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mesurées respectivement par ν et μ, l'identité

$$\varphi(x) \,.\, \nu(x) = \psi(x) \,.\, \mu(x) \qquad (10)$$

veut signifier que à chaque instant x il y a une situation d'équilibre entre les deux forces φ et ψ: (10) est, en quelques sortes, l'équation de l'équilibre vital, pour un groupe donné en un temps déterminé.

Enfin, les quelques mots que nous avons dédiés aux forces biologiques de vitalité et de mortalité peuvent suffire, ce me semble, pour y construire là-dessus une théorie de la mortalité, sans recours à nulle autre hypothèse.

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A note about premiums of disability benefits in connection with life insurance contract.

Karel Šťastný.

Latterly, the influence of a change of technical basis on all actuarial values, was the object of many treatises. In the branch of disability benefits insurance it is necessary to mention especially Karup's work „Neue Versicherungsformen der Gothaer Lebensversicherungsbank A. G.", the later one written by dr. Urech. „Sur les bases techniques de l'assurance collective" which will both be referred to below and dr. Haldy's very interesting paper „Influence des variations de l'invalidité sur les réserves mathématiques" (Bulletin de l'Association des Actuaires suisses, vol. 25, 26 and 27), which — in contradistinction to this note — deals with another theme (the influence on reserves) and another method (investigation of the influence of a constant-changing on the analytic expression of probability rates).

The matter of this note will be a supplement to the proof of formulas given by Karup and dr. Urech for disability benefits in life assurance.

Disability assurance is practised only in a few forms by private insurance companies in addition to the ordinary whole life and endowment assurance contracts. The most prevalent form is an exemption from further payment of premiums on a life policy in case of disability, if the insured is also the policy-holder. Other forms are assurance of a life annuity and assurance of a sum, both payable in case of disability. All these three forms of assurances can be considered as an assurance of a life annuity payable in case of disability; in the first case (waiver of premiums) an annuity equal to the yearly premium on the main assurance and in the third one equal to the rent of the assured sum as from the time of disablement until the end of the main contract. (But it is necessary to calculate each of these three forms of assurances by application of three different scales of disability rates, as was proved by Mattson, Riedel and others.[1]) It will therefore be sufficient to limit our problem only to annuity assurance in case of disability.

In the additional Disability Assurance, the problem of determination of the just premiums for persons of different occupations with different risk of disablement, is of fundamental importance.

By solving this problem we suppose that we know the probabilities of disability for a certain category of occupation and, on the other hand, that the mortality of disabled persons is the same, regardless of the category in which those persons were before being disabled. We can assume that in the branch of additional assurance these two conditions are approximately fulfilled. There are groups of persons menaced in different way by risk of disability in accordance with their profession and work in Social Assurance too; but for fixing the premiums this fact is irrelevant (except in Accident Insurance, which is more like responsibility insurance); In Social Assurance changes of disability rates for all persons insured by a certain institute are investigated as for a whole. The opinion has been advanced that the rise in disability rates — whether it is the result of a more moderate judgement of disability or of granting annuities to unemployed persons (although not invalid) — leads to lower mortality rates among invalids and higher rates of withdrawal of invalids due to the above - mentioned moderate judgement of disability. The same assumption may be expressed also in the additional disability assurance; Smolensky, for exemple, applies this supposition in his treatise[2]) submited to the Eighth Int. Congress of Act. held in London, to the calculation of the probability of disability, but it would be an error to lose sight of the circumstance that higher disability rates are the consequence of more dangerous occupations; and there is no reason to suppose that the mortality of persons disabled in a less dangerous occupation is lower than the mortality of invalids from a more dangerous occupation.

It is a matter for regret that there is a lack of statistical data for individual categories of occupations, from which it would be possible to derive disability rates depending on age. To this circumstance was due the practice of accepting certain professions as a basis and for this basis a suitable scale of disability rates was applied. The probabilities of becoming disabled were considered for more dangerous occupations as a fixed multiple of those disability rates.

Karup, dealing with Additional Disability Assurance in ,,Neue Ver-

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[1]) Mattson: ,,Disability Benefits in Life Assurance Contracts" (Transactions of the Eight International Congress of Actuaries held in London 1927). — Riedel: ,,Der technische Aufbau der Invaliditätszusatzversicherung in Verbindung mit der Lebensversicherung" (Translation from Transaction of the Second Italian Congress of Actuaries, Trieste 1932).

[2]) Disability Benefic in Life Assurance Contract.

sicherungsformen der Gothaer Lebensversicherungsbank" wanted to find out the influence of raised disability probabilities on the yearly premiums payable for this assurance; he raised the disability probabilities, for all ages, by multiplying them by the coefficient 1·3, and verified that the new premiums are higher in comparison with the original ones, as the following table shows.

Age at entry	Annuity of disability payble till the age of 55 years	of 60 years
25	1.326	1.316
30	1.329	1.320
35	1.332	1.325
40	1.335	1.330
45	1.338	1.336
50	—	1.341

(See Karup-Andrae „Neue Versicherungsformen der Gothaer Lebensversicherungsbank A. G." page 510).

This result (and certainly others too) caused Karup to state that probabilities of disability, made higher by a certain percentage, lead to premiums (required for the insurance of annuity for invalids) raised approximately by the same percentage. Dr. Urech, in his article „Sur les bases techniques de l'assurance collective" verified also a penetrative influence of increased disability probabilities on premiums for the additional assurance of annuity for invalids. He did that in a way analogous to Karup's; that is to say by exchanging probabilities of disability and keeping other basis unalterable. (However he did not consider disability probabilities of one table as a multiple of probabilities of another table, but he used tables derived from statistical observations.) In this note evidence will be produced of the statement made by Karup and Dr. Urech in the above - mentioned empiric way.

Let us suppose that an annuity of α is insured payable in case of invalidity; its value at the origin of disability is $\alpha \bar{a}^{i}_{[n+t],\, \overline{n-t|}}$. The premium for this insurance payable while not disabled, for n years as maximum is

$$P = \frac{\alpha \int\limits_0^n D^{aa}_{x+t}\, \nu_{x+t}\, \bar{a}^{i}_{[x+t]\, \overline{n-t|}}\, dt}{\int\limits_0^n D^{aa}_{x+t}\, dt}.$$

Let us suppose that $'\nu_{x+t} = (1 + k)\, \nu_{x+t}$ for all t and intensity of disbility raised in this way corresponds to a premium $'P$. Whereas we consider that mortality rates of disabled and those of able-bodied persons remain unchanged. Therefore it wil be

$$P_{\prime}\int\limits_0^{n} e^{-\int\limits_0^t [\mu^a_{x+u} + (1+k)\nu_{x+u}]\mathrm{d}u}\, v^t\,\mathrm{d}t =$$

$$= \alpha \int\limits_0^{n} e^{-\int\limits_0^t [\mu^a_{x+u} + (1+k)\nu_{x+u}]\mathrm{d}u}\,(1+k)\,\nu_{x+t}\,\bar{a}^i_{[x+t]\,\overline{n-t|}}\,v^t\,\mathrm{d}t$$

in another form

$$'P\int\limits_0^{n} e^{-\int\limits_0^t (\mu^a_{x+u} + \nu_{x+u})\mathrm{d}u}\, v^t\, e^{-k\int\limits_0^t \nu_{x+u}\mathrm{d}u}\,\mathrm{d}t =$$

$$= \alpha(1+k)\int\limits_0^{n} e^{-\int\limits_0^t (\mu^a_{x+u} + \nu_{x+u})\mathrm{d}u}\, e^{-k\int\limits_0^t \nu_{x+u}\mathrm{d}u}\,\nu_{x+t}\,\bar{a}^i_{(x+t)\,\overline{n-t|}}\,v^t\,\mathrm{d}t.$$

Substituing in this formula D^{aa}_{x+t} in its usual meaning and denoting ${}_nD^{ai}_{x+t} = D^{aa}_{x+t}\,\nu_{x+t}\bar{a}^i_{[x+t]\,\overline{n-t|}}$ we can write

$$P = \alpha\,\frac{\int\limits_0^n {}_nD^{ai}_{x+t}\,\mathrm{d}t}{\int\limits_0^n D^{aa}_{x+t}\,\mathrm{d}t}, \quad 'P = \alpha(1+k)\,\frac{\int\limits_0^n {}_nD^{ai}_{x+t}\, e^{-k\int\limits_0^t \nu_{x+u}\mathrm{d}u}\,\mathrm{d}t}{\int\limits_0^n D^{aa}_{x+t}\, e^{-k\int\limits_0^t \nu_{x+u}\mathrm{d}u}\,\mathrm{d}t}.$$

We form the fraction $\frac{'P}{P}$ and transforming it in a suitable way we obtain

$$\frac{'P}{P} = (1+k)\,\frac{\int\limits_0^n {}_nD^{ai}_{x+t}\, e^{-k\int\limits_0^n \nu_{x+u}\mathrm{d}u}\,\mathrm{d}t}{\int\limits_0^n {}_nD^{ai}_{x+t}\,\mathrm{d}t} : \frac{\int\limits_0^n D^{aa}_{x+t}\, e^{-k\int\limits_0^n \nu_{x+u}\mathrm{d}u}\,\mathrm{d}t}{\int\limits_0^t D^{aa}_{x+t}\,\mathrm{d}t}.$$

In the dividend and also in the divisor there are weight averages of the quantity $e^{-k\int\limits_0^t \nu_{x+t}\mathrm{d}u}$, but only the weights are different. In the average in the dividend are the weights ${}_nD^{ai}_{x+t} = D^{aa}_{x+t}\,\nu_{x+t}\,\bar{a}^i_{[x+t]\,\overline{n-t|}}$ and the average in the divisor has the weights D^{aa}_{x+t}. Let us write in accordance with the mean value theorem (of integral calculation)

$$\int_0^n {}_nD^{ai}_{x+n}\, e^{-k\int_0^t v_{x+u}} \mathrm{d}t = e^{-k\int_0^{\tau_1} v_{x+u}\mathrm{d}u} \int_0^n {}_nD^{ai}_{x+t}\mathrm{d}t, \qquad 0 < \tau_1 < n.$$

$$\int_0^n D^{aa}_{x+t}\, e^{-k\int_0^t v_{x+u}\mathrm{d}u} \mathrm{d}t = e^{-k\int_0^{\tau_2} v_{x+u}\mathrm{d}u} \int_0^n D^{aa}_{x+t}\,\mathrm{d}t, \qquad 0 < \tau_2 < n.$$

After having abridged we can write

$$\frac{'P}{P} = (1 + k) \frac{e^{-k\int_0^{\tau_1} v_{x+u}\mathrm{d}u}}{e^{-k\int_0^{\tau_2} v_{x+u}\mathrm{d}u}} = (1 + k)\, e^{k\int_{\tau_1}^{\tau_2} v_{x+u}\mathrm{d}u}$$

v_{x+u} is small enough (Zimmermann's i_x for railway employees is for the age of 40 years 3,8% and rises to 9,8% only at the age of 65 years). k is usually smaller than 1. Then $k\int_{\tau_1}^{\tau_2} v_{x+u}\,\mathrm{d}u$ is also a small value, for τ_1 and τ_2 do not differ much from one another, according to the known experience of weight averages, so that $e^{k\int_{\tau_1}^{\tau_2} v_{x+u}\mathrm{d}u}$ is near to 1 and therefore $\frac{'P}{P} \doteq 1 + k$; that is to say, the rise in disability rates in this way that $v'_{x+t} = (1 + k)\, v_{x+t}$ results in the above defined heightening of premiums. We have just made use of our knowledge that a weight average and a simple average do not differ much. That was a basis for M. Jabob's and L. Riedel's transactions submited to the Tenth International Congress of Actuaries held in Rome and for the treatise in „Blätter für Versicherungsmathematik" written also by L. Riedel.

This experience could facilitate the calculation of premiums for Additional Disability Assurance having regard to occupation for it suffices to multiply the premiums determined for occupations of lowest disability probabilities by a factor expressing the relation between disability probabilities for perso ns in other more dangerous occupations and probabilities taken as a basis. Of course, this factor would be an average.

We could investigate the influence of the change in mortality of able-bodied persons on premiums in a way analogous to the one used above for the change in disability rates. We should obtain the fraction $\frac{'P}{P}$ in a similar form, but the coefficient $1 + k$ would not appear and the fraction would approach the unit. That is conformable with the experience that the change in mortality of able - bodied persons has almost no influence on premiums for Additional Disability Assurance.