s \in D \\ N[0, \det F(s)] = \textstyle\sum_{i=1}^{m} N\{0, [1 + q_i(s)]\} = n_q\end{gathered} \tag{10}$$

where $F(s) = (I + Q(s))$ and n_q is the number of unstable poles of $Q(s)$.

Problem Formulation. Consider an uncertain system with m subsystems given by a nominal model and the model uncertainty described by (1), (2) and (3). Let the nominal model $G_0(s)$ can be split into the diagonal part (representing mathematical models of decoupled subsystems) and the off-diagonal part (representing interactions between subsystems)

$$G_0(s) = G_d(s) + G_m(s) \tag{11}$$

where $G_d(s) = \text{diag}\{G_0(s)\}_{m\times m}$ and $G_m(s) = G_0(s) - G_d(s)$.

A decentralized controller

$$R(s) = \text{diag}\{R_i(s)\}_{m\times m}, \quad \det R(s) \neq 0 \quad \forall\, s \in D \tag{12}$$

is to be designed with $R_i(s)$ being transfer function of the ith local controller. The designed controller has to guarantee stability of the whole family of plants specified by (1), (2), (3) or (4) (i. e. robust stability, RS) and a specified performance of the nominal model (nominal performance, NP).

3. DECENTRALIZED CONTROLLER DESIGN FOR NOMINAL PERFORMANCE AND ROBUST STABILITY

3.1. Decentralized controller design for nominal performance

The proposed DC design technique evolves from factorization of the closed-loop characteristic polynomial in terms of the split nominal system (11) under the decentralized controller (12)

$$\begin{aligned} \det F(s) &= \det\{I + [G_d(s) + G_m(s)]R(s)\} \\ &= \det[R(s)^{-1} + G_d(s) + G_m(s)] \det R(s). \end{aligned} \tag{13}$$

Existence of $R(s)^{-1}$ is implied by the assumption (12) that $\det R(s) \neq 0$. Denote

$$F_1(s) = R(s)^{-1} + G_d(s) + G_m(s). \tag{14}$$

With respect to (13)–(14) the necessary and sufficient stability conditions of Theorem 2 modify as follows.

Corollary 1. The closed-loop comprising the nominal system $G_0(s)$ and the decentralized controller (12) is stable if and only if

$$\det F_1(s) \neq 0 \quad \forall\, s \in D \tag{15}$$

$$N[0, \det F_1(s)] + N[0, \det R(s)] = n_q. \tag{16}$$

The first two terms in (14) are diagonal matrices comprising information on the dynamics of decoupled subsystems. Denote

$$P(s) = R(s)^{-1} + G_d(s) \tag{17}$$

where $P(s) = \operatorname{diag}\{p_i(s)\}_{m\times m}$ is a matrix that changes with $R(s)$ and conversely, by specifying $P(s)$ through $R(s)^{-1}$ it is possible to affect performance of individual subsystems (including stability). A simple manipulation of (17) yields

$$I + R(s)[G_d(s) - P(s)] = I + R(s)G^{\text{eq}}(s) = 0 \tag{18}$$

where

$$G^{\text{eq}} = G_d(s) - P(s) = \operatorname{diag}\{G_i(s) - p_i(s)\}_{m\times m} = \operatorname{diag}\{G_i^{\text{eq}}\}_{m\times m} \tag{19}$$

is a diagonal matrix of equivalent subsystems. For individual subsystems, (18) yields

$$1 + R_i(s)G_i^{\text{eq}}(s) = 0 \quad i = 1, 2, \dots, m \tag{20}$$

which are the m equivalent characteristic equations.

Substituting (17) into (14) we obtain

$$\det F_1(s) = \det[P(s) + G_m(s)]. \tag{21}$$

In the context of the independent design philosophy, the design parameters $p_i(s),\ i = 1, 2, \dots, m$ represent the bounds for individual designs. To be able to provide closed-loop stability of the full system using a DC controller, $p_i(s),\ i = 1, 2, \dots, m$ are to be chosen so as to appropriately cope with the interactions $G_m(s)$.

3.1.1. Development of conditions for nominal stability and performance

According to (8), characteristic functions $g_i(s)$ of $G_m(s)$ are defined as follows

$$\det[g_i(s)I - G_m(s)] = 0 \quad i = 1, 2, \dots, m. \tag{22}$$

Substituting for $P(s) = p_i(s)I$ in (21) and equating to zero yields

$$\det[p_i(s)I + G_m(s)] = 0 \quad i = 1, 2, \dots, m. \tag{23}$$

Hence the $p_i(s)$ satisfying (23) are actually the m characteristic functions of $[-G_m(s)]$.

If the diagonal entries of $P(s)$ are identical and equal to any of the characteristic functions of $[-G_m(s)]$ then the following reasoning can be made (assuming that (10) b) holds):

1. If $P(s) = -g_k(s)I$ where fixed $k \in \{1, 2, \dots, m\}$ then

$$\det F_1(s) = \prod_{i=1}^{m}[-g_k(s) + g_i(s)] = 0 \quad \forall\, s \in D. \tag{24}$$

In this case the closed-loop system is at the limit of instability; the equivalent subsystems generated by the selected $g_k(s)$ are

$$G_{ik}^{\mathrm{eq}}(s) = G_i(s) + g_k(s) \quad i - 1, 2, \dots, m. \tag{25}$$

2. If $P(s-\alpha) = -g_k(s-\alpha)I$, fixed $k \in \{1, 2, \dots, m\}$, then

$$\det F_1(s-\alpha) = \prod_{i=1}^{m}[-g_k(s-\alpha) + g_i(s-\alpha)] = 0 \quad \forall\, s \in D. \tag{26}$$

In this case the closed-loop system would be at the limit of instability "shifted to $-\alpha$", i. e. having just poles with $\mathrm{Re}\, s \le -\alpha$; thus its degree of stability is α and the equivalent subsystems generated by the selected $g_k(s-\alpha)$ are

$$G_{ik}^{\mathrm{eq}}(s-\alpha) = G_i(s-\alpha) + g_k(s-\alpha) \quad i = 1, 2, \dots, m. \tag{27}$$

3. If $P(s-\alpha) = -g_k(s-\alpha)I$ fixed $k \in \{1, 2, \dots, m\}$, $0 \le \alpha \le \alpha_m$, where α_m denotes the maximum achievable degree of stability for the given plant under a decentralized controller (it may depend on plant dynamics and/or existing fixed modes of system) then the following inequality holds

$$\det F_{1k} = \prod_{i=1}^{m}[-g_k(s-\alpha) + g_i(s)] = \prod_{i=1}^{m} r_{ik}(s) \ne 0 \quad \forall\, s \in D. \tag{28}$$

Note that for any $\alpha_1 : 0 \le \alpha_1 \le \alpha_m$ the characteristic loci $g_i(s-\alpha_1)$ and $g_k(s-\alpha_m)$ must not intersect.

Hence by suitably choosing $g_k(s-\alpha)$ it is possible to specify the required closed-loop performance under the decentralized controller in terms of the degree of stability.

Definition 1. Consider a fixed $k \in \{1, 2, \ldots, m\}$ and $\alpha > 0$. The characteristic locus $g_k(s-\alpha)$ of the matrix $G_m(s-\alpha)$ will be called a proper characteristic locus if it satisfies conditions (24), (26) and (28). The set of all proper characteristic loci will be denoted P_S.

Lemma 1. The closed-loop in Figure 1 comprising nominal system $G_0(s)$ and a decentralized controller $R(s)$ (12) is stable if and only if the following conditions are satisfied $\forall\, s \in D$, $\alpha \geq 0$

1. $g_k(s-\alpha) \in P_S$, fixed $k \in \{1, 2, \ldots, m\}$;
2. all equivalent characteristic polynomials $\mathrm{CLCP}_i^{\mathrm{eq}} = 1 + R_i(s)\, G_i^{\mathrm{eq}}(s)$, $i = 1, 2, \ldots, m$ have roots with $\mathrm{Re}(s) \leq -\alpha$;
3. $N[0, \det F(s-\alpha)] = n_{q\alpha}$ where $F(s-\alpha) = I + G(s-\alpha)\, R(s-\alpha)$ and $n_{q\alpha}$ is the number of open loop poles with $\mathrm{Re}\, s > -\alpha$.

Proof of Lemma 1 is evident from previous considerations.

3.1.2. Illustrative examples

Following simple examples illustrate applicability of the above theoretical results. To design local controllers for equivalent subsystems any frequency domain method of SISO controller design can be used. Here, a modification of the *Neymark D*-partition method was applied [16]. Commonly, D-partition is applied as a conformal mapping of $s = j\omega$, $\omega \in \langle -\infty, \infty \rangle$ onto the plane of parameters of the closed loop characteristic polynomial (CLCP); in our case the two-dimensional space of PI controller parameters is considered, i. e. the (r_0, r_1) plane if considering parallel form of the PI controller $R(s) = r_0 + \frac{r_1}{s}$. The proposed modification consists in mapping $s = -\alpha + j\omega$, $\omega \in \langle 0, \omega_{\max} \rangle$, $\alpha \in \langle 0, \alpha_m \rangle$ onto the (r_0, r_1) plane of individual equivalent closed loop characteristic polynomials $\mathrm{CLCP}_i^{\mathrm{eq}}$, $i = 1, \ldots, m$ yielding a family of D_α-plots for each. Local controller parameters chosen from the D_α-plot specific for some feasible α guarantee the same degree of stability α for the full system under the designed decentralized controller; it is necessary to note that if using point-by-point calculation of individual D_α-plots it may not be feasible to recognize singular lines and properly identify regions corresponding to individual degrees of stability ("α stability regions").

Example 1. (stable subsystems/stable interactions) Consider a MIMO system described by

$$\begin{bmatrix} \frac{1}{(s+1)(s+a)} & \frac{0.5}{0.7s+b} \\ \frac{0.8}{0.5s+c} & \frac{1.3}{(1.2s+1)(1.4s+1)} \end{bmatrix} \tag{29}$$

with $a = 0.5, b = 1, c = 1$.

CL's of $G_m(s-\alpha)$ plotted for $\alpha \in \{0.0, 0.1, 0.2, 0.3, 0.4\}$ are in Figures 4–5; for the controller design procedure, the CL $g_2(s-\alpha)$ has been chosen. To check its properness, it remains to verify condition (28). Corresponding curves are plotted in Figure 6, whereby the "large" curve verifies the lumped condition (28), the two "small" curves correspond to the individual factors. Obviously, $g_2(s-\alpha) \in P_S$.

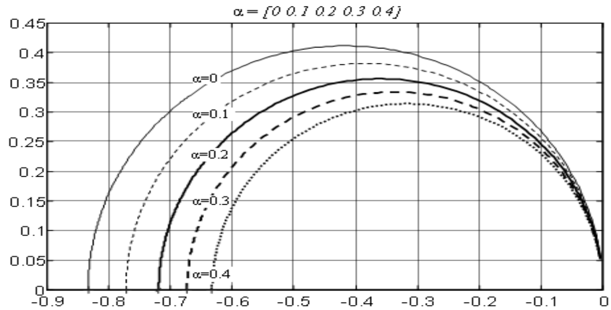

Fig. 4. Characteristic loci $g_1(s-\alpha)$.

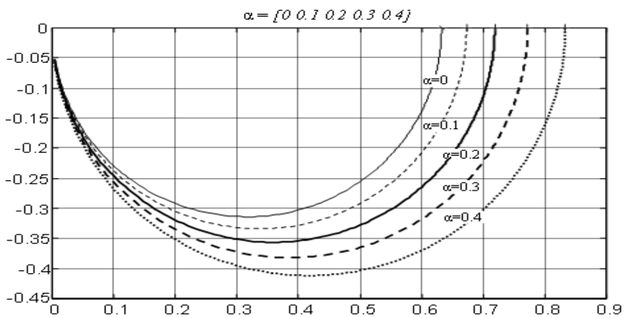

Fig. 5. Characteristic loci $g_2(s-\alpha)$.

Nyquist plots of equivalent subsystems generated by $g_2(s-\alpha)$ according to (25) are in Figures 7 – 8. The D_α-plots of individual equivalent subsystems in the (r_0, r_1) plane plotted for $\alpha \in \{0.0, 0.1, 0.2, 0.3, 0.4\}$ are in Figures 9 – 10 (zoomed on regions of feasible controller parameters).

Resulting local PI controllers constituting the decentralized controller $R(s) = \text{diag}\{R_i(s)\}_{2\times 2}$ were designed for $\alpha = 0.3$.

$$R_1(s) = 0.691 + \frac{0.300}{s} \quad R_2(s) = 0.782 + \frac{0.388}{s}.$$

Closed-loop stability and performance have been verified by calculating closed loop poles and checking the encirclement condition 3 of Lemma 1 for $\det F(s-0.3)$. Closed-loop poles of the full system under the designed controller are

$$\Lambda = \{-0.3; -0.32 \pm 0.672j; -0.3958; -0.4109; -0.7057 \pm 0.5443j; -1.3538; -2.3017\}.$$

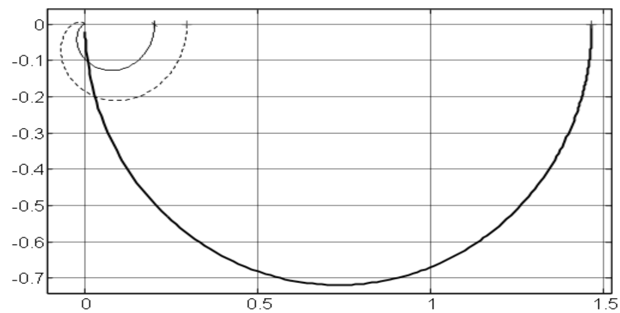

Fig. 6. Verification of condition (28) for $g_2(s-\alpha), \alpha = \{0, 0.3\}$.

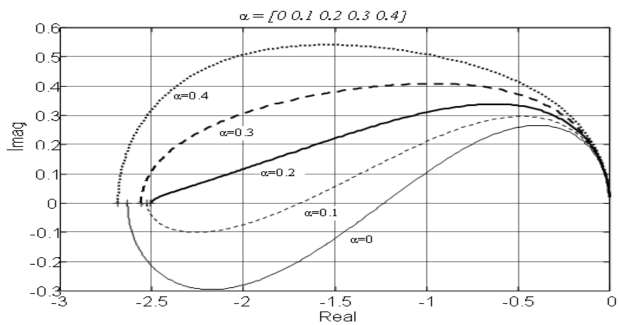

Fig. 7. Nyquist plots of equivalent subsystems $G_{12}^{\mathrm{eq}}(s-\alpha)$.

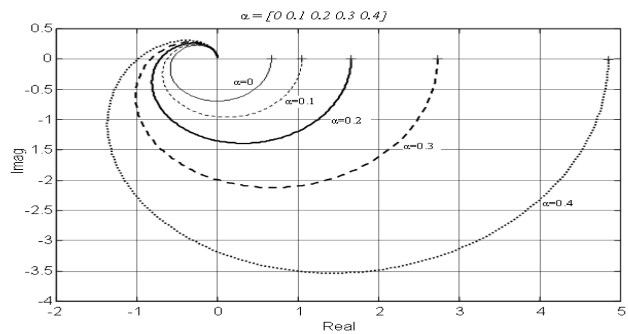

Fig. 8. Nyquist plots of equivalent subsystems $G_{22}^{\mathrm{eq}}(s-\alpha)$.

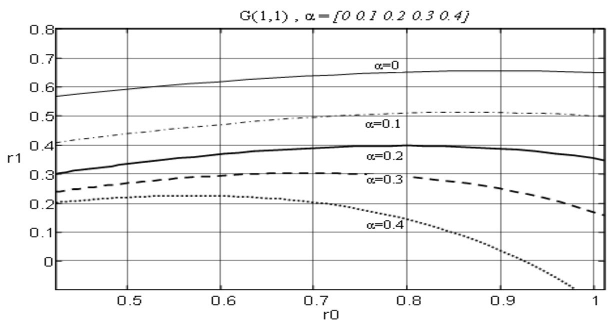

Fig. 9. D_α-plots for $G_{12}^{\mathrm{eq}}(s-\alpha)$.

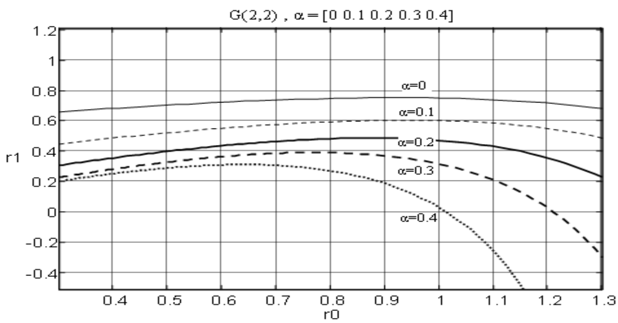

Fig. 10. D_α-plots for $G_{22}^{\mathrm{eq}}(s-\alpha)$.

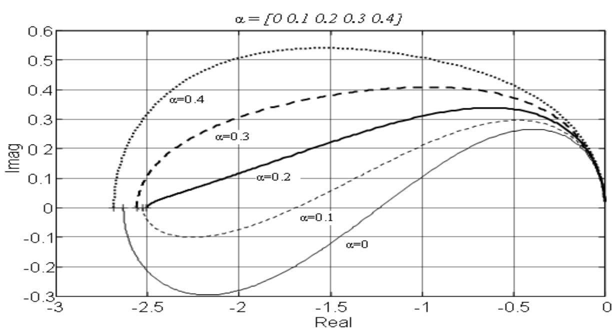

Fig. 11. Nyquist plot of $\det F(s-0.3)$.

Poles of $G(s)$ are the poles of its individual entries, i. e. $\{-0.5; -0.7143; -0.833; -1.0; -1.429; -2\}$; $R(s)$ has a double pole in $s = 0$, hence $n_{q,0.3} = 2$ and $n_q = 0$.

The closed loop system under the DC will be stable with a guaranteed degree of stability $\alpha = 0.3$ if $N[0, \det F(s - 0.3)] = n_{q,0.3} = 2$ and $N[0, \det F(s)] = n_q = 0$. Nyquist plot of $\det F(s - 0.3)$ in Figure 11 proves that the required closed-loop performance in terms of the degree of stability has been achieved (the point marked by asterisk corresponds to $\omega \to \pm\infty$).

Example 2. (Unstable subsystem, stable interactions) Consider $G(s)$ in (29) with $a = -0.5, b = 1, c = 1$, i. e. the $1st$ subsystem is unstable and the interactions are stable with the same $G_m(s)$ as in Example 1; hence characteristic loci of $G_m(s)$ and verification of the CL properness condition (28) are in Figures 4−6. From the D_α-plots of individual equivalent subsystems in Figures 12−13 plotted for $\alpha \in \{0.0, 0.1, 0.2, 0.3, 0.4\}$ it is evident that the maximum achievable degree of stability is $\alpha_m = 0.1$ (if $\alpha > 0.1$ the values r_1 for $G_{12}^{eq}(s - \alpha)$ are negative and hence unfeasible).

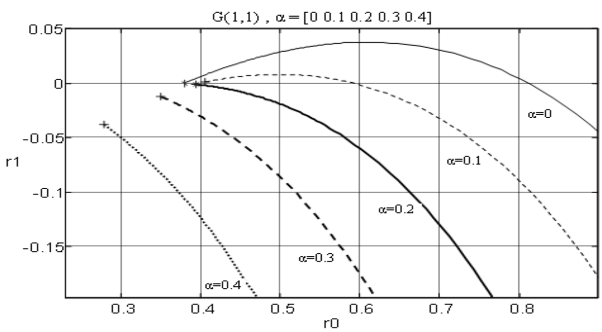

Fig. 12. D_α-plots for the $G_{12}^{eq}(s - \alpha)$.

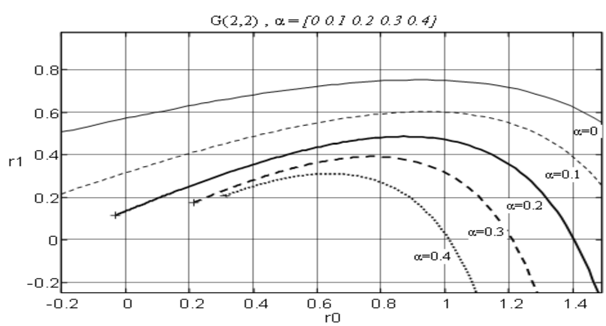

Fig. 13. D_α-plots for the $G_{22}^{eq}(s - \alpha)$.

Local PI controllers designed for $\alpha = 0.1$ are as follows

$$R_1(s) = 0.55 + \frac{0.011}{s} \qquad R_2(s) = 1.194 + \frac{0.582}{s}.$$

Closed-loop poles are

$$\Lambda = \{-0.14; -0.2479 \pm 0.2672j; -0.3188 \pm 0.6872j; -0.4714; -1.3136; -2.417\}.$$

Poles of $G(s)$ are now $\{0.5; -0.7143; -0.833; -1.0; -1.429; -2\}$ and $R(s)$ has a double pole in $s = 0$; hence $n_{q,0.1} = 3$ and $n_q = 1$. The closed loop system under the DC is stable with guaranteed degree of stability $\alpha = 0.1$ if $N[0, \det F(s-0.1)] = n_{q,0.1} = 3$ and $N[0, \det F(s) = n_q = 1$. Corresponding Nyquist plots in Figure 14 (for the sake of clarity plotted just for $\omega > 0$, hence encirclements of $[0, 0j]$ are to be doubled) confirm achieving of the required closed-loop performance in terms of the degree of stability $\alpha = 0.1$.

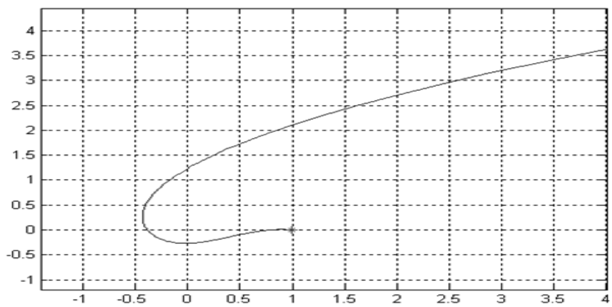

Fig. 14. Nyquist plot of $\det F(s)$.

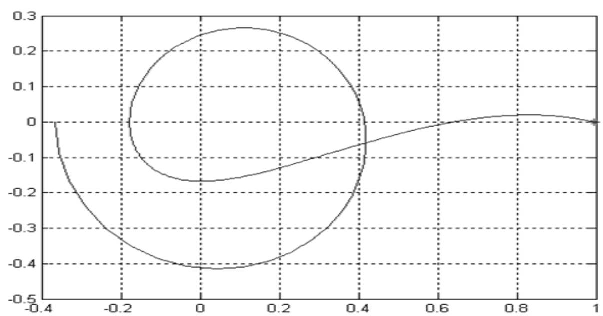

Fig. 15. Nyquist plot of $\det F(s - 0.1)$.

Lemma 1 and the above examples show that the required performance in terms of the degree of stability α achieved for individual equivalent subsystems guarantee achieving of that degree of stability for the full closed-loop system.

Using the proposed design philosophy, local controllers for equivalent subsystems can be designed using any frequency-domain controller design method for SISO systems. For example, according to Lemma 1 and using Nyquist plots of equivalent subsystems with $\alpha = 0$, local controllers guaranteeing performance of equivalent subsystems in terms of phase and gain margins can be obtained. However, interpretation of various performance measures (other than degree of stability) achieved in equivalent subsystems in terms of performance measure of the full closed-loop system is subject of ongoing research.

3.2. Decentralized controller design for robust stability

Let the possible realizations of the uncertain plant $G(s)$ be given as a set of N transfer function matrices corresponding to N different operating points, hence

$$G^k(s) = \{G^k_{ij}(s)\}_{m\times m} \quad k = 1, 2, \ldots, N \tag{30}$$

with

$$G^k_{ij}(s) = \frac{y^k_i(s)}{u^k_j(s)} \quad i, j = 1, 2, \ldots, m$$

where $y^k_i(s)$ is the ith output and $u^k_j(s)$ is the jth input of the plant in the kth operating point.

In this case, individual perturbation forms and related families of plants $\Pi_k, k = a, i, o, ia$ are obtained using relations (1), (2) and (3) modified as follows:

- Additive uncertainty

$$\Pi_a : G(s) = G_0(s) + l_a(s)\Delta(s) \tag{31}$$
$$l_a(s) = \max_k \sigma_M\{G^k(s) - G_0(s)\}. \tag{32}$$

- Multiplicative input uncertainty

$$\Pi_i : G(s) = G_0(s)[I + l_i(s)\Delta(s)] \tag{33}$$
$$l_i(s) = \max_k \sigma_M\{G_0(s)^{-1}[G^k(s) - G_0(s)]\}. \tag{34}$$

- Multiplicative output uncertainty

$$\Pi_o : G(s) = [I + l_o(s)\Delta(s)]G_0(s) \tag{35}$$
$$l_o(s) = \max_k \sigma_M\{[G^k(s) - G_0(s)]G_0(s)^{-1}\}. \tag{36}$$

- Inverse additive uncertainty

$$\Pi_{ia} : \quad G(s) = (I + l_{ia}G_0(s)\Delta(s))^{-1}G_0(s) \tag{37}$$
$$l_{ia} = \max_k \sigma_M\{(G^k(s))^{-1}(s)(G^k(s) - G_0(s))G_0(s)^{-1}). \tag{38}$$

To examine robust stability, conditions (5) and the relations (6) are applied in a usual way.

The resulting robust controller design procedure has the following main steps.

1. Choice of nominal model $G_0(s) = G_d(s) + G_m(s)$, computation and plotting uncertainty bounds $l_k, k = a, i, o, ia$ according to (31), (33), (35) or (37).
2. Design of local controllers $\frac{R_i(s)}{\delta_i}$, $i = 1, 2, \dots, m$ for equivalent subsystems with $\delta_i = 1$, $i = 1, \dots, m$ according to Lemma 1. The decentralized controller has the form
$$R(s) = \begin{bmatrix} \frac{R_1(s)}{\delta_1} & 0 & 0 \\ 0 & \cdots & 0 \\ 0 & 0 & \frac{R_m(s)}{\delta_m} \end{bmatrix}$$
3. Verification of the robust stability condition (5). If satisfied, the design procedure stops, otherwise the design procedure repeats to find δ_i which have the most beneficial effect on the robust stability condition improvement. Usually, δ_i, $i = 1, 2, \dots, m$ is increased and the procedure repeats with step 2.

Remark. Finding δ_i, $i = 1, 2, \dots, m$ is performed in m steps. In each step a single δ_i is increased while the other δ's remain unchanged, and the robust stability condition is verified. The single $\delta_k, k \in \{1, 2, \dots, m\}$ which contributes to the robust stability improvement in the most significant way, is then chosen to modify parameters of the local controller $R_k(s)$.

4. CASE STUDY

The quadruple tank laboratory process consists of four tanks [8]. Its linearized dynamics can be described by a transfer function matrix

$$G(s) = \begin{bmatrix} \frac{3.7\gamma_1}{62s+1} & \frac{3.7(1-\gamma_2)}{(23s+1)(62s+1)} \\ \frac{4.7(1-\gamma_1)}{(30s+1)(90s+1)} & \frac{4.7\gamma_2}{90s+1} \end{bmatrix}$$

where $\gamma_1, \gamma_2 \in (0, 1)$. The system is minimum phase if $1 < \gamma_1 + \gamma_2 < 2$.

A decentralized PI controller is to be designed guaranteeing:

1. robust stability over the whole operating range specified by three working points WPi, $i = 1, 2, 3$ chosen from inside of the "minimum phase" region;
2. specified nominal performance.

$$\text{WP1}: \gamma_1 = 0.4; \gamma_2 = 0.8 \qquad G_1(s) = \begin{bmatrix} \frac{1.48}{62s+1} & \frac{0.74}{(23s+1)(62s+1)} \\ \frac{2.82}{(30s+1)(90s+1)} & \frac{3.76}{90s+1} \end{bmatrix}$$

$$\text{WP2}: \gamma_1 = 0.8; \gamma_2 = 0.4 \qquad G_2(s) = \begin{bmatrix} \frac{2.96}{62s+1} & \frac{2.22}{(23s+1)(62s+1)} \\ \frac{0.94}{(30s+1)(90s+1)} & \frac{1.88}{90s+1} \end{bmatrix}$$

$$\text{WP3}: \gamma_1 = 0.8; \gamma_2 = 0.8 \qquad G_3(s) = \begin{bmatrix} \frac{2.96}{62s+1} & \frac{0.74}{(23s+1)(62s+1)} \\ \frac{0.94}{(30s+1)(90s+1)} & \frac{3.76}{90s+1} \end{bmatrix}$$

The nominal model obtained as a mean value parameter model (corresponding to $\gamma_1 = 0.6667; \gamma_2 = 0.6667$) is

$$G_0(s) = \begin{bmatrix} \frac{2.4667}{62s+1} & \frac{1.2333}{(23s+1)(62s+1)} \\ \frac{1.5667}{(30s+1)(90s+1)} & \frac{3.1333}{90s+1} \end{bmatrix}$$

Characteristic loci $g_2(s-\alpha)$ plotted for $\alpha \in \{0; 0.005; 0.007; 0.009; 0.011; 0.013\}$ are in Figure 16.

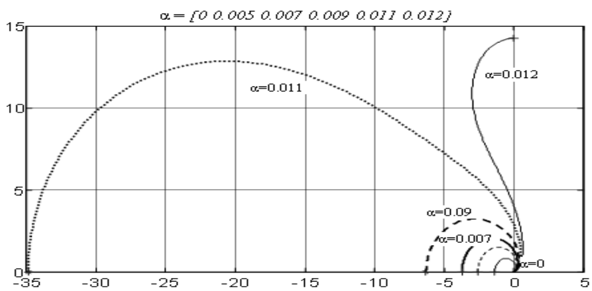

Fig. 16. Characteristic loci $g_2(s-\alpha)$.

The D_α-plots of individual equivalent subsystems plotted for $\alpha \in \{0; 0.005; 0.007; 0.009; 0.011; 0.013\}$ in the (r_0, r_1) plane are in Figures 17−18.

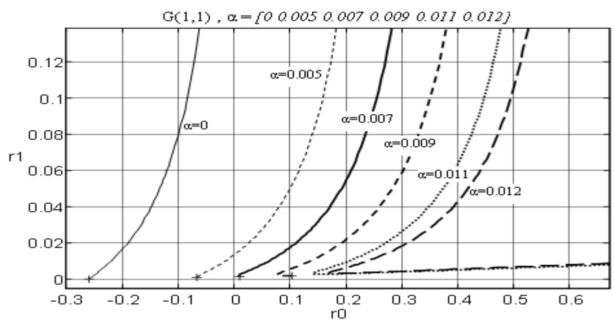

Fig. 17. D_α-plots for the $G_{12}^{\mathrm{eq}}(s-\alpha), \alpha \in \{0; 0.005; 0.007; 0.009\}$.

The decentralized controller has been designed for $\alpha = 0.009$. The resulting local controllers are

$$R_1(s) = 0.18 + \frac{0.01734}{s} \qquad R_2(s) = 0.3 + \frac{0.02306}{s}.$$

Closed-loop poles are

$$\Lambda = \{-0.009 \pm 0.0304j; -0.011; -0.017 \pm 0.01571j; -0.025; -0.0448\}.$$

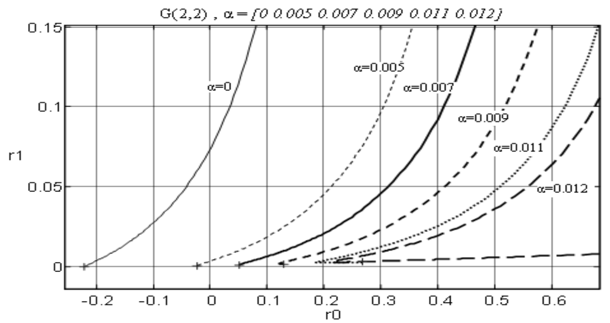

Fig. 18. D_α-plots for the $G^{\mathrm{eq}}_{22}(s-\alpha), \alpha \in \{0; 0.005; 0.007; 0.009\}$.

Nominal closed-loop step responses and step responses in the three working points are in Figures 19 – 22 (in all cases, step change in the 1st reference occurs in 0 s, step change in the 2nd reference in 150 s).

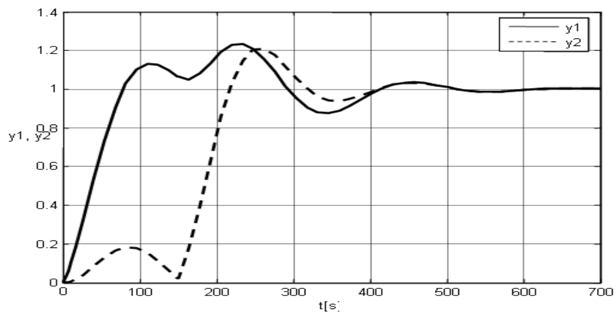

Fig. 19. Nominal closed-loop step response.

Verification of the robust stability condition (4) for three uncertainty types is in Figure 23 for $\delta_1 = \delta_2 = 1$.

The designed decentralized PI controller guarantees the prescribed degree of stability $\alpha = 0.009$ for the mean value parameter nominal model, and stability over the whole operating range of the plant specified by the three working points.

5. CONCLUSION

In this paper a novel frequency-domain approach to the decentralized controller design for performance and robust stability has been proposed. Its main advantage consists in that the plant interactions are included in the design of local controllers so as to achieve required closed-loop performance of the full system (in this paper the required performance was specified in terms of the degree of stability). Local controllers are designed using the independent design approach applied for

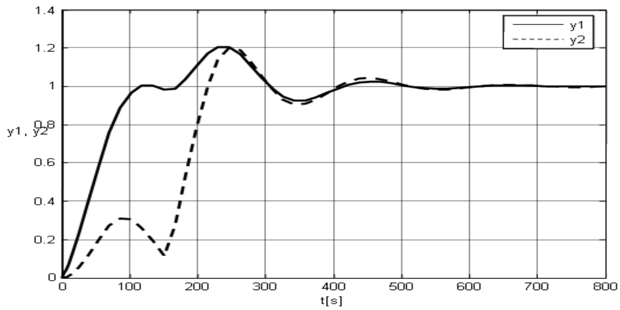

Fig. 20. Closed-loop step response for WP1.

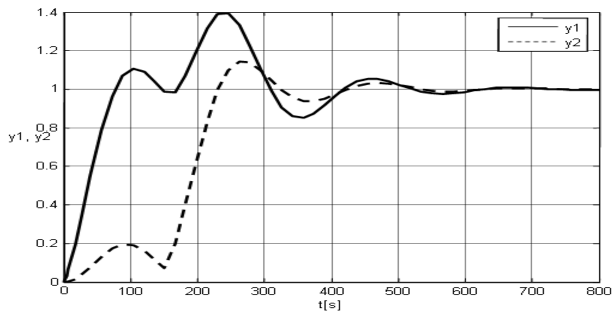

Fig. 21. Closed-loop step response for WP2.

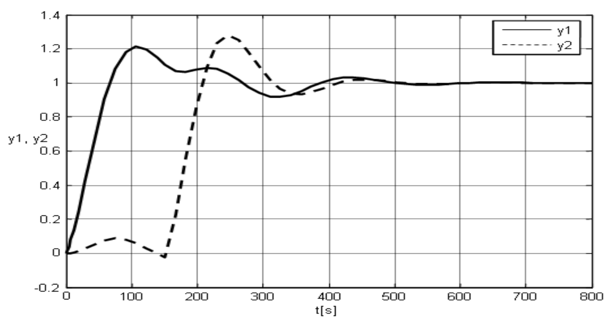

Fig. 22. Closed-loop step response for WP3.

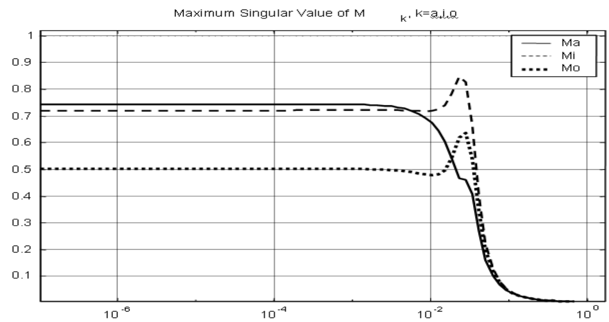

Fig. 23. Verification of the robust stability condition $\sigma_M(M_k(j\omega), k = a, i, o$.

the equivalent subsystems that are actually Nyquist plots of individual decoupled subsystems modified using a selected characteristic locus of the plant interaction matrix. Local controllers designed for equivalent subsystems guarantee fulfilment of performance requirements imposed on the full system without any performance deterioration brought about by the effect of interactions. To guarantee robust stability, the $M - \Delta$ stability conditions are used. Unlike standard robust approaches, the proposed technique considers full nominal model thus reducing conservativeness of resulting robust stability conditions. The developed frequency domain design procedure is graphical, interactive and insightful. Theoretical results are supported by several examples and a case study.

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REFERENCES

[1] M. S. Chiu and Y. Arkun: A methodology for sequential design of robust decentralized control systems. Automatica *28* (1992), 997–1001.

[2] M. Hovd and S. Skogestad: Improved independent design of robust decentralized controllers. In: 12th IFAC World Congress, Vol. 5, Sydney 1993, pp. 271–274.

[3] M. Hovd and S. Skogestad: Sequential design of decentralized controllers. Automatica *30*, (1994), 1601–1607.

[4] A. İftar: Decentralized robust control nased on overlapping decompositions. In: 10th IFAC Symposium om Large Scale Systems, Osaka 2004, pp. 605–609.

[5] A. İftar: Decentralized robust control of large-scale time-delay systems. In: 17th IFAC World Congress, Seoul 2008, CD-ROM.

[6] A. İftar and U. Özgüner: Decentralized LQG/LTR controller design for interconnected systems. In: Proc. American Control Conference, Minneapolis 1987, pp. 1682–1687.

[7] A. İftar and U. Özgüner: Local LQG/LTR controller design for decentralized systems. IEEE Trans. Automat. Control *AC-32* (1987), 926–930.

[8] K. H. Johansson: Interaction bounds in multivariable control systems. Automatica *38* (2002), 1045–1051.

[9] A. Kozáková: Robust decentralized control of complex systems in the frequency domain. In: 2nd IFAC Workshop New Trends in Design of Control Systems, Elsevier Kidlington UK, 1998.

[10] A. Kozáková and V. Veselý: A frequency domain design technique for robust decentralized controllers. In: 16th IFAC World Congress, Prague 2005, Mo-E21-TO/6, CD-ROM.

[11] A. Kozáková and V. Veselý : Improved tuning technique for robust decentralized PID controllers. In: 11th IFAC/IFORS/IMACS/IFIP Symposium on Large Scale Systems Theory and Applications, Gdansk 2007, CD-ROM.

[12] A. Kozáková and V. Veselý: Robust decentralized controller design with additive affine-type uncertainty. Internat. J. Innovative Computing, Information and Control (IJICIC), *3* (2007), 5, 1109–1120.

[13] W. L. Luyben and A. Jutan: Simple method for tuning SISO controllers in multivariable system. Indust. Eng. Chem. Process Development *25* (1986), 654–660.

[14] A. G. J. MacFarlane and I. Postlethwaite: The generalized Nyquist stability criterion and multivariable root loci. Internat. J. Control *25* (1977), 81–127.

[15] D. Q. Mayne: The design of linear multivariable systems. Automatica *9* (1973), 201–207.

[16] J. J. Neymark: Dynamical Systems and Controlled Processes. Nauka, Moscow 1978 (in Russian).

[17] X. Qiang, C. Wen-Jian, and H. Ming: A practical decentralized auto-tuning method for TITO systems under closed-loop control. Internat. J. Innovative Computing, Information and Control (IJICIC) *2* (2006), 305–322.

[18] H. Schmidt and E. W. Jacobsen: Selecting control configurations for performance with independent design. Comput. Chem. Engrg. *27* (2003), 101–109.

[19] S. Skogestad and M. Morari: Robust performance of decentralized control systems by independent designs. Automatica *25* (1989), 119–125.

[20] S. Skogestad and I. Postlethwaite: Multivariable Feedback Control: Analysis and Design. Third edition. Wiley, Chichester – New York – Brisbane – Toronto – Singapore 1996.

[21] V. Veselý: Large scale dynamic system stabilization using the principle of dominant subsystem approach. Kybernetika *29* (1993), 1, 48–61.

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