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Acta Universitatis Carolinae. Mathematica et Physica, Vol. 15 (1974), No. 1-2, 157--158

Persistent URL: http://dml.cz/dmlcz/142347

1974 ACTA UNIVERSITATIS CAROLINAE - MATHEMATICA ET PHYSICA NO. 1-2 PAG. 157-158

Note About the Parallel Projection Method for Solution of System of Linear Algebraic Equations

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This article is only a summary of the paper [3]. The purpose of this paper is to describe a direct projection method for solution of linear algebraic equations. The method is suitable also for systems with singul atrices and for execution on a parallel computer.

I. Introduction

Well-known projection methods, the method of orthogonal projections of Kaczmarz [1, 2] and the method of Cimmino [1], are iterative methods which in many cases require many operations to achieve sufficient accuracy of solution. We now describe a direct projection method i.e. a projection method which needs finite number of projections for solution of system of linear algebraic equations. Let us consider system

$$A\bar{x} = \bar{b} \tag{1}$$

where A is a regular $n \times n$ matrix, and $\bar{b}$ is n-vector. We will consider system (1) in the form

$$r_i(\bar{x}) = \{b_i - \sum_{j=1}^{n} a_{ij}x_j\} = 0\,; \quad i = 1, 2, \ldots, n \tag{2}$$

where r_i represent n hyperplanes in n-dimensional Euclidean space E_n.

2. Mathematical Description of the Algorithm

Let us consider system (2) in the form

$$r_i(\bar{x}) = \{b_i - (\bar{a}_i, \bar{x})\} = 0\,; \quad i = 1, 2, \ldots, n$$

Definition: Let $\bar{x}_0^{(0)}, \bar{x}_0^{(1)}, \bar{x}_0^{(2)}, \ldots, \bar{x}_0^{(n)}$ be $n + 1$ linear independent points of the space E_n, then the algorithm for solution of system (1) is defined as follows:

(1) Choose the point

$$\bar{x}_{i-1}^{(p)} \in \{\bar{x}_{i-1}^{(s)}\}_{s=i}^{n} \text{ that}$$

$$(\bar{v}_{i-1}^{(p)}, \bar{a}_i) \neq 0$$

where
$$\bar{v}_{i-1}^{(p)} = \bar{x}_{i-1}^{(p)} - \bar{x}_{i-1}^{(i-1)}\,.$$

(2) Make the transformation

$$\bar{x}^{(p)}_{i-1} \rightarrow \bar{x}^{(i)}_{i-1}, \ \bar{x}^{(i)}_{i-1} \rightarrow \bar{x}^{(p)}_{i-1} \quad \text{if} \quad p \neq i\,.$$

(3) Make the calculation by the reccurent relation

$$\bar{x}^{(k)}_i = \bar{x}^{(k)}_{i-1} + \bar{v}^{(i)}_{i-1} \frac{r_i(\bar{x}^{(k)}_{i-1})}{(\bar{v}^{(i)}_{i-1}, \bar{a}_i)} \tag{3}$$

where

$$\bar{v}^{(i)}_{i-1} = \bar{x}^{(i)}_{i-1} - \bar{x}^{(i-1)}_{i-1}$$

for $i = 1, 2, \ldots, n, \quad k = i, i + 1, \ldots, n$

In the paper [3] following theorems are proved:

Theorem 1: The point $\bar{x}^{(n)}_n$ defined by algorithm (3) is the solution of system (1).

Theorem 2: The vector $\bar{v}^{(n)}_{n-1}$ defined by algorithm (3) is orthogonal to $n - 1$ linear independent vectors $\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_{n-1}$.

Theorem 3: The vectors $\bar{v}^{(1)}_0, \bar{v}^{(2)}_1, \ldots, \bar{v}^{(n)}_{n-1}$ defined by algorithm (3) define the solution of system (1) for arbitrary non-zero vector $\bar{b}$.

The method is suitable also for systems with singular matrices and can be used for calculation of the eigenvectors. The algorithm (3) requires $n/2(n + 1)$ projections i.e.

$n^3 + 5n^2/2 + n/2$	operations of multiplications
$n^3 + 3n^2/2 - 3n/2$	operations of additions
n	operations of divisions

The first two steps of the algorithm (3) are only formal steps and number of operations which they require are from computation point of view negligible. The number of operations by parallel performance is equivalent to $O(n^2)$ arithmetical operations. In the paper [4] is shown, how the algorithm (3) can be modified by suitable way to an algorithm for minimization.

References

[1] Gastinel, N.: Lineare numerische Analysis, pp. 162—172. VEB Deutscher Verlag der Wissenschaften, Berlin (1972).

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