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NINETY YEARS OF JAROSLAV KURZWEIL

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A prominent Czech scientist, Jaroslav Kurzweil, an excellent mathematician, reaches ninety years of age on May 7, 2016. His previous anniversaries were mentioned among other in the papers [1]–[5]; their electronic versions are now available online at DML-CZ: The Czech Digital Mathematics Library[1]. In particular, [5] contains a detailed survey of Kurzweil's results achieved before 2006 as well as his complete bibliography up to 2006.

Jaroslav Kurzweil significantly contributed e.g. to the metric theory of Diophantine approximations, geometry of Banach spaces, stability theory of differential equations, theory of differential inclusions, control theory, invariant manifolds flows or global solutions of functional differential equations. Particularly valuable is his impact on the theory of differential equations. The lecture notes [9] are still very actual and widely used for advanced courses of ordinary differential equations.

However, in mathematical world he is now famous primarily as the creator of a new approach to the integration and qualitative theory of differential equations. His Riemann-type definition of an integral was first published in 1957 in the Czechoslovak Mathematical Journal (see [11]). Its main idea is similar to the classical Riemann's approach: The integral of a function over an interval $[a, b]$ is approximated by the sum of the lengths of subintervals of a division of $[a, b]$ multiplied by the value of the function in particular points called the *tags*. The novelty is that the tags are chosen first, while the division points are allowed to vary in a controlled neighborhood of the tag. This made it possible to control the singularities and integrate very general classes of functions. This integral, now generally called the Kurzweil (or more often Henstock-Kurzweil) integral has proved to be very strong and inspiring, not only for the integration theory itself but also for differential and integral equations. It includes the classical concepts of the Riemann, Newton, Lebesgue and

[1] `http://project.dml.cz`

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Perron integrals as well as their improper modifications. In particular, it can integrate non-absolutely integrable functions. At the same time the Kurzweil integral calculus is surprisingly simple. Despite these facts and despite that Kurzweil published his first monograph [10] on the integration theory in 1980, it took about forty years before the Kurzweil integral made its way through. Its research and application became an important part of mathematical analysis. It is now worldwide used as a new approach to teach theory of integration. The answer to the query in scientific databases after work dealing with Kurzweil integrals includes these days more than 1500 items, including more than a dozen monographs, published mostly during the last few decades. After 2000 J. Kurzweil himself published three monographs [6]–[8] which had and apparently will long have remarkable impact in the international mathematical community.

Of course, the new approach to the general integration theory was growing up from the needs of ordinary differential equations. The main goal of the paper [11], where J. Kurzweil introduced his new integral, was to obtain new results on the continuous dependence on a parameter of solutions to systems of nonlinear differential equations. In particular, when rapidly oscillating external forces were present on the right hand sides, it turned out that, under reasonable conditions, the solutions of the approximating systems converge to a function that need not be absolutely continuous and thus it cannot be a solution of any ordinary differential equation (in the sense of Carathéodory which is based on the Lebesgue integral). However, using the new integral, it was possible to observe that this limit function satisfies a certain integral equation. Since then, this new kind of equations is called *generalized ordinary differential equations*. They were later studied by several authors, see e.g. the monographs [13] and [12]. The drawback of those results was that they were restricted only to solutions of bounded variation, and thus the fast oscillating right hand sides were not allowed.

Already in 2004, being encouraged by W. N. Everitt to develop a theory of generalized differential equations covering also the cases of rapidly oscillating right hand sides, J. Kurzweil started the preparation of a new monograph. A significant motivation came from the study of motion of Kapitza's pendulum, i.e., an inverted pendulum whose support oscillates in a vertical direction. The first version of the monograph devoted to generalized differential equations with non-absolutely continuous solutions was ready for print in 2009, and contained numerous new results. However, as Kurzweil was dissatisfied with this version, he essentially reorganized the manuscript and extended it with five new chapters. He completed the work in 2011, and the book [8] was published by World Scientific in 2012. It starts with three chapters devoted to the equation of Kapitza's pendulum and related problems. The next chapters of Kurzweil's book focus on generalized differential equations whose solutions are regulated but, in contrast to the well-known monograph [12] by Š. Schwabik, need not have bounded variation. The main results are new theorems on the existence and uniqueness of solutions, as well as continuous dependence on the right-hand side. Since the solutions can take values in arbitrary Banach spaces, it was necessary to introduce and develop a new concept of integral for vector-valued functions, namely the strong Kurzweil-Henstock integral. Therefore, the book also represents an important contribution to integration theory. For example, it includes the proof of the integration by parts formula for strong Kurzweil-Henstock integrals, and discusses the integrability of products of functions. Moreover, it contains a new version of the Gronwall inequality for the Kurzweil-Stieltjes integral; this result plays an essential role when dealing with generalized differential equations.

Sixty five years lasting devoted scientific research by Kurzweil led to numerous results in several branches of mathematics which have been recognized by the worldwide mathematical community for their richness and depth. His contributions are characterized by a rare combination of high creativity and exceptional technical power. Many generations of Czech and Slovak mathematicians were influenced by his work and often profited directly from his expert advice. When dealing with younger or less experienced colleagues he never showed his unquestionable superiority and was always ready to consider everybody's problem. And mostly he had a something to say to it even if the problem was outside his main field of interest.

One of the main handicaps of mathematicians to the east of the "iron curtain" was that personal contacts with colleagues from the West were made almost totally impossible. Jaroslav Kurzweil, together with another outstanding Czech mathematician (and also this year's nonagenarian) Ivo Babuška succeeded in founding and organizing a series of international scientific conferences bearing the name EQUADIFF. Since 1962 they have been held every four years alternately in Praha, Bratislava and Brno. Participation of many excellent mathematicians from all over the world helped to maintain contacts and collaboration with the world mathematical community.

Jaroslav is a man with deep devotion and love for mathematics but also one with broad interests in music, literature, history. He has not been indifferent to the world surrounding him and feels strongly any dishonesty and unfairness both in public and personal affairs. It was also a sense of humor of his own that helped him to get over the absurdities of the period. Let us just recall the opening ceremony of the EQUADIFF 7 Conference in 1989 (still before the "velvet revolution" in Czechoslovakia) at which Kurzweil delivered an opening address. He started quite innocently: "Today we celebrate an extraordinary anniversary." Nevertheless, the audience (at least the Czechs and Slovaks, but many foreigners as well) held their breath: it was

August 21, the day of Soviet invasion to the country in 1968. After a well-timed pause, Kurzweil went on: "Exactly two hundred years passed since the birth of one of the greatest mathematicians of all times, Augustin Cauchy..."

Among other awards that Jaroslav Kurzweil received (and already mentioned in [5]), let us point out the state decoration of the Czech Republic "Medal of Merit (First Grade)" for meritorious service to the state awarded in 1997. Furthermore, in 2006 he was awarded the National Prize of the Government of the Czech Republic "Czech Brain", also nicknamed, with a touch of humor but with serious respect, "the Czech Nobel Prize". During the award ceremony he said: "I want to say that mathematics is beautiful and has beautiful characteristics. Above all, it does not allow any shifting of the meaning of symbols and words. Second, in mathematics, what was true yesterday is true today. Third, when someone says something, it does not matter who said it, but what he or she said. May we all get these principles into our blood." The audience applauded him sincerely (including several top politicians). Thanks to this success he was even asked to deliver a New Year's toast for 2007 broadcasted by the Czech Television.

As father and grandfather, J. Kurzweil has had ample opportunities to watch how children accept, perceive and absorb mathematical knowledge and liked to discuss it with friends. He disliked hasty reforms and formalism and although the possibilities were rare, he tried to point out the worst abberations to the authorities. When he was appointed Head of Department of Didactics of Mathematics, he did not take it as a formal task or even a nuisance but did his best to convince the authorities (as well as many "pure" mathematicians) of the importance of the field, and on the other hand to warn those who saw it as an easy way to promotion.

Mathematica Bohemica is proud to mention that Jaroslav Kurzweil was its chief editor in the period 1956–1970 (that time the name of the journal was Časopis pro pěstování matematiky) and continued to be a member of its editorial board till 2007.

We join the world mathematical community in expressing Jaroslav our respect, admiration and friendship. We wish him firm health, happy and joyful family life with his wife Stefania, children and grandchildren, as well as pleasant times spent over new mathematical results, be it of himself or of his younger colleagues, friends and students.

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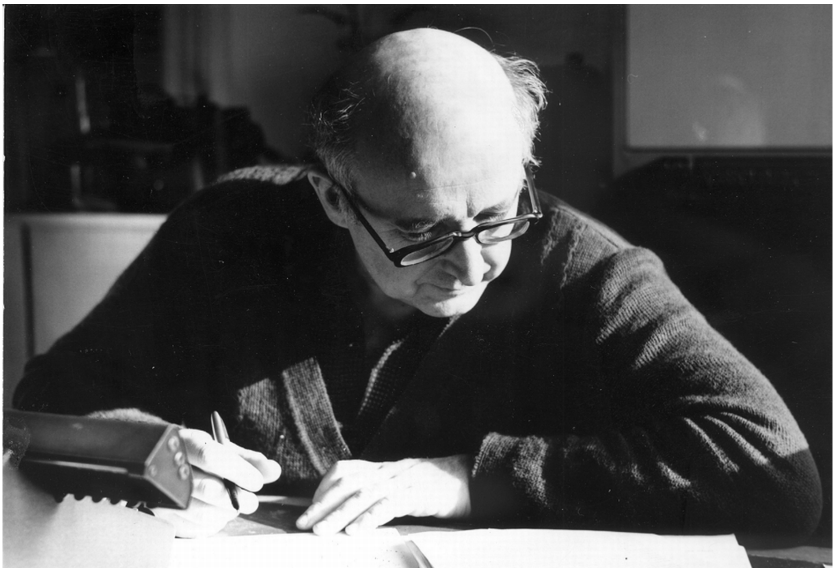
Photos [a]–[c] and [q] were taken by Štefan Schwabik.



[a]

[b]

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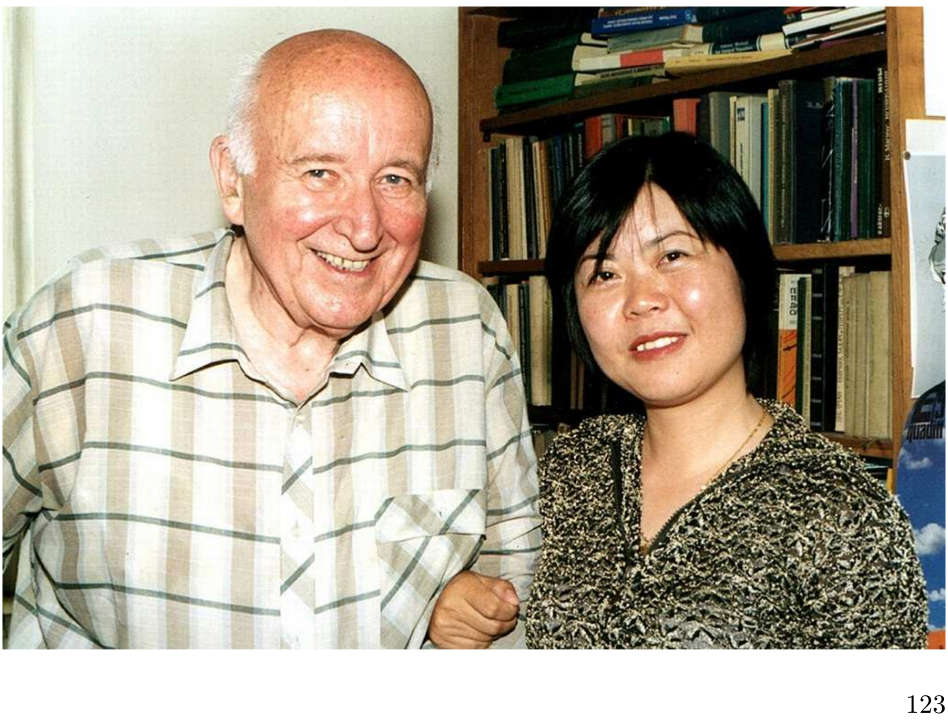
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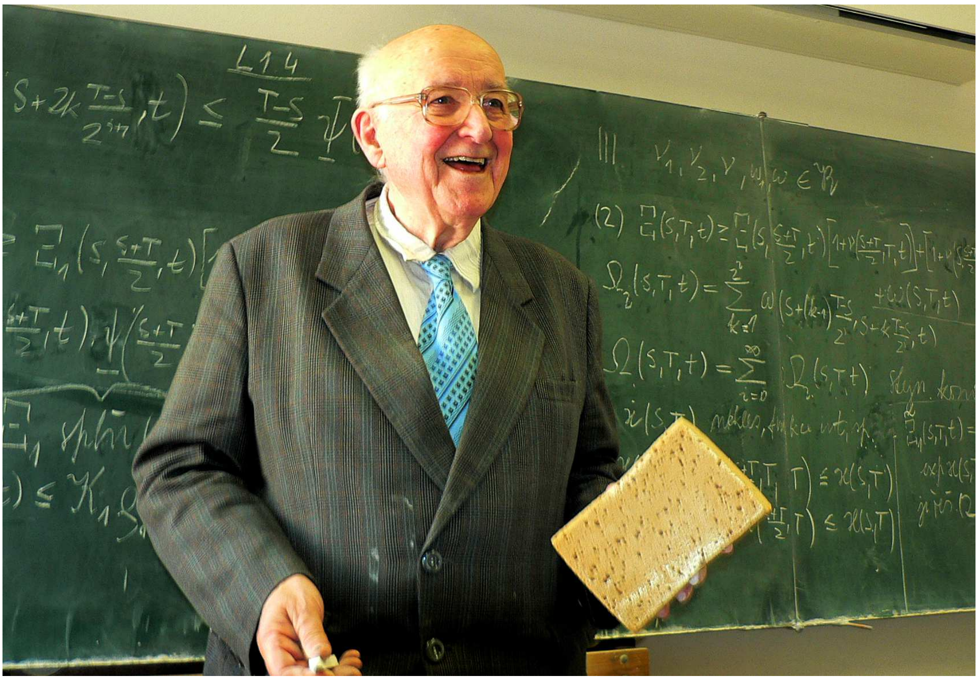


[h]


Δ se nazývá systém.
Je-li $K \in Iv$, $\bigcup_\Delta T = K$, pak Δ je dělení K.
$\delta : I \to (0, \infty)$, Δ je δ-jemné, když
$T \subset [t - \delta(t), T + \delta(t)]$ pro $(t, T) \in \Delta$.
$\operatorname{Del} \mathscr{L} = \{\Delta\}$,
$\operatorname{Del} \mathscr{HK} = \{\Delta \in \operatorname{Del} \mathscr{L};\ t \in T \in Iv$ pro $(t, T) \in$
$\operatorname{Del} \mathscr{HK} \subset \operatorname{Del} \mathscr{X} \subset \operatorname{Del} \mathscr{L}$
10

[i]


JSOU ÚPLNÉ
NEJSOU ÚPLNÉ

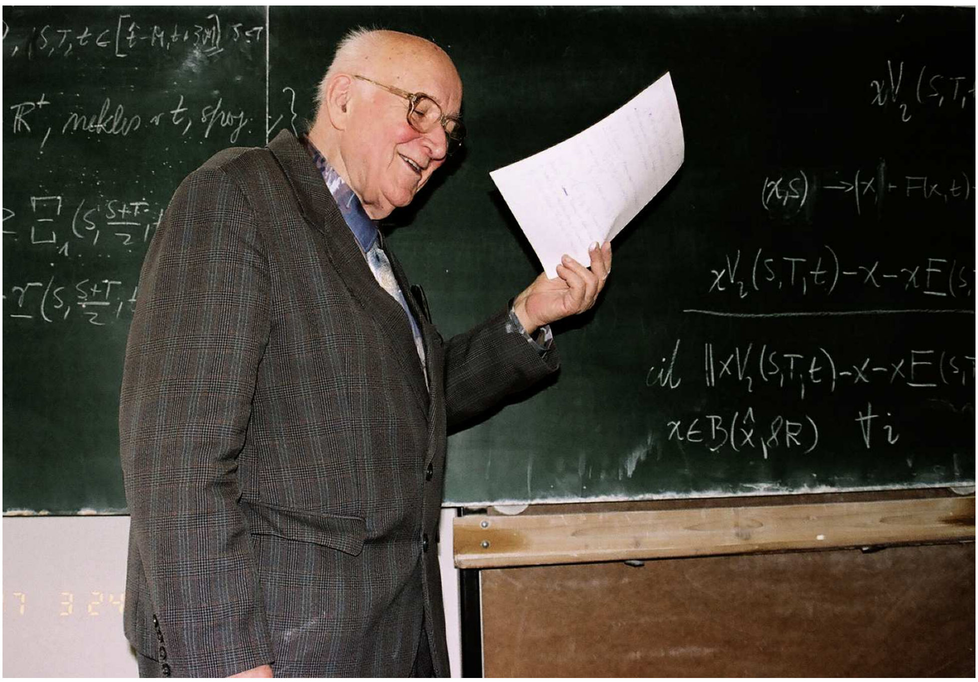
[j]


(20) applies to (23) and (24)
are close to solutions of
Back to classical
solutions of (22) are close
(25)

[k]

[l]

[m]



[n]

[o]

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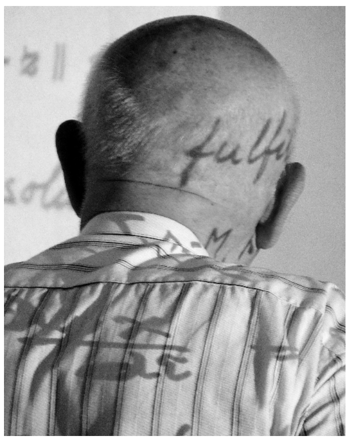

[r]