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ON MAIN CHARACTERISTICS OF THE $M/M/1/N$ QUEUE WITH SINGLE AND BATCH ARRIVALS AND THE QUEUE SIZE CONTROLLED BY AQM ALGORITHMS

Wojciech M. Kempa

In the article finite-buffer queueing systems of the $M/M/1/N$ type with queue size controlled by AQM algorithms are considered, separately for single and batch arrivals. In the latter case two different acceptance strategies: WBAS (Whole Batch Acceptance Strategy) and PBAS (Partial Batch Acceptance Strategy) are distinguished.

Three essential characteristics of the system are investigated: the stationary queue-size distribution, the number of consecutively dropped packets (batches of packets) and the time between two successive accepted packets (batches of packets).

For these characteristics the formulae which can be easily numerically treated are derived. Numerical results obtained for three sample dropping functions are attached as well.

Keywords: active queue management (AQM), drop function, finite buffer, queue size

Classification: 90B22, 60K25

1. MOTIVATION

The traditional tool for the controlling the queue size in finite-buffer systems is connected with using the Tail Drop (TD) algorithm. In this approach the arriving packets are lost only when all "places" in the buffer are occupied (see [2]). To avoid congestion of the buffer the Active Queue Management (AQM) procedures were proposed. In the AQM scheme a drop function is introduced and successive packets are accepted for service with a probability that usually depends on the current queue size.

For the first time the AQM approach in the control of finite-buffer queues was described in [4]. In practice different types of dropping functions are used. The most popular is a linear dropping function (Random Early Detection (RED) algorithm) which was introduced in [4]. One can also find some applications of the RED scheme in [7], [9], [10], [12] and [13]. In particular, in [7] the RED scheme is applied to the $G/M/1/N$ queueing system with batch arrivals and next some measures for assessing the performance of this algorithm are introduced. One can find some other types of dropping functions e.g. in [5], [8] (Gentle Random Early Detection (GRED) scheme) and [1] (Random Exponential Marking (REM) algorithm). In [2] the queueing model of the $M/M/1/N$ type with single arrivals is considered and some characteristics for TD and AQM schemes are compared. The case of AQM algorithms applied in the queueing system with small buffer capacity is studied in [3].

It seems that the influence of AQM algorithms for main characteristics of finite-buffer queueing systems is not sufficiently investigated analytically. In particular, queues with batch arrivals, which can be helpful in modelling the Internet traffic, are inadequately studied. Besides, majority of results is of numerical nature only.

In the article we consider separately systems of the $M/M/N/1$ type with single and batch arrivals, with AQM algorithms applied for the controlling the queue size. For batch arrivals we analyze two types of acceptance strategies: WBAS (Whole Batch Acceptance Strategy) and PBAS (Partial Batch Acceptance Strategy) which will be described in details in the next section. Three main characteristics of such systems are investigated: the steady-state queue size, the number of consecutively lost packets (or batches in the case of batch arrivals) and the time between two successive accepted packets (batches of packets). These characteristics are of great importance in the analyzing the influence of AQM algorithms for the risk of buffer congestions. We treat them analytically, considering separately cases of individual and group arrivals. Explicit equations for all characteristics which can be used in numerical practice are derived. As an illustration we present numerical results obtained for three different sample drop functions.

Thus, the article is organized as follows. In the next Section 2 we describe models and introduce some necessary notations. In Section 3 we investigate queue-size distribution in the steady state of the system. Section 4 is devoted to the distribution of the number of consecutively lost packets (batches of packets) – we present a useful algorithm for computing proper probabilities. In Section 5 we derive results for the distribution of the time between two successive accepted packets (batches of packets). The last Section 6 contains numerical results.

2. DESCRIPTION OF MODELS

Let us consider the $M/M/1/N$ queueing system in that interarrival times and service times are mutually independent and exponentially distributed random variables with means λ^{-1} and μ^{-1} respectively. The system capacity is assumed to be N (there are $N-1$ places in the buffer queue and one place for service). We investigate separately systems with individual and batch arrivals. Thus, let us denote by p_k the probability that the arriving batch size is exactly k, where $\sum_{k=1}^{\infty} p_k = 1$.

As a tool for controlling the queue size in the system we introduce a drop function d_k depending on the actual queue size k. For the case of single arrivals d_k gives the probability of rejection the arriving packet that finds k packets present in the system. Of course $d_k = 1$ for $k \geq N$. For group arrivals we distinguish two acceptance strategies (see [6] for more details):

- the Whole Batch Acceptance Strategy (WBAS) in which the arriving batch is always lost (without using a drop function) when the batch size is larger than the number of free waiting places in the buffer;
- the Partial Batch Acceptance Strategy (PBAS) in which the arriving packet

can be partially accepted (according to the dropping procedure) even when the batch size is larger than the number of available free positions in the buffer.

In the case of batch arrivals we additionally assume that all packets belonging to the same arriving batch are dropped with the same probability depending on the queue size just before the batch arrival.

3. STEADY-STATE QUEUE-SIZE DISTRIBUTION

In this section we deal with the steady-state queue size distribution. We consider cases of individual and batch arrivals separately.

3.1. Individual arrivals

Let us call the "ordinary" one the system without dropping of packets. The steady-state queue size probabilities π_k, $k = 0, 1, \ldots, N$ for the "ordinary" system can be obtained from the following equilibrium equations:

$$\begin{cases} \lambda\pi_0 &= \mu\pi_1, \\ (\lambda+\mu)\pi_k &= \lambda\pi_{k-1} + \mu\pi_{k+1}, \quad 1 \le k \le N-1, \\ \mu\pi_N &= \lambda\pi_{N-1} \end{cases} \tag{1}$$

and hence the following well known result (see for example [11]) can be obtained:

$$\pi_k = \frac{(1-\varrho)\varrho^k}{1-\varrho^{N+1}}, \qquad 0 \le k \le N, \tag{2}$$

where $\varrho = \frac{\lambda}{\mu}$ denotes the occupation rate of the system.

Now let us assume that the arriving packet which finds k packets present in the system is dropped with probability d_k, $k = 0, 1, \ldots, N$. The system of equilibrium equations for the steady-state queue-size probabilities $\widehat{\pi}_k$ takes the following form:

$$\begin{cases} \lambda(1-d_0)\widehat{\pi}_0 &= \mu\widehat{\pi}_1, \\ \big[\lambda(1-d_k)+\mu\big]\widehat{\pi}_k &= \lambda(1-d_{k-1})\widehat{\pi}_{k-1} + \mu\widehat{\pi}_{k+1}, \quad 1 \le k \le N-1, \\ \mu\widehat{\pi}_N &= \lambda(1-d_{N-1})\widehat{\pi}_{N-1}. \end{cases} \tag{3}$$

Hence we get

$$\widehat{\pi}_k = \frac{\lambda^k}{\mu^k}\prod_{i=0}^{k-1}(1-d_i)\widehat{\pi}_0, \;\; k = 1, 2, \ldots N. \tag{4}$$

From the condition

$$\sum_{k=0}^{N}\widehat{\pi}_k = 1$$

we easily find

$$\widehat{\pi}_0 = \Big(1 + \sum_{k=1}^{N}\varrho^k\prod_{i=0}^{k-1}(1-d_i)\Big)^{-1} \tag{5}$$

and hence, finally,

$$\widehat{\pi}_k = \frac{\varrho^k \prod_{i=0}^{k-1}(1-d_i)}{1+\sum_{j=1}^{N} \varrho^j \prod_{i=0}^{j-1}(1-d_i)}, \qquad k=0,1,\dots,N. \tag{6}$$

Let us note that (6) is equivalent to (2) if $d_i \equiv 0$.

3.2. Group arrivals

Now let us consider the case of packets entering in groups. Denoting by Π_k^W and Π_k^P the steady-state queue-size probabilities with respect to the strategies WBAS and PBAS respectively, we can write down the following systems of equlibrium equations in the "ordinary" system:

$$\begin{cases} \lambda\Pi_0^W \sum_{i=1}^{N} p_i &= \mu\Pi_1^W, \\ (\lambda\sum_{i=1}^{N-k} p_i + \mu)\Pi_k^W &= \lambda\sum_{i=0}^{k-1} \Pi_i^W p_{k-i} + \mu\Pi_{k+1}^W, \quad k=1,\dots,N-1, \\ \mu\Pi_N^W &= \lambda\sum_{i=0}^{N-1} \Pi_i^W p_{N-i} \end{cases} \tag{7}$$

and

$$\begin{cases} \lambda\Pi_0^P &= \mu\Pi_1^P, \\ (\lambda+\mu)\Pi_k^P &= \lambda\sum_{i=0}^{k-1} \Pi_i^P p_{k-i} + \mu\Pi_{k+1}^P, \quad k=1,\dots,N-1, \\ \mu\Pi_N^P &= \lambda(\sum_{i=N}^{\infty} p_i\Pi_0^P + \sum_{i=N-1}^{\infty} p_i\Pi_1^P + \ldots + \sum_{i=1}^{\infty} p_i\Pi_{N-1}^P). \end{cases} \tag{8}$$

Now, let us introduce a drop function d_k to the model. Assume that each packet in the arriving group which can be joined to the queue, has the same probability of dropping and is dropped independently on the other packets in the same batch. First, let us consider the WBAS strategy. It is obvious that the queue size in the system at fixed moment t can be described by a continuous-time Markov chain with the state space $\{0,1,\dots,N\}$. It is easy to check that for $0 \le k < l \le N-1$ we have

$$\widehat{q}_{k,l}^W = \lambda(1-d_k)^{l-k}\Big(p_{l-k} + \sum_{i=l-k+1}^{N-k} p_i \binom{i}{l-k} d_k^{i-(l-k)}\Big), \tag{9}$$

where $\widehat{q}_{k,l}^W$ denotes the intensity of transition from state k to l. Besides we have

$$\widehat{q}_{k,N}^W = \lambda(1-d_k)^{N-k} p_{N-k}, \qquad 0 \le k \le N-1. \tag{10}$$

Hence we can find stationary probabilities $\widehat{\Pi}_k^W$ in the case of WBAS from the following system of equations:

$$\begin{cases} \widehat{\Pi}_0^W \sum_{i=1}^{N} \widehat{q}_{0,i}^W &= \mu\widehat{\Pi}_1^W, \\ (\sum_{i=k+1}^{N} \widehat{q}_{k,i}^W + \mu)\widehat{\Pi}_k^W &= \sum_{i=0}^{k-1} \widehat{\Pi}_i^W \widehat{q}_{i,k}^W + \mu\widehat{\Pi}_{k+1}^W, \quad k=1,\dots,N-1, \\ \mu\widehat{\Pi}_N^W &= \sum_{i=0}^{N-1} \widehat{\Pi}_i^W \widehat{q}_{i,N}^W. \end{cases} \tag{11}$$

For the steady-state probabilities $\widehat{\Pi}_k^P$ in the PBAS we have, in fact, the same system of equations

$$\begin{cases} \widehat{\Pi}_0^P \sum_{i=1}^N \widehat{q}_{0,i}^P &= \mu \widehat{\Pi}_1^P, \\ (\sum_{i=k+1}^N \widehat{q}_{k,i}^P + \mu)\widehat{\Pi}_k^P &= \sum_{i=0}^{k-1} \widehat{\Pi}_i^P \widehat{q}_{i,k}^P + \mu \widehat{\Pi}_{k+1}^P, \quad k = 1, \dots, N-1, \\ \mu \widehat{\Pi}_N^P &= \sum_{i=0}^{N-1} \widehat{\Pi}_i^P \widehat{q}_{i,N}^P, \end{cases} \tag{12}$$

where proper transition intensities $\widehat{q}_{k,l}^P$ for $0 \le k < l \le N-1$ are defined as follows:

$$\widehat{q}_{k,l}^P = \lambda(1-d_k)^{l-k}\Big(p_{l-k} + \sum_{i=l-k+1}^{\infty} p_i \binom{i}{l-k} d_k^{i-(l-k)}\Big) \tag{13}$$

and, for $0 \le k \le N-1$,

$$\widehat{q}_{k,N}^P = \lambda(1-d_k)^{N-k} \times$$
$$\times \Big[p_{N-k} + \sum_{i=1}^{\infty} p_{N-k+i}\Big(1 + \sum_{j=1}^{i} \binom{N-k}{N-k-j} \sum_{l=j}^{i} \binom{l-1}{j-1} d_k^l\Big)\Big]. \tag{14}$$

4. NUMBER OF CONSECUTIVELY LOST PACKETS

In this section our aim is to find the explicit representation for the probability function of the number of consecutively lost packets (batches of packets) in the steady state of the system. As in the previous section, we consider cases of individual and group arrivals separately.

4.1. Individual arrivals

Let us denote by γ the number of consecutively dropped packets in the steady state of the system. If X and Y are random variables denoting interarrival time and service time respectively, then (having in mind the memoryless of exponential distributions)

$$\alpha = \mathbf{P}\{X < Y\} = \frac{\lambda}{\lambda + \mu} \tag{15}$$

is the probability that an arrival epoch precedes a departure one. Let $\mathbf{P}\{\gamma \ge k \,|\, i\}$ denote the probability that at least k packets are lost consecutively on condition that the "series of losses" begins with i packets present in the system.

Applying the formula of total probability we obtain

$$\mathbf{P}\{\gamma \ge 1 \,|\, i\} = d_i \quad \text{and hence} \quad \mathbf{P}\{\gamma \ge 1\} = \sum_{i=0}^{N} d_i \widehat{\pi}_i. \tag{16}$$

Let us briefly comment (16). In the first equation the probability that at least one packet will be lost, if the series of losses begins with i packets present, equals the probability d_i that the first arriving packet is dropped. The second equation in (16) is the consequence of the first one by using the formula of total probability.

One can construct a useful algorithm for computing probabilities $\mathbf{P}\{\gamma \geq k \,|\, i\}$ for successive k. Let us note that the formula of total probability gives

$$\mathbf{P}\{\gamma \geq k \,|\, i\} = d_i\big[\alpha\mathbf{P}\{\gamma \geq k-1 \,|\, i\} + (1-\alpha)\mathbf{P}\{\gamma \geq k-1 \,|\, i-1\}\big], \tag{17}$$

where $i = 1, 2, \ldots, N$, and besides

$$\mathbf{P}\{\gamma \geq k \,|\, 0\} = d_0^k, \tag{18}$$

where d_0^k denotes the probability that at least k successive losses occur on condition that the series of losses begins when the system is empty.

Of course now

$$\mathbf{P}\{\gamma = k\} = \sum_{i=0}^{N} \widehat{\pi}_i \mathbf{P}\{\gamma = k \,|\, i\} = \sum_{i=0}^{N} \widehat{\pi}_i \big(\mathbf{P}\{\gamma \geq k \,|\, i\} - \mathbf{P}\{\gamma \geq k+1 \,|\, i\}\big). \tag{19}$$

4.2. Group arrivals

Let us consider the system with dropping of packets and group arrivals. Let Γ^W and Γ^P denote numbers of consecutively dropped batches in the system with WBAS and PBAS strategy respectively. Introducing conditional probabilities $\mathbf{P}\{\Gamma^W \geq k \,|\, i\}$ and $\mathbf{P}\{\Gamma^P \geq k \,|\, i\}$ we obtain

$$\mathbf{P}\{\Gamma^W \geq 1 \,|\, i\} = \sum_{k=1}^{N-i} p_k d_i^k + \sum_{k=N-i+1}^{\infty} p_k, \quad i = 0, 1, \ldots, N. \tag{20}$$

Indeed, if the size of the arriving batch exceeds the number of free places in the system at the pre-arrival epoch, then the batch is rejected as a whole "by definition" (the second summand in (20)). Otherwise, particular packets can be lost by using a drop function (the first summand in (20)).

Next we get for $i = 1, 2, \ldots, N$

$$\begin{aligned}\mathbf{P}\{\Gamma^W \geq n \,|\, i\} = \Big(&\sum_{k=1}^{N-i} p_k d_i^k + \sum_{k=N-i+1}^{\infty} p_k\Big) \\ &\times \big(\alpha\mathbf{P}\{\Gamma^W \geq n-1 \,|\, i\} + (1-\alpha)\mathbf{P}\{\Gamma^W \geq n-1 \,|\, i-1\}\big)\end{aligned} \tag{21}$$

and

$$\mathbf{P}\{\Gamma^W \geq n \,|\, 0\} = \Big(\sum_{k=1}^{N} p_k d_i^k + \sum_{k=N+1}^{\infty} p_k\Big)^n, \tag{22}$$

where $n \geq 1$.

In the PBAS strategy the arriving batch is rejected as a whole if all packets in this batch are dropped, thus for the PBAS model we obtain

$$\mathbf{P}\{\Gamma^P \geq 1 \,|\, i\} = \sum_{k=1}^{\infty} p_k d_i^k, \quad i = 0, 1, \ldots, N. \tag{23}$$

Hence we get for $i = 1, 2, \dots, N$

$$\mathbf{P}\{\Gamma^P \geq n \,|\, i\} = \sum_{k=1}^{\infty} p_k d_i^k \big[\alpha \mathbf{P}\{\Gamma^P \geq n-1 \,|\, i\} + (1-\alpha)\mathbf{P}\{\Gamma^P \geq n-1 \,|\, i-1\}\big] \tag{24}$$

and

$$\mathbf{P}\{\Gamma^P \geq n \,|\, 0\} = \Big(\sum_{k=1}^{\infty} p_k d_0^k\Big)^n, \quad n \geq 1. \tag{25}$$

Lastly we can write

$$\mathbf{P}\{\Gamma^\star = k\} = \sum_{i=0}^{N} \widehat{\pi}_i \mathbf{P}\{\Gamma^\star = k \,|\, i\} = \sum_{i=0}^{N} \widehat{\Pi}_i^\star \big(\mathbf{P}\{\Gamma^\star \geq k \,|\, i\} - \mathbf{P}\{\Gamma^\star \geq k+1 \,|\, i\}\big), \tag{26}$$

where the notation $\star$ stands for "W" or "P".

5. TIME BETWEEN ACCEPTED PACKETS (BATCHES OF PACKETS)

5.1. Individual arrivals

Let i be the number of packets in the system just after the arrival of accepted packet. Let us denote by τ the time from this moment to the nearest arrival of accepted packet. There is a natural relationship between random variables τ and γ. In the case of k consecutive losses, τ has the Erlang distribution with $k+1$ degrees of freedom. The rule of total probability leads to the following formula:

$$\mathbf{P}\{\tau > x \,|\, i\} = \mathbf{P}\{\sum_{k=1}^{\gamma+1} X_k > x \,|\, i\} = \sum_{m=1}^{\infty} \mathbf{P}\{\sum_{k=1}^{m+1} X_k > x\}\mathbf{P}\{\gamma = m \,|\, i\} =$$
$$= e^{-\lambda x} \sum_{m=1}^{\infty} \sum_{k=0}^{m} \frac{(\lambda x)^k}{k!} \mathbf{P}\{\gamma = m \,|\, i\}, \tag{27}$$

where $X_1, X_2, \dots$ are successive interarrival times. Hence we easily obtain

$$\mathbf{P}\{\tau > x\} = \sum_{i=0}^{N} \widehat{\pi}_i \mathbf{P}\{\tau > x \,|\, i\}. \tag{28}$$

5.2. Group arrivals

Similarly, let us denote by T the time between two successive completely accepted batches. The distribution of T has the same form for WBAS and PBAS strategies. We have

$$\mathbf{P}\{T > x \,|\, i\} = \mathbf{P}\{\sum_{k=1}^{\Gamma^\star+1} X_k > x \,|\, i\} = \sum_{m=1}^{\infty} \mathbf{P}\{\sum_{k=1}^{m+1} X_k > x\}\mathbf{P}\{\Gamma^\star = m \,|\, i\} =$$
$$= e^{-\lambda x} \sum_{m=1}^{\infty} \sum_{k=0}^{m} \frac{(\lambda x)^k}{k!} \mathbf{P}\{\Gamma^\star = m \,|\, i\}, \tag{29}$$

where the notation $\star$ was defined earlier. Finally we get

$$\mathbf{P}\{T > x\} = \sum_{i=0}^{N} \widehat{\Pi}_i^{\star} \mathbf{P}\{T > x \,|\, i\}. \tag{30}$$

6. NUMERICAL EXAMPLES

All formulae obtained above can be easily numerically treated. In the following illustrative examples we compare the queue-size distributions in systems with and without dropping of packets for cases of individual and batch arrivals. Besides, for systems with drop functions, we evaluate distributions of consecutively lost packets (batch of packets). We derive results for the system of capacity $N = 8$ and for three different drop functions d_n:

- an increasing linear dropping function (RED) of the form

$$d_n = \begin{cases} 0, & n \leq 3, \\ 0.2n - 0.6, & 3 < n < 8, \\ 1, & n \geq 8; \end{cases} \tag{31}$$

- an exponential dropping function (REM) of the form

$$d_n = \begin{cases} 0, & n \leq 4, \\ -1.0187e^{-n+4} + 1.0186, & 4 < n < 8, \\ 1, & n \geq 8; \end{cases} \tag{32}$$

- an increasing broken line as a dropping function (GRED) of the form

$$d_n = \begin{cases} 0, & n \leq 2, \\ 0.1n - 0.2, & 2 < n \leq 5, \\ 0.2333n - 0.8664, & 5 < n < 8, \\ 1, & n \geq 8. \end{cases} \tag{33}$$

The described above drop functions are presented in Figure 1 below.

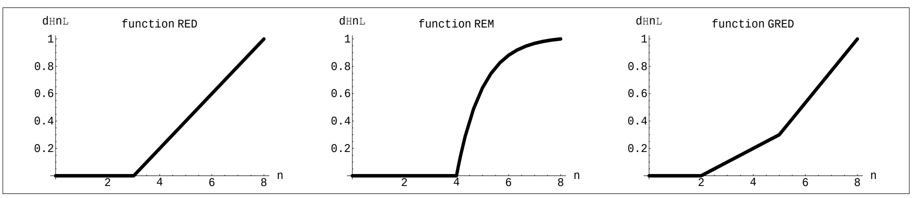

Fig. 1. Three different sample drop functions: RED, REM and GRED.

For the need of examples we assume that the arrival rate (of single customers or batches) is $\lambda = 2$, the service rate is $\mu = 3$ and the size of arriving groups (in the

Queue size (k)	π_k	$\widehat{\pi}_k$ (RED)	$\widehat{\pi}_k$ (REM)	$\widehat{\pi}_k$ (GRED)
0	0.342236	0.361642	0.360954	0.364943
1	0.228157	0.241195	0.240636	0.243296
2	0.152105	0.160730	0.160424	0.162197
3	0.101403	0.107153	0.106949	0.108131
4	0.067602	0.071436	0.071300	0.064879
5	0.045068	0.038099	0.047533	0.034602
6	0.030045	0.015240	0.011286	0.016148
7	0.020030	0.004064	0.000897	0.005023
8	0.013354	0.000542	0.000019	0.000781

Tab. 1. Comparison of queue-size distributions for the single arrival system with and without AQM algorithms.

k	$\mathbf{P}\{\gamma = k\}$ (RED)	$\mathbf{P}\{\gamma = k\}$ (REM)	$\mathbf{P}\{\gamma = k\}$ (GRED)
1	0.029977	0.025394	0.036201
2	0.008262	0.009768	0.007733
3	0.002686	0.003802	0.002243
4	0.000956	0.001492	0.000770
5	0.000357	0.000589	0.000286
6	0.000138	0.000233	0.000111
7	0.000054	0.000093	0.000043
8	0.000021	0.000037	0.000017

Tab. 2. Distribution function of the number of consecutively lost packets for different-type drop functions.

case of batch arrivals) is geometrically distributed with probability function $p_k = \frac{1}{2^k}$ for $k \geq 1$. Thus, we investigate the underloaded system (with $\varrho = 0.(6) < 1$) in the case of single arrivals and the overloaded system (with $\varrho = 1.(3) > 1$) in the case of batch arrivals. For the latter case we will consider disciplines WBAS and PBAS separately. In all computations we take the precision of 10^{-6}.

6.1. The system with single arrivals

In Table 1 we present the comparison of the queue-size distributions for the case of "ordinary" system and with different-type drop functions. Results from Table 1 are presented geometrically in Figure 2.

As one can see, for different sample drop functions, the REM one most efficiently reduces the risk of buffer's congestion.

Table 2 contents results for the distribution of the number of consecutively lost packets k for different drop functions (we take k from 1 to 8).

For three fixed sample drop functions, for the GRED one the probability of losing (consecutively) big number of packets is the lowest one.

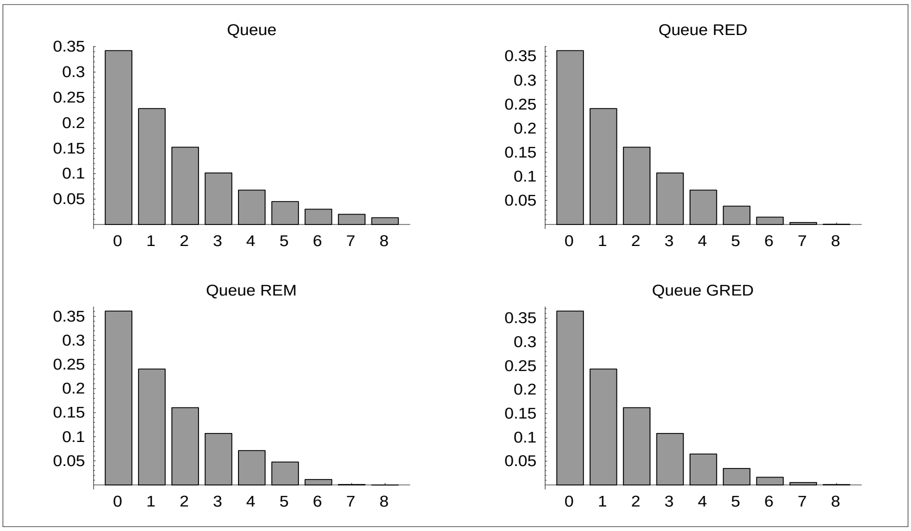

Fig. 2. Comparison of queue-size distributions for the single arrival system with and without AQM algorithms.

6.2. The batch arrival system with WBAS discipline

The stationary queue-size distribution function for the system with and without drop function is presented in Table 3 and Figure 3. In the table we use notations introduced earlier.

Queue size (k)	Π_k^W	$\widehat{\Pi}_k^W$ (RED)	$\widehat{\Pi}_k^W$ (REM)	$\widehat{\Pi}_k^W$ (GRED)
0	0.118411	0.144968	0.149103	0.145368
1	0.078632	0.096268	0.099014	0.096533
2	0.091174	0.111622	0.114806	0.111930
3	0.105061	0.128623	0.132293	0.128979
4	0.119549	0.146360	0.150535	0.142239
5	0.132564	0.151454	0.166924	0.144973
6	0.139192	0.128829	0.124201	0.129051
7	0.129250	0.073589	0.050531	0.079832
8	0.086167	0.018287	0.012592	0.021096

Tab. 3. Comparison of queue-size distributions for the batch arrival system with and without dropping of packets (WBAS).

It is easy to note that the REM function (among sample ones) is the best one for the avoiding buffer's congestions.

Distributions of numbers of consecutively lost batches of packets k for different drop functions in the system with WBAS service discipline are presented in Table 4 for $k = 1, \dots, 8$.

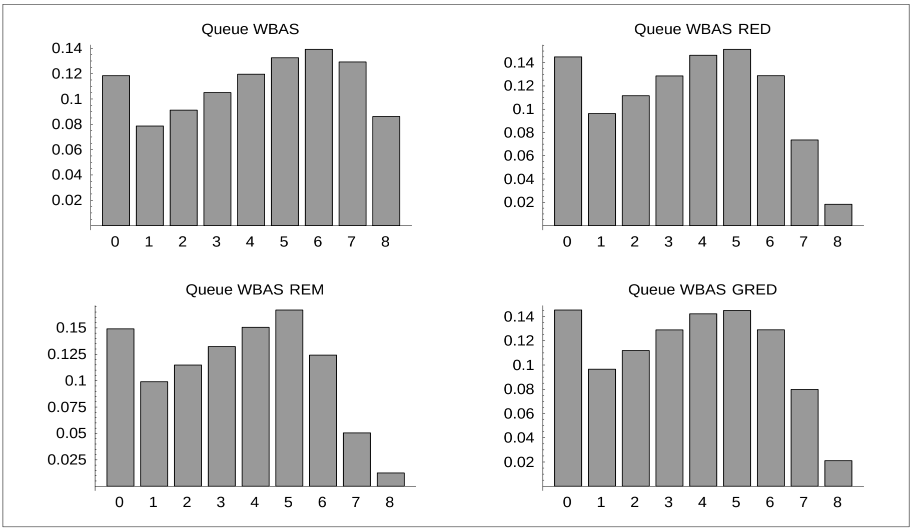

Fig. 3. Comparison of queue-size distributions for the batch arrival system with and without dropping of packets (WBAS).

As one can see, if we are interested in the minimizing the risk of dropping of big number of batches, the RED dropping function is the better one (of course, taking into consideration three functions defined earlier).

6.3. The batch arrival system with PBAS discipline

The steady-state queue-size distribution for the system with and without dropping function in the case of PBAS strategy is presented in Table 5 and Figure 4.

Probability of the buffer's congestion is the smallest one for the case of REM functions (taking into consideration three sample functions).

Distributions of numbers of consecutively lost batches of packets k for different

k	$\mathbf{P}\{\Gamma^W = k\}$ (RED)	$\mathbf{P}\{\Gamma^W = k\}$ (REM)	$\mathbf{P}\{\Gamma^W = k\}$ (GRED)
1	0.133259	0.123609	0.135496
2	0.060902	0.070050	0.058020
3	0.031188	0.041570	0.028844
4	0.015889	0.023867	0.014405
5	0.007843	0.013175	0.006981
6	0.003742	0.007001	0.003272
7	0.001733	0.003595	0.001490
8	0.000783	0.001793	0.001490

Tab. 4. Distribution of the number of consecutively lost batches of packets for different-type drop functions (WBAS).

Queue size (k)	Π_k^P	$\widehat{\Pi}_k^P$ (RED)	$\widehat{\Pi}_k^P$ (REM)	$\widehat{\Pi}_k^P$ (GRED)
0	0.093207	0.123240	0.133494	0.124841
1	0.062138	0.082160	0.088996	0.083227
2	0.072494	0.095854	0.103829	0.097098
3	0.084576	0.111829	0.121134	0.113281
4	0.098672	0.130467	0.141323	0.128035
5	0.115118	0.142548	0.164876	0.137074
6	0.134304	0.138252	0.140172	0.135676
7	0.156688	0.110976	0.076297	0.108522
8	0.182803	0.064674	0.029879	0.072245

Tab. 5. Comparison of queue-size distributions for the batch arrival system with and without dropping of packets (PBAS).

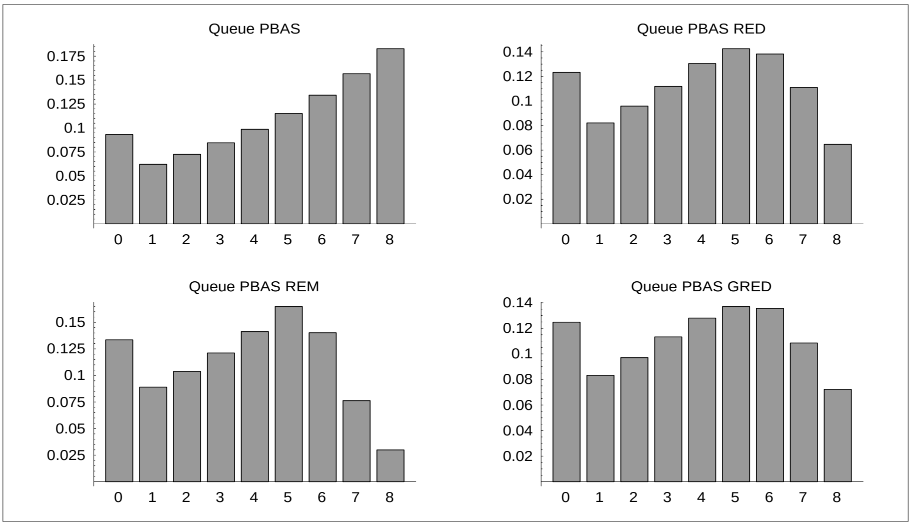

Fig. 4. Comparison of queue-size distributions for the batch arrival system with and without dropping of packets (PBAS)

drop functions in the system with PBAS service discipline are presented in Table 6 for $k = 1, 2, 3, 4$.

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k	$\mathbf{P}\{\Gamma^P = k\}$ (RED)	$\mathbf{P}\{\Gamma^P = k\}$ (REM)	$\mathbf{P}\{\Gamma^P = k\}$ (GRED)
1	0.131921	0.119620	0.129158
2	0.059951	0.072035	0.054883
3	0.029745	0.043518	0.026718
4	0.014489	0.025571	0.012691

Tab. 6. Distribution of the number of consecutively lost batches of packets for different-type drop functions (PBAS).

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