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SLIDING-MODE PINNING CONTROL OF COMPLEX NETWORKS

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In this paper, a novel approach for controlling complex networks is proposed; it applies sliding-mode pinning control for a complex network to achieve trajectory tracking. This control strategy does not require the network to have the same coupling strength on all edges; and for pinned nodes, the ones with the highest degree are selected. The illustrative example is composed of a network of 50 nodes; each node dynamics is a Chen chaotic attractor. Two cases are presented. For the first case the whole network tracks a reference for each one of the states; afterwards, the second case uses the backstepping technique to track a desired trajectory for only one state. Tracking performance and dynamical behavior of the controlled network are illustrated via simulations.

Keywords: complex network, pinning control, sliding mode, backstepping, trajectory tracking

Classification: 05C82, 93D05, 93C10

1. INTRODUCTION

The study of various complex dynamical networks is currently pervading all kinds of sciences, such as physics, biology, and social sciences; its impact is significant for science and technology in different fields [3, 5, 30]. Interest in complex networks has been increasing since the time of Euler in the sixteenth-century to recent studies. Different topologies and graphic characteristics in complex networks are described by Erdös and Rényi (ER) random graph model [9], Watts and Strogatz (WS) small-world model [29] and Barabási and Albert (BA) scale-free model [1]. Concerning the special structures of complex networks, a simple and effective control strategy named pinning control is discussed in [10, 21], which was developed by applying local control to a small fraction of network nodes. Recent research results propose many quantitative measurements of complex networks, where three concepts play a key role: average path length, clustering coefficient and degree distribution [3]. For achieving the best selection of pinned nodes and a desired behavior, the present paper uses node degrees, and their distribution.

On the other hand, the sliding-mode control is a well-known discontinuous feedback control technique, which has been reviewed in several books and many journal arti-

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cles. Theoretical analysis are presented by Emelyanov [8], Utkin et al. [11], and Utkin [27]. Relative simplicity for design, control of independent motion (maintaining sliding conditions), invariance to process dynamics, and external disturbances robustness are main characteristics of sliding-mode control [11]. Recently, synchronization has become a subject for study with increasing attention. For most of chaos synchronization techniques, the master-slave or drive-response approach is used, where the basic idea is to design a controller to accomplish the slave system states track the master system ones asymptotically. Different control techniques have been developed for synchronization of chaotic systems, among them, sliding-mode control [20, 31, 32]. For finite-time synchronization of complex dynamical networks, theoretical analysis are reviewed in [14, 24, 34]. In [14], fixed-time synchronization of complex dynamical networks with non-identical nodes in the presence of bounded uncertainties and disturbances using sliding-mode control technique was developed. Furthermore, [24] applies nonsingular terminal sliding-mode control technique to realize the novel combination-combination synchronization between combination of two chaotic systems as drive system and combination of two chaotic systems as response system with unknown parameters in a finite time. Finally [34], introduces the idea of combination synchronization into complex networks; based on sliding-mode control principle, the finite-time combination synchronization of four uncertain complex networks is investigated. In this regard, the main disadvantage to the above works is that all network nodes require a controller.

The novelty of the present paper consists in developing a new controller based on slide mode combined with pinning control for trajectory tracking of complex networks. The proposed control strategy allows to achieve trajectory tracking even for chaotic systems, this control strategy does not require the network to have the same coupling strength on all edges and these strengths can vary randomly; its application is illustrated via simulations. Two cases are presented; the first case considers that all network nodes follows a reference system for each one of the respective states, and the second case uses backstepping technique to track a desired trajectory for only one state of each network node.

The paper is organized as follows: In section 2, mathematical preliminaries are provided. Section 3 presents the main contribution of this paper, where the control scheme is proposed. Simulation results are reported in Section 4, using a scale-free network of chaotic Chen oscillators with the chaotic Lorenz system as the reference for trajectory tracking. Finally, conclusions are drawn in Section 5.

Notations: $diag(\ldots)$ denotes a block-diagonal matrix. A^T and A^{-1} denote the transpose and the inverse of the matrix A, respectively. Write $A > 0$ ($A < 0$) if A is positive (negative) definite. $\|\cdot\|_1$ stands for the 1-norm and $\|\cdot\|_2$ for the Euclidean norm. $\lambda_{\min}(\cdot)$ ($\lambda_{\max}(\cdot)$) represents the minimum (maximum) eigenvalue of the corresponding matrix. $sign(\cdot)$ denotes the signum function that extracts the sign of a real number. $A \otimes B$ represents their Kronecker product.

2. MATHEMATICAL PRELIMINARIES

This section presents a brief review on sliding-mode control [23, 28], complex networks [5], and pinning methodology [33].

2.1. Sliding-mode control

Consider a multi-variable nonlinear system of the form

$$\dot{\mathbf{x}} = f(\mathbf{x}) + G(\mathbf{x})\mathbf{u} + \omega(\mathbf{x}), \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $\mathbf{u} \in \mathbb{R}^m$ is the input vector, $\omega(\mathbf{x}) \in \mathbb{R}^m$ characterizes the unknown disturbances, and $f : \mathbb{R}^n \to \mathbb{R}^n$ and $G : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ are smooth nonlinear functions of $\mathbf{x}$. The design of sliding-mode control consists basically of two stages. The first stage of design is the selection of the sliding manifold $\mathbf{s}$, obtained as the intersection of m smooth manifolds:

$$\mathbf{s} = \{x \in \mathbb{R}^n \quad | \quad s_i(x) = 0 \in \mathbb{R}, \quad i = 1, 2, \ldots, m\},$$

where this manifold represents a desired system dynamics, which is of lower order than the given system (whose dimension is $n-m$). The second stage is to find a discontinuous control law such that it drives the trajectories to the sliding manifold in finite time and then maintains them on this surface, such that the reaching modes satisfy the reachability condition

$$\mathbf{s}^T\dot{\mathbf{s}} < 0.$$

There are several methods for describing the dynamics of the sliding mode; one of them is named as the equivalent control approach. For this method, the conditions in the sliding mode are analyzed at $\mathbf{s} \equiv 0$ and $\dot{\mathbf{s}} = 0$. The equivalent control is obtained from

$$\dot{s}(\mathbf{x}) = \frac{\partial s}{\partial \mathbf{x}^T}\left(f(\mathbf{x}) + G(\mathbf{x})\mathbf{u_{eq}}(\mathbf{x}) + \omega(\mathbf{x})\right) = 0,$$

assuming that $\frac{\partial s}{\partial \mathbf{x}^T} G(\mathbf{x}) \neq 0$, $\forall \mathbf{x}$, as

$$\mathbf{u}_{eq}(\mathbf{x}) = -\left[\frac{\partial s}{\partial \mathbf{x}^T}G(\mathbf{x})\right]^{-1}\left(\frac{\partial s}{\partial \mathbf{x}^T}f(\mathbf{x}) + \frac{\partial s}{\partial \mathbf{x}^T}\omega(\mathbf{x})\right). \tag{2}$$

The resulting dynamics on the sliding mode is then written as

$$\dot{\mathbf{x}} = \left[I - G(\mathbf{x})\left[\frac{\partial s}{\partial \mathbf{x}^T}G(\mathbf{x})\right]^{-1}\frac{\partial s}{\partial \mathbf{x}^T}\right]f(\mathbf{x}) + \left[I - G(\mathbf{x})\left[\frac{\partial s}{\partial \mathbf{x}^T}G(\mathbf{x})\right]^{-1}\frac{\partial s}{\partial \mathbf{x}^T}\right]\omega(\mathbf{x}). \tag{3}$$

According to (3), the ideal sliding dynamics are invariant with respect to the disturbances if, and only if, the vector $\omega(\mathbf{x})$ belongs to the span of $G(\mathbf{x})$ (matched condition), i. e.,

$$\left[I - G(\mathbf{x})\left[\frac{\partial s}{\partial \mathbf{x}^T}G(\mathbf{x})\right]^{-1}\frac{\partial s}{\partial \mathbf{x}^T}\right]\omega(\mathbf{x}) = 0.$$

Then, a non-zero vector function $\vartheta(\mathbf{x})\mathbb{R}^m$ exists, such that

$$\omega(\mathbf{x}) = G(\mathbf{x})\vartheta(\mathbf{x}).$$

A classical control strategy, which achieves the control objective, is to use

$$\mathbf{u} = \bar{\mathbf{u}}_{eq} - K_c sign\left(\mathbf{s}\right), \tag{4}$$

where $\bar{\mathbf{u}}_{eq}$ represents the equivalent control (2) (without including $\frac{\partial s}{\partial \mathbf{x}^T}\omega(\mathbf{x})$, since the disturbance is unknown), K_c is a control gain, and

$$sign\,(\mathbf{s}) = [sign(s_1), sign(s_2), \ldots, sign(s_m)]^T \in \mathbb{R}^m$$

is a vector input function whose ith component is

$$sign(s_i) = \begin{cases} 1, & s_i \geq 0 \\ -1, & s_i < 0. \end{cases}$$

The described sliding-mode methodology can ensure finite-time convergence, as established in the following theorem.

Theorem 2.1. (Bhat and Bernstein [2]) Suppose there exists a continuous function $V : \mathcal{D} \to \mathbb{R}$, such that the following conditions hold:

(i) V is positive definite.

(ii) There exist real numbers $c > 0$ and $\alpha \in (0, 1)$ and an open neighborhood $\nu \subseteq \mathcal{D}$ of the origin, such that

$$\dot{V}(x) + c(V(x))^\alpha \leq 0, x \in \nu \backslash \{0\}. \tag{5}$$

Then, the origin is a finite-time-stable equilibrium of $\dot{y}(t) = f(y(t))$. Moreover, if $\mathcal{N}$ is an open neighborhood $\mathcal{N} \subseteq \mathcal{D}$ and $\mathbf{T}$ is the settling-time function, then

$$\mathbf{T}(x) \leq \frac{1}{c(1-\alpha)} V(x)^{1-\alpha}, x \in \mathcal{N}, \tag{6}$$

and $\mathbf{T}$ is continuous on $\mathcal{N}$. If in addition $\mathcal{D} = \mathbb{R}^n$, V is proper, and $\dot{V}$ takes negative values on $\mathbb{R}^n \backslash \{0\}$, then the origin is a globally finite-time-stable equilibrium of $\dot{y}(t) = f(y(t))$.

A detailed proof of this theorem is discussed in [2].

2.2. Complex networks

Consider a weighted network consisting of N linearly and diffusively coupled identical nodes, with each node being an n-dimensional dynamical system. The state equations of this network are given by

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + \sum_{j=1,\, j\neq i}^{N} c_{ij} a_{ij} \mathbf{T}(\mathbf{x}_j - \mathbf{x}_i), \qquad i = 1, 2, \ldots, N, \tag{7}$$

where $\mathbf{x}_i = [\mathbf{x}_{i_1}, \mathbf{x}_{i_2}, \ldots, \mathbf{x}_{i_n}]^T \in \mathbb{R}^n$ are the state vectors of node i, $c_{ij} > 0$ are constants representing the coupling strengths between node i and node j, and $\mathbf{T} = diag\{\tau_1, \tau_2, \ldots, \tau_n\}$ is the inner coupling matrix describing the connections among different components of a state vector.

In (7), the coupling matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ represents the coupling configuration of the network. If there is a connection between node i and node j for $i \neq j$, then $a_{ij} = a_{ji} = 1$; otherwise, $a_{ij} = a_{ji} = 0$ for $i \neq j$. If the degree k_i of node i is defined to be the number of its outreaching connections, then

$$\sum_{j=1, j\neq i}^{N} a_{ij} = \sum_{j=1, j\neq i}^{N} a_{ji} = k_i, \qquad i = 1, 2, \ldots, N.$$

Let the diagonal elements of A be $a_{ii} = -k_i$, $i = 1, 2, \ldots, N$.

2.2.1. Pining control strategy

For network (7), the objective of control is to stabilize it onto a homogeneous stationary state $\bar{x}$, which satisfies $f(\bar{x}) = 0$:

$$x_1 = x_2 = \cdots = x_N = \bar{x}.$$

This objective is achieved by applying local linear feedback injections to a small fraction of the nodes in the network, which are called pinned, for notational simplicity, these nodes be labeled as $1, 2, \ldots, l$, where $1 \leq l \leq N$, and l can actually be as small as one. Thus, the controlled network can be written as

$$\begin{aligned} \dot{\mathbf{x}}_i &= f(\mathbf{x}_i) + \sum_{j=1}^{N} c_{ij} a_{ij} \mathbf{T} \mathbf{x}_j + \mathbf{u}_i, && i = 1, 2, \ldots, l, \\ \dot{\mathbf{x}}_i &= f(\mathbf{x}_i) + \sum_{j=1}^{N} c_{ij} a_{ij} \mathbf{T} \mathbf{x}_j, && i = l+1, \ldots, N. \end{aligned} \tag{8}$$

Using the local linear negative feedback control law

$$\mathbf{u}_i = -c_{ii} d_i \mathbf{T}(\mathbf{x}_i - \bar{x}), \tag{9}$$

where $d_i > 0$ is the feedback control gain, $\mathbf{u}_i \in \mathbb{R}^n$ is the local linear single-state feedback control law, $i = 1, 2, \ldots, l$, and the coupling strengths c_{ii} satisfy

$$c_{ii} a_{ii} + \sum_{j=1, j\neq i}^{N} c_{ij} a_{ij} = 0.$$

Define the following matrices:

$$\begin{aligned} D &= diag\,(d_1, d_2, \ldots, d_l, 0, \cdots, 0) \in \mathbb{R}^{N\times N}, \\ D' &= diag\,(c_{11} d_1, c_{22} d_2, \ldots, c_{ll} d_l, 0, \cdots, 0) \in \mathbb{R}^{N\times N}. \end{aligned}$$

Moreover, using the Kronecker notation, one can write

$$\dot{\mathbf{X}} = I_N \otimes f(\mathbf{x}_i) - [(G + D) \otimes \mathbf{T}]\, \mathbf{X} + (D' \otimes \mathbf{T})\, \overline{\mathbf{X}}, \tag{10}$$

where $\mathbf{X} = [\mathbf{x}_1^T, \ldots, \mathbf{x}_N^T]^T$, $\overline{\mathbf{X}} = [\bar{x}^T, \ldots, \bar{x}^T]^T$, $G = (g_{ij}) \in \mathbb{R}^{N\times N}$ is a symmetric and semi-positive definite matrix, with $g_{ij} = -c_{ij}a_{ij}$, $i, j = 1, 2, \ldots, N$, and $G+D$ is positive definite [6] with the minimal eigenvalue

$$\lambda_{\min}(G + D) > 0.$$

Lemma 2.2. (Li, Wang, and Chen [18]) Assume that the node $\dot{x}_i = f(x_i)$ is chaotic for all $i = 1, 2, \ldots, N$, with the maximum positive Lyapunov exponent $h_{\max} > 0$. If $c_{ij} = c$, $d_i = cd$ and $\mathbf{T} = I_n$, then the controlled network (8) is locally asymptotically stable about the homogeneous state $\bar{x}$, provided that

$$c > \frac{h_{\max}}{\lambda_{\min}(-A + diag(d, \ldots, d, 0, \ldots, 0))}, \tag{11}$$

where $\lambda_{\min}$ is the minimal non-zero eigenvalue of the matrix.

3. SLIDING-MODE PINNING CONTROL

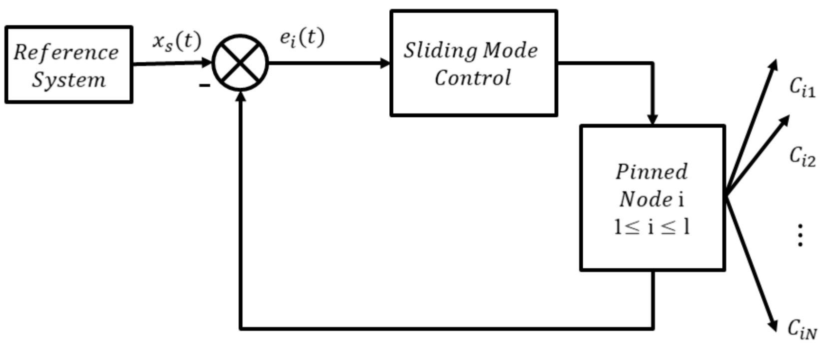

Fig. 1. The proposed control scheme.

This section develops the main contributions of the paper. Sliding-mode pinning control is proposed for trajectory tracking of complex networks. This scheme is visualized in Figure 1. To establish a control law, based on the sliding-mode technique consider a general complex network with pining control as in (8). Define $\mathbf{x}_s$ as the desired nonlinear reference system given by

$$\dot{\mathbf{x}}_s = f_s(\mathbf{x}_s), \quad \mathbf{x}_s \in \mathbb{R}^n. \tag{12}$$

For trajectory tracking, suppose that the pinned node dynamics are known. On the other hand, the tracking error is defined as $\mathbf{e}_{\mathbf{s_i}} = \mathbf{x}_i - \mathbf{x}_s$.

The following theorem is established to guarantee trajectory tracking using sliding-mode pinning control.

Theorem 3.1. Consider the general complex network with pinning control as in (8), with $\mathbf{T} = \mathbf{I}_n$, a reference system defined by (12) with $f_s(\mathbf{x}_s)$ being a chaotic system, $h_{\max}$ being the maximum positive Lyapunov exponent of $f_i(\mathbf{x}_i) - f_s(\mathbf{x}_s)$. If $c_{\min}$, the minimal coupling strength of the whole network, fulfills

$$c_{\min} > \frac{h_{\max}}{\lambda_{\min}(-A_0 + \hat{D}_0)}\,, \tag{13}$$

where A_0 is obtained from the coupling matrix $A = [a_{ij}] \in \mathbb{R}^{N\times N}$ by removing those rows and columns that correspond to the controlled nodes $i = 1, 2, \ldots, l$, and $\hat{D}_0 = diag(\hat{d}_{0_{l+1}}, \ldots, \hat{d}_{0_N})$ with $\hat{d}_{0_i} = \sum_{j=1}^{l} a_{ij}$, then the sliding-mode pinning control law given by

$$\mathbf{u}_i = \bar{\mathbf{u}}_{eq_i} - K_{s_i}\, sign\left(\boldsymbol{\sigma}_i(\mathbf{x}_i)\right), \quad i = 1, 2, \ldots, l, \tag{14}$$

with $\bar{\mathbf{u}}_{eq_i} = f_s(\mathbf{x}_s) - f(\mathbf{x}_i)$, $K_{s_i} \in \mathbb{R}$ is a control gain, and $\boldsymbol{\sigma}_i = \mathbf{e}_{\mathbf{s_i}} = 0 \in \mathbb{R}^n$ the desired sliding manifold, ensures the tracking error, along the trajectories of (12), is locally asymptotically stable.

Proof. Substitute the control law (14) into (8) to get

$$\begin{aligned}\dot{\boldsymbol{\sigma}}_i =& \sum_{j=1}^{N} c_{ij} a_{ij} \mathbf{T}\mathbf{x}_j - K_{s_i}\, sign\left(\boldsymbol{\sigma}_i(\mathbf{x}_i)\right), \quad i = 1, 2, \ldots, l,\\ \triangleq& f(\mathbf{x}) + G(\mathbf{x})\mathbf{u} + \omega(\mathbf{x}),\end{aligned}$$

where $f(\mathbf{x}) = 0$, $G(\mathbf{x}) = I_n$, and $\omega(\mathbf{x}) = \sum_{j=1}^{N} c_{ij} a_{ij} \mathbf{T}\mathbf{x}_j$ are the influence of other network nodes, which are considered as a disturbance.

Assume now that the following bound is satisfied:

$$\left\| \sum_{j=1}^{N} c_{ij} a_{ij} \mathbf{T}\mathbf{x}_j \right\|_1 \leq q_i, \quad q_i > 0, \quad i = 1, 2, \ldots, l. \tag{15}$$

Select the controller (14) with K_{s_i} chosen as

$$K_{s_i} \geq q_i + \frac{\Upsilon_i}{\sqrt{n}}, \quad \Upsilon_i > 0, \quad i = 1, 2, \ldots, l.$$

Defining the Lyapunov function candidate as

$$V(\boldsymbol{\sigma}_i) = \frac{1}{2}\boldsymbol{\sigma}_i^T \boldsymbol{\sigma}_i \Rightarrow 2V = \boldsymbol{\sigma}_i^T \boldsymbol{\sigma}_i \Rightarrow \sqrt{2V} = \|\boldsymbol{\sigma}_i\|_2,$$

and taking the time derivative, one gets

$$
\begin{aligned}
\dot{V}(\boldsymbol{\sigma}_i) &= \boldsymbol{\sigma}_i^T\left(\sum_{j=1}^{N} c_{ij}a_{ij}\mathbf{T}\mathbf{x}_j - K_{s_i}\, sign\,(\boldsymbol{\sigma}_i)\right)\\
\dot{V}(\boldsymbol{\sigma}_i) &\leq \left\|\dot{V}(\boldsymbol{\sigma}_i)\right\|_1\\
&\leq \|\boldsymbol{\sigma}_i\|_1\left\|\sum_{j=1}^{N} c_{ij}a_{ij}\mathbf{T}\mathbf{x}_j\right\|_1 - K_{s_i}\|\boldsymbol{\sigma}_i\|_1\|\, sign\,(\boldsymbol{\sigma}_i)\,\|_1\\
&\leq \|\boldsymbol{\sigma}_i\|_1\left\|\sum_{j=1}^{N} c_{ij}a_{ij}\mathbf{T}\mathbf{x}_j\right\|_1 - K_{s_i}\,\|\boldsymbol{\sigma}_i\|_1\,.
\end{aligned}
$$

Using (15) and $\|\cdot\|_2 \leq \|\cdot\|_1 \leq \sqrt{n}\,\|\cdot\|_2$ [13, p. 648], one obtains

$$
\begin{aligned}
\dot{V}(\boldsymbol{\sigma}_i) &\leq -(K_{s_i} - q_i)\sqrt{n}\,\|\boldsymbol{\sigma}_i\|_2\\
&\leq -\Upsilon_i\,\|\boldsymbol{\sigma}_i\|_2\\
&\leq -\Upsilon_i\sqrt{2}\,(V(\boldsymbol{\sigma}_i))^{1/2} < 0.
\end{aligned} \tag{16}
$$

$V(\boldsymbol{\sigma}_i) > 0$ is positive definite, the real number $c = \Upsilon_i\sqrt{2} > 0$ for $K_{s_i} > q_i$, and $\alpha = \frac{1}{2}$ such that

$$
\dot{V}(\boldsymbol{\sigma}_i) + \Upsilon_i\sqrt{2}\,(V(\boldsymbol{\sigma}_i))^{1/2} \leq 0.
$$

is satisfied. According to the Theorem 2.1, the control law (14) guarantees the convergence of pinned nodes motion to the manifold $\boldsymbol{\sigma}_i = 0$ in a finite time defined by the settling time function

$$
\mathbf{T}(\sigma_i) \leq \frac{2}{\Upsilon_i\sqrt{2}}V\,(\sigma_i)^{1/2}\,.
$$

In order to analyze the sliding modes dynamics stability ($\boldsymbol{\sigma}_i = \mathbf{x}_i - \mathbf{x}_s \equiv 0$, $\dot{\boldsymbol{\sigma}}_i = 0$), the sliding mode equation is written as

$$
\begin{aligned}
&\mathbf{x}_i = \mathbf{x}_s, && i = 1, 2, \ldots, l,\\
&\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + \sum_{j=l+1}^{N} c_{ij}a_{ij}\mathbf{T}\mathbf{x}_j + \sum_{j=1}^{l} c_{ij}a_{ij}\mathbf{T}\mathbf{x}_s, && i = l+1, \ldots, N.
\end{aligned} \tag{17}
$$

Considering

$$
a_{ii} = -\sum_{j=l,j\neq i}^{N} a_{ij} = -\left(\sum_{j=l+1,j\neq i}^{N} a_{ij} + \sum_{j=1}^{l} a_{ij}\right), \tag{18}
$$

network (17) can be rewritten as

$$
\begin{aligned}
&\mathbf{x}_i = \mathbf{x}_s, && i = 1, 2, \ldots, l,\\
&\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + \sum_{j=l+1}^{N} c_{ij}a_{0_{ij}}\mathbf{T}\mathbf{x}_j - \sum_{j=1}^{l} c_{ij}a_{ij}\mathbf{T}(\mathbf{x}_i - \mathbf{x}_s), && i = l+1, \ldots, N,
\end{aligned} \tag{19}
$$

where $A_0 = [a_{0_{ij}}] \in \mathbb{R}^{(N-l)\times(N-l)}$ is obtained from the coupling matrix A by removing those rows and columns that correspond to the controlled nodes $i = 1, 2, \ldots, l$, i. e.,

$$a_{0_{ij}} = a_{ij}, \qquad\qquad j \neq i, \quad j = l+1, \ldots, l, \quad i = l+1, \ldots, l,$$
$$a_{0_{ij}} = -\sum_{j=l+1, j\neq i}^{N} a_{ij}, \quad i = l+1, \ldots, N,$$

The proof follows the same procedure as in [18, 25]. The controlled network (19) is linearized for the unpinned nodes $i = l+1, \ldots, N$, so that

$$\dot{\mathbf{e}}_{\mathbf{s}} = \mathbf{e}_{\mathbf{s}} \left[Df_{is} \left(\mathbf{x}_s \right) \right] - \hat{B} \mathbf{e}_{\mathbf{s}}, \tag{20}$$

where $Df_{is}(\mathbf{x}_s) \in \mathbb{R}^{n\times n}$ is the Jacobian of $(f_i - f_s)$ at $\mathbf{x}_s$,

$$\mathbf{e}_{\mathbf{s}} = \left[\mathbf{e}_{s_{l+1}}, \mathbf{e}_{s_{l+2}}, \ldots, \mathbf{e}_{s_N} \right]^T \in \mathbb{R}^{(N-l)n},$$

with $\mathbf{e}_{\mathbf{s}i}(t) = \mathbf{x}_i(t) - \mathbf{x}_s(t)$, $i = l+1, l+2, \ldots, N$, and $\hat{B} = (G_0 + \hat{D}) \in \mathbb{R}^{(N-l)\times(N-l)}$, where $G_0 = [-c_{ij} a_{0_{ij}}] \in \mathbb{R}^{(N-l)\times(N-l)}$, $\hat{D} = diag(\hat{d}_{l+1}, \ldots, \hat{d}_N)$ with $\hat{d}_i = \sum_{j=1}^{l} a_{ij} c_{ij}$. According to [22], one has

$$0 \quad < \quad \sigma_{\min} \left(c_{\min} \left[-A_0 + \hat{D}_0 \right] \right) \leq \sigma_{\min} \left(G_0 + \hat{D} \right),$$

where $\hat{D}_0 = diag(\hat{d}_{0_{l+1}}, \ldots, \hat{d}_{0_N})$ with $\hat{d}_{0_i} = \sum_{j=1}^{l} a_{ij}$. The above inequality is obtained based on the fact that $c_{ij} \geq c_{\min} > 0$ for all c_{ij} in G_0. Then, the coupling strengths can be different for different nodes.

Furthermore, the Transversal Lyapunov Exponents (TLEs) denoted by $\mu_k(\lambda_i)$, for each eigenvalue λ_i, $i = l+1, l+2, \ldots, N$, is given by [17]

$$\mu_k(\lambda_i) = h_k - c_{ij}\lambda_i, \quad k = 1, 2, \ldots, n,$$

where h_k is the respective Lyapunov exponent. The TLEs determine the stability of the controlled states [25], hence the local stability of the controlled network (8), ensures negative TLEs. Thus, the following condition must be satisfied:

$$\mu_{\max}(\lambda_{\min}) = h_{\max} - c_{\min}\lambda_{\min}(-A_0 + \hat{D}_0) < 0, \tag{21}$$

(21) is equivalent to condition (13). Then, with the proposed control law (14), trajectory tracking is achieved. □

On the other hand, consider to control the network (8), to track a reference output $\mathbf{x}_s \in \mathbb{R}^m$ $(m < n)$ with pinning control $\mathbf{u}_i \in \mathbb{R}^m$. Assume that the pinned node dynamics are known. The tracking error is defined as $\mathbf{e}_{\mathbf{s_i}} = \mathbf{x_i} - \mathbf{x}_s$. In this case, the backstepping technique is used to design a sliding manifold such that the resulting sliding mode dynamics is described by a desired linear system. Next, one can synthesize a discontinuous control law which enforce sliding-mode motion into the sliding manifold designed by using backstepping control. For more details about backstepping control see [15].

This approach is feasible if the nonlinear system can be transformed into a special state-space form named as block feedback form given by the following definition

Definition 3.2. (Block-feedback form BFF, Krstic et al. [15]) The system

$$\begin{aligned}
\dot{\mathbf{x}}_j &= f_j(\bar{\mathbf{x}}_j) + g_j(\bar{\mathbf{x}}_j)x_{j+1} + \omega_j(t), \qquad j = 1, 2, \ldots, r-1, \\
\dot{\mathbf{x}}_r &= f_r(\mathbf{x}) + g_r(\mathbf{x})u + \omega_r(t), \\
\dot{\mathbf{x}}_{r+1} &= f_{r+1}(\mathbf{x}) + g_{r+1}(\mathbf{x})u + \omega_{r+1}(t), \\
y &= \mathbf{x}_1,
\end{aligned} \tag{22}$$

where $\mathbf{x} = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_{r+1}]^T$, $\mathbf{x}_{r+1}$ represents the zero dynamics of the system, $\bar{\mathbf{x}}_j = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_j]^T$, $j = 1, 2, \ldots, r$, $\mathbf{x}_j \in \mathbb{R}^{nj}$, r is the number of blocks, $\omega(t) \in \mathbb{R}^{nj}$ is the bounded unknown disturbance vector, then there exists a constant $\bar{\omega}_j$ such that $\|\omega_j(t)\| \leq \bar{\omega}_j$, for $0 < t < \infty$, system (22) has the block-feedback form with Zero Dynamics, if

$$rank\ [g_j] = n_j \quad \forall \mathbf{x} \in \mathbb{R}^n \wedge t \in [0, \infty), \quad j = 1, 2, \ldots, r.$$

The numbers $n_1, n_2, \ldots, n_r$ are the controllability indexes and satisfies

$$n_1 \leq n_2 \leq \ldots \leq n_r \leq m,$$

with $\sum_{j=1}^{r+1} n_j = n.$

It is easy to see that the relative degree of system (22) is r.

The following Lemma is established to guarantee output tracking for the whole network, using sliding-mode pinning control.

Lemma 3.3. Assume that the complex network (8) with pinning control law defined as

$$\mathbf{u}_i = \bar{\mathbf{u}}_{eq_i} - K_{r_i} sign\left(z_{r_i}(\mathbf{x}_i)\right) \quad \in \mathbb{R}^m, \quad i = 1, 2, \ldots, l, \tag{23}$$

where $\bar{\mathbf{u}}_{eq_i}$ is the equivalent control, K_{r_i} a control gain, and $z_{r_i} \in \mathbb{R}^m$ the desired sliding manifold, can be transformed to BFF, its zero dynamics is stable, and condition (13) is fulfilled, then the tracking error, along the trajectories of $\mathbf{x}_s$, is locally ultimately bounded.

P r o o f. Consider system (8) transformed in BFF form

$$\begin{aligned}
\dot{\mathbf{x}}_{i_j} &= f_{ij}(\overline{\mathbf{x}}_{i_j}) + g_{ij}(\overline{\mathbf{x}}_{i_j})\mathbf{x}_{j+1_i} + \omega_{ij}(t), && j = 1, 2, \ldots, r-1 \wedge i = 1, 2, \ldots, l, \\
\dot{\mathbf{x}}_{i_r} &= f_{ir}(\mathbf{x}_i) + g_{ir}(\mathbf{x}_i)u + \omega_{ir}(t), && i = 1, 2, \ldots, l, \\
\dot{\mathbf{x}}_{i_{r+1}} &= f_{ir+1}(\mathbf{x}_i) + g_{ir+1}(\mathbf{x}_i)u + \omega_{ir+1}(t), && i = 1, 2, \ldots, l, \\
\dot{\mathbf{x}}_i &= f(\mathbf{x}_i) + \sum_{j=1}^{N} c_{ij}a_{ij}\boldsymbol{T}\mathbf{x}_j, && i = l+1, \ldots, N,
\end{aligned} \tag{24}$$

where $\omega_{ij}(t) = \sum_{j=1}^{N} c_{ij}a_{ij}\mathbf{T}\mathbf{x}_j$, and $\bar{\omega}_{ij} \geq |\omega_{ij}(t)|$, $\forall i = 1, 2, \ldots, N \wedge j = 1, 2, \ldots, n$, are considered disturbances. Initially, the sliding manifold is designed for each pinned node ($i = 1, 2, \ldots, l$) using the backstepping technique, which is described step-by-step as follows.

Step 1: Let z_1 be the error between $\mathbf{x}_1$ and its desired value $\mathbf{x}_s$:

$$z_1 = \mathbf{x}_1 - \mathbf{x}_s.$$

Define the Lyapunov function candidate as

$$V(z_1) = \frac{1}{2}\|z_1\|^2 > 0,$$

$$\dot{V}(z_1) = z_1^T \dot{z}_1 = z_1^T (f_1(\bar{\mathbf{x}}_1) + g_1(\bar{\mathbf{x}}_1)x_2 + \omega_1(t) - \dot{\mathbf{x}}_s). \tag{25}$$

The objective is to design a virtual control $\mathbf{x}_2 = \alpha_1$ which forces $z_1 \to 0$. This control is proposed as

$$\alpha_1 = g_1^{-1}(\bar{\mathbf{x}}_1)(-f_1(\bar{\mathbf{x}}_1) + \dot{\mathbf{x}}_r - k_1 z_1) \qquad \text{with} \qquad k_1 > 0.$$

Then, (25) becomes

$$\dot{V}(z_1) = -k_1\|z_1\|^2 + z_1^T\omega_1 + g_1(\bar{\mathbf{x}}_1)z_1^T z_2.$$

Step j: The error dynamics for z_j is derived

$$z_j = \mathbf{x}_j - \alpha_{j-1},$$

which represents the error between the actual and virtual controls. Select the augmented Lyapunov function candidate as

$$V(z_1,\ldots,z_j) = \frac{1}{2}\|z_1\|^2 + \cdots + \frac{1}{2}\|z_j\|^2 = V(z_1,\ldots,z_{j-1}) + \frac{1}{2}\|z_j\|^2,$$

and its derivative

$$\begin{aligned}\dot{V}(z_1,\ldots,z_j) &= z_1^T\dot{z}_1 + \cdots + z_j^T\dot{z}_j \qquad (26)\\ &= \dot{V}(z_1,\ldots,z_{j-1}) + z_j^T\left(f_j(\bar{\mathbf{x}}_j) + g_j(\bar{\mathbf{x}}_j)x_{j+1} + \omega_j(t) - \dot{\alpha}_{j-1}\right).\end{aligned}$$

Analogously, the objective at jth step is to design a virtual control $x_{j+1} = \alpha_j$ in order to stabilize the error $z_j = 0$, which is proposed as

$$\alpha_j = g_j^{-1}(\bar{\mathbf{x}}_j)\left(-f_j(\bar{\mathbf{x}}_j) - \dot{\alpha}_{j-1} - g_{j-1}(\bar{\mathbf{x}}_{j-1})z_{j-1} - k_j z_j\right), \text{ with } \quad k_j > 0. \tag{27}$$

Replacing (27) in (26) then

$$\dot{V}(z_1,\ldots,z_j) = -\sum_{k=1}^{j} k_k\|z_k\|^2 + \sum_{k=1}^{j} z_k^T\omega_k + g_j(\bar{\mathbf{x}}_j)z_j z_{j+1}.$$

Step r: The sliding manifold is selected as $z_r = \mathbf{x}_s - \alpha_{r-1}$. The dynamics for pinned nodes (24) in z-variables is

$$\begin{aligned}
\dot{z}_1 &= -k_1 z_1 + g_1(\bar{\mathbf{x}}_1) z_2 + \omega_1, \\
&\vdots \\
\dot{z}_j &= -g_{j-1}(\bar{\mathbf{x}}_{j-1}) z_{j-1} - k_j z_j + g_j(\bar{\mathbf{x}}_j) z_{j+1} + \omega_j, \qquad j = 2, 3, \ldots, r-1, \\
&\vdots \\
\dot{z}_{r-1} &= -g_{r-2}(\bar{\mathbf{x}}_{r-2}) z_{r-2} - k_{r-1} z_{r-1} + g_{r-1}(\bar{\mathbf{x}}_{r-1}) z_r + \omega_{r-1}, \\
\dot{z}_r &= f_r(\mathbf{x}) + g_r(\mathbf{x}) u + \omega_r(t) - \dot{\alpha}_{r-1}, \\
\dot{\mathbf{x}}_{r+1} &= f_{r+1}(\mathbf{x}) + g_{r+1}(\mathbf{x}) u + \omega_{r+1}(t).
\end{aligned}$$

Now, defining u as a discontinuous feedback control

$$\begin{aligned}
u &= g_r^{-1}(\mathbf{x}) \left(-f_r(\mathbf{x}) + \dot{\alpha}_{r-1} \right) - g_r^{-1}(\mathbf{x}) k_r sign(z_r) \\
u &= \bar{u}_{eq} - K_r \; sign \; (z_r(\mathbf{x}_i)),
\end{aligned}$$

where $\bar{u}_{eq} = g_r^{-1}(\mathbf{x}) \left(-f_r(\mathbf{x}) + \dot{\alpha}_{r-1} - g_{r-1}(\bar{\mathbf{x}}_{r-1}) z_{r-1} \right)$, and $K_r = g_r^{-1}(\mathbf{x}) k_r$, a candidate Lyapunov function is selected as $V(z_r) = \frac{1}{2} z_r^T z_r$, the derivative of $V(z_r)$ is computed as

$$\begin{aligned}
\dot{V}(z_r) &= z_r^T \dot{z}_r \\
&= z_r(-k_r \; sign \; (z_r) + \omega_r) \\
&\leq -k_r \|z_r\|_1 + \bar{\omega_r} \|z_r\|_1, \quad k_r \geq \frac{u_0}{\sqrt{n_r}} + \bar{\omega_r}, \\
&\leq -u_0 \|z_r\|_2 \leq -u_0 \sqrt{2} (V(z_r))^{\frac{1}{2}} < 0.
\end{aligned}$$

Then, control law (23) guarantees the convergence to the manifold $z_r = 0$ in a finite time.

To analyze the stability on sliding modes, the sliding-mode equation for the whole network is rewritten as

$$\begin{aligned}
\dot{\boldsymbol{\zeta}}_i &= A_i \boldsymbol{\zeta}_i + E_i(\boldsymbol{\zeta}_i) + \boldsymbol{\omega}_i, & & \\
\dot{\mathbf{x}}_{i_{r+1}} &= f_{ir+1}(\mathbf{x}_i) + g_{ir+1}(\mathbf{x}_i) \bar{u}_{eq_i}, & & i = 1, \ldots, l, \\
\dot{\mathbf{x}}_i = f(\mathbf{x}_i) &+ \sum_{j=l+1}^{N} c_{ij} a_{0_{ij}} \mathbf{T} \mathbf{x}_j - \sum_{j=1}^{l} c_{ij} a_{ij} \mathbf{T} (\mathbf{x}_i - \boldsymbol{\alpha}), & & i = l+1, \ldots, N,
\end{aligned} \tag{28}$$

where $\boldsymbol{\zeta}_i = [z_i^{(1)}, \ldots, z_i^{(r-1)}]^T$, $A_i = diag(-k_i^{(1)}, \ldots, -k_i^{(r-1)})$, $\boldsymbol{\omega}_i = [\omega_i^{(1)}, \ldots, \omega_i^{(r-1)}]^T$,

$$E_i(\boldsymbol{\zeta}_i) = [g_{i1}(\bar{\mathbf{x}}_{i_1}) z_i^{(2)}, -g_{i1}(\bar{\mathbf{x}}_{i_1}) z_i^{(1)} + g_{i2}(\bar{\mathbf{x}}_{i_2}) z_i^{(3)}, \ldots, -g_{ir-2}(\bar{\mathbf{x}}_{i_{r-2}}) z_i^{(r-2)}],$$

correspond to the crossed terms, $\mathbf{x}_{i_{r+1}}$ correspond to zero dynamics of pinned nodes, $\boldsymbol{\alpha} = [\mathbf{x}_s, \alpha_1, \ldots, \alpha_{r-1}]^T$.

$\boldsymbol{\zeta}_i$ can be consider as a linear stable one, perturbed by the last terms. Since, these terms are bounded, we can conclude that the trajectories of $\boldsymbol{\zeta}$ are ultimately bounded.

Moreover, it is assumed that the zero dynamics for pinned nodes is stable. For stability of unpinned nodes ($i = l+1, \dots, N$), first, it is defined $\boldsymbol{z}_i = \mathbf{x}_i - \boldsymbol{\alpha}$; the dynamics of these nodes are linearized on an outer neighborhood of the ultimate bound, so that

$$\dot{\eta} = \eta \left[D f_i \left(\boldsymbol{\alpha} \right) \right] - \hat{B} \eta, \tag{29}$$

where , $D f_i (\boldsymbol{\alpha}) \in \mathbb{R}^{n \times n}$ is the Jacobian of f_i at $\boldsymbol{\alpha}$,

$$\eta = \left[\eta_{l+1}, \eta_{l+2}, \dots, \eta_N \right]^T \in \mathbb{R}^{(N-l)n},$$

with $\eta_i(t) = \boldsymbol{z}_i(t)$, $i = l+1, l+2, \dots, N$. From this point, the same procedure of Theorem 3.1 can be applied in the linear region, which implies that the tracking error for whole network is ultimately bounded. $\square$

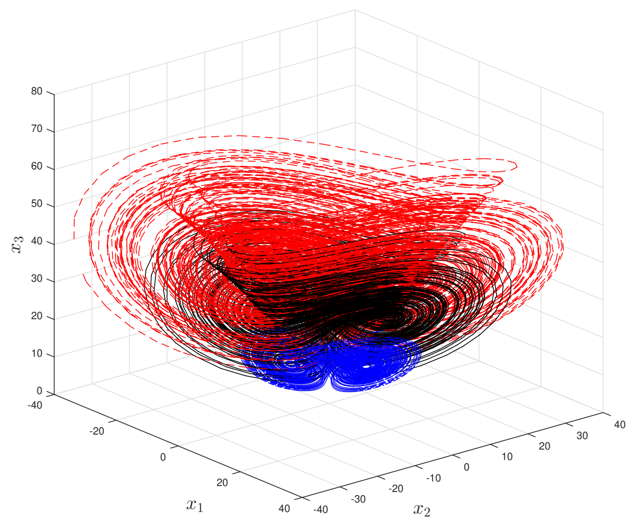

Fig. 2. Phase portrait of Chen's oscillator for different parameters: black line ($a_C = 35$, $b_C = 3$ and $c_C = 28$), blue line ($a_C = 30$, $b_C = 3$ and $c_C = 20$), and red line ($a_C = 50$, $b_C = 4$ and $c_C = 40$).

4. SIMULATION RESULTS

To illustrate tracking performance and dynamical behavior of the controlled network, two cases are included. For the first one, the whole network tracks a reference for each one of the states by means of the control inputs for pinned nodes ($u_i(t) \in \mathbb{R}^n, \quad i = 1, 2, \dots, l$); on the other hand, the second case uses the backstepping technique to track a desired trajectory for only one state; in this case the control is scalar for each pinned nodes ($u_i(t) \in \mathbb{R}, \quad i = 1, 2, \dots, l$).

4.1. Case 1

Consider a scale-free network of chaotic Chen's oscillators [4] with degree distribution $\delta(K_i) \approx {k_i}^{-2}$ and $N = 50$ nodes. The Chen's oscillator is described by

$$\begin{aligned} \dot{x} &= a_C(y - x) \\ \dot{y} &= (c_C - a_C)x - xz + c_C y \\ \dot{z} &= xy - b_C z, \end{aligned} \tag{30}$$

where $a_C = 35$, $b_C = 3$ and $c_C = 28$. Then, the maximum positive Lyapunov exponent is $h_{\max} \approx 2.018$, which renders a chaotic behavior. Figure 2 presents phase portrait of Chen's oscillator for different parameters: black line ($a_C = 35$, $b_C = 3$ and $c_C = 28$), blue line ($a_C = 30$, $b_C = 3$ and $c_C = 20$), and red line ($a_C = 50$, $b_C = 4$ and $c_C = 40$).

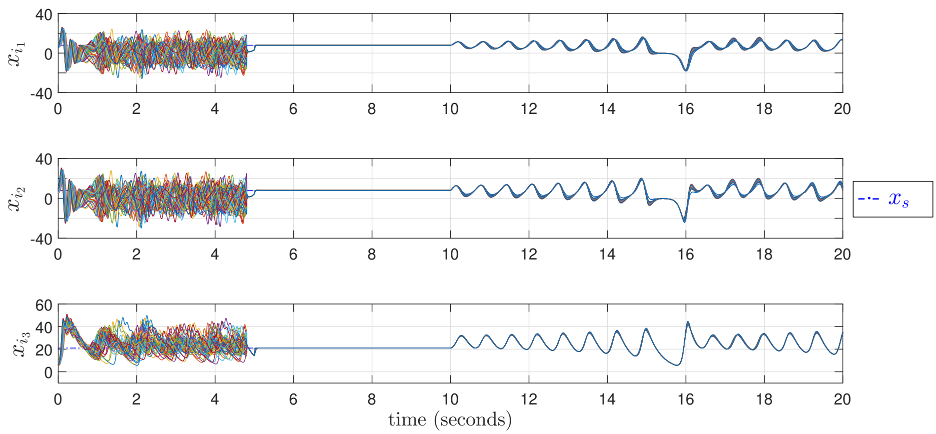

Fig. 3. Network states evolutions.

Suppose that $\mathbf{T} = diag(1, 1, 1)$. We want the network to track a desired reference system (12) applying the sliding-mode pinning control only to the node with the highest degree ($l = 1$). Defining the state variables as: $x_1 = x$, $x_2 = y$, $x_3 = z$, the equation of the pinned node $\mathbf{x}_1$ is

$$\begin{aligned} \dot{x}_{1_1} &= a_C(x_{1_2} - x_{1_1}) + \sum_{j=1}^{50} c_{1j} a_{1j} \mathbf{T} x_{j_1} + u_{1_1} \\ \dot{x}_{1_2} &= (c_C - a_C)x_{1_1} - x_{1_1}x_{1_3} + c_C x_{1_2} + \sum_{j=1}^{50} c_{1j} a_{1j} \mathbf{T} x_{j_2} + u_{1_2} \\ \dot{x}_{1_3} &= x_{1_1}x_{1_2} - b_C x_{1_3} + \sum_{j=1}^{50} c_{1j} a_{1j} \mathbf{T} x_{j_3} + u_{1_3}, \end{aligned} \tag{31}$$

where $\mathbf{u}_1 = f_s(\mathbf{x}_s) - f(\mathbf{x}_1) - K_{s_1}\, sign\,(\boldsymbol{\sigma}_1(\mathbf{x}_1))$, with $\boldsymbol{\sigma}_1(\mathbf{x}_1) = \mathbf{x}_1 - \mathbf{x}_s$.

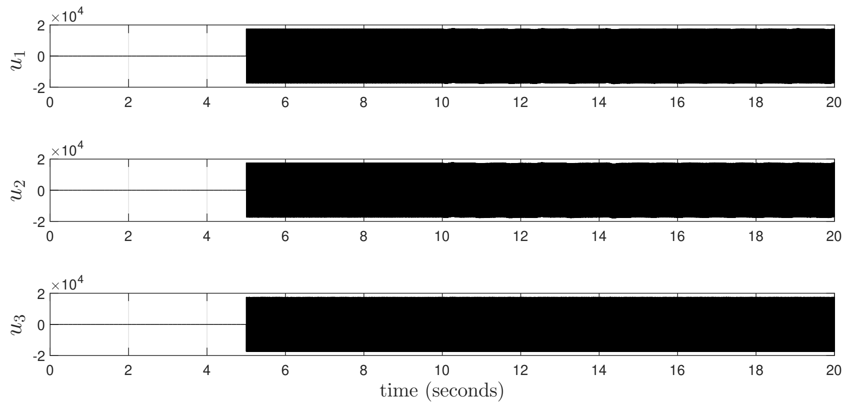

Fig. 4. Control input signals for the pinned node.

Simulations are done using MATLAB [1] Simulink with the Euler solver and a fixed step size of 0.5×10^{-4}. Simulations are performed as follows. From $t = 0s$ to $t = 4.8s$, the network runs without any connection ($c_{ij} = 0$). From $t = 4.8s$, the coupling strengths are a random set that it is time-variant, chosen with $c_{ij} > 20$. Then, at $t = 5s$, the proposed control law is applied; the network is stabilized at a constant reference, which is selected as the unstable equilibrium point, $\mathbf{x}_s = [7.9373, 7.9373, 21]^T$. Afterwards, at $t = 10s$, a reference selected as a the chaotic Lorenz attractor [19] is incepted to generate the desired trajectory $\mathbf{x}_s(t)$, which is defined as

$$
\begin{aligned}
\dot{x}_{s_1} &= a_L(x_{s_2} - x_{s_1}) \\
\dot{x}_{s_2} &= x_{s_1}(b_L - x_{s_3}) - x_{s_2} \\
\dot{x}_{s_3} &= x_{s_1}x_{s2} - c_L x_{s_3},
\end{aligned}
\tag{32}
$$

where $a_L = 10$, $b_L = 28$ and $c_L = 8/3$. From $t = 15s$, plant parameters changes are done for the network odd nodes as follows:

$$
a_C = \begin{cases} 30, & 15 < t \leq 16 \quad \text{and} \quad 19 < t \leq 20 \\ 35, & 0 < t \leq 15, \ 16 < t \leq 17 \quad \text{and} \quad 18 < t \leq 19 \\ 50, & 17 < t \leq 18 \end{cases}
$$

$$
b_C = \begin{cases} 3, & 0 < t \leq 17 \quad \text{and} \quad 18 < t \leq 20 \\ 4, & 17 < t \leq 18 \end{cases}
$$

$$
c_C = \begin{cases} 20, & 15 < t \leq 16 \quad \text{and} \quad 19 < t \leq 20 \\ 28, & 0 < t \leq 15, \ 16 < t \leq 17 \quad \text{and} \quad 18 < t \leq 19 \\ 40, & 17 < t \leq 18. \end{cases}
$$

[1]MATLAB & Simulink are registered trademarks of MathWorks Inc.,Natick, Massachusetts, U.S.A.

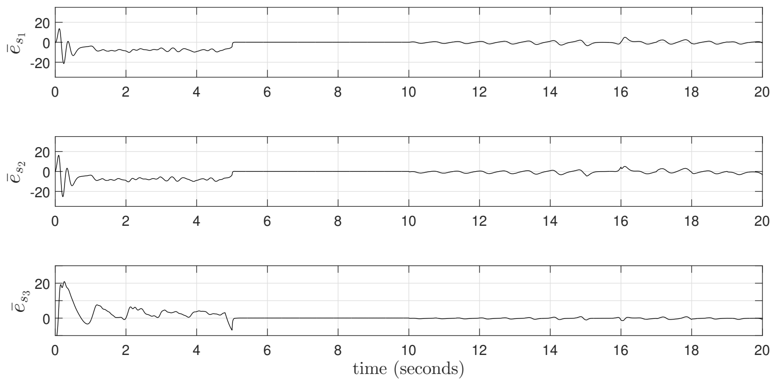

Fig. 5. Average trajectory error $\bar{e}_{s_i}(t)$, $i = 1, 2, 3$.

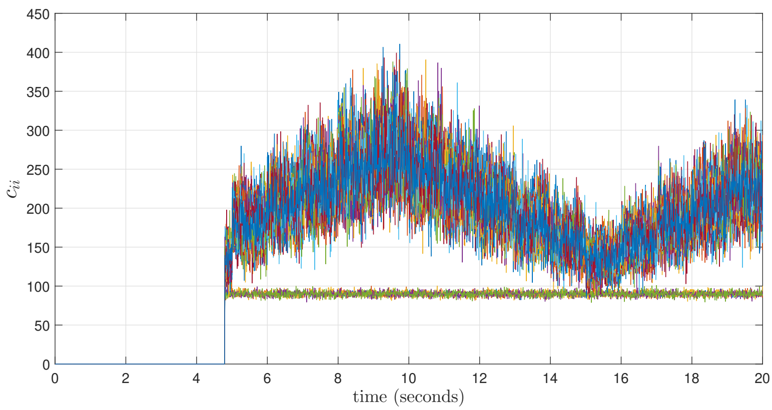

Fig. 6. Coupling strengths c_{ii}, $i = 1, 2, \ldots, 50$.

For all the described events c_{ij} always fulfill equation (13). Figure 3 displays the states evolutions of the entire network. Before $t = 5s$, trajectory evolves freely without control action; when the proposed control law is applied, the complex network tracks the desired trajectory. As can be seen, tracking is achieved for both a constant reference and a chaotic one even in presence of plant parameters changes, with 0.75% as the mean square error (MSE), illustrating robustness of the proposed controller. Figure 4 control input signal $u_i(t)$ applied to the pinned node, displaying the typical chattering characteristic for discontinuous control actions based on sliding modes. To eliminate or at least reduce chattering, boundary layer, observer-based, regular form, and disturbance rejection techniques can be used as in [12, 16, 26]. Moreover, Figure 5 display simulation results for the average trajectory error. Figure 6 presents the values of the coupling strengths $c_{ii} = \frac{1}{k_i} \sum_{j=1, j \neq i}^{N} c_{ij} a_{ij}$. Finally, Figure 7 displays plant parameters changes

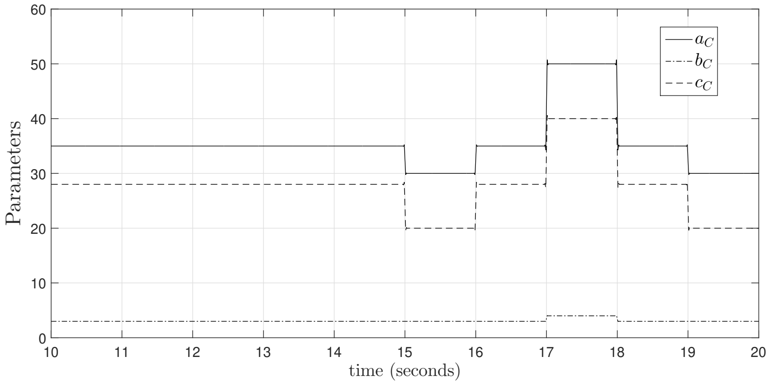

Fig. 7. Time evolution of Chen's oscillator parameters (a_c, b_C, and c_C).

associated with the values presented in Figure 2. The effects of the interconnected nodes and the coupling strengths variations can be seen as unmodeled dynamics and external disturbances.

Based on theoretical and simulation results, it is concluded that one main advantage of the proposed controller is to achieve trajectory tracking successfully even in presence of plant parameters changes, unmodeled dynamics, and bounded external disturbances.

4.2. Case 2

Consider a scalar control $u_i \in R$, which is applied to only one state of the whole network. Then, the equation of the pinned node is

$$\begin{aligned}
\dot{x}_{1_1} &= a_C(x_{1_2} - x_{1_1}) + \sum_{j=1}^{50} c_{1j} a_{1j} \mathbf{T} x_{j_1} \\
\dot{x}_{1_2} &= (c_C - a_C)x_{1_1} - x_{1_1}x_{1_3} + c_C x_{1_2} + \sum_{j=1}^{50} c_{1j} a_{1j} \mathbf{T} x_{j_2} \\
\dot{x}_{1_3} &= x_{1_1}x_{1_2} - b_C x_{1_3} + \sum_{j=1}^{50} c_{1j} a_{1j} \mathbf{T} x_{j_3} + u_1,
\end{aligned} \tag{33}$$

where $u_1 = \bar{u}_{eq} - k_{3_1}\ sign(z_{3_1})$, with

$$\begin{aligned}
z_{1_1} &= x_{1_1} - x_s \\
z_{1_2} &= x_{1_2} - \alpha_1, \quad \alpha_1 = \frac{1}{a_C}(a_C x_{1_1} + x_{1_r} - k_1 z_{1_1}) \\
z_{1_3} &= x_{1_3} - \alpha_2, \quad \alpha_2 = \frac{1}{x_{1_1}}\left((c_C - a_C)x_{1_1} + c x_{1_2} - \dot{\alpha}_1 + a_C z_{1_1} + k_2 z_{1_2}\right),
\end{aligned}$$

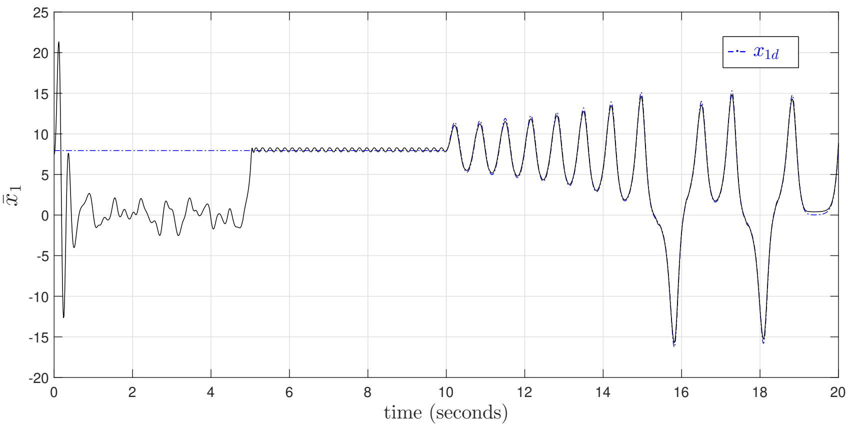

Fig. 8. Time response for only one state of the whole network.

and $\bar{u}_{eq} = -x_1x_2 + b_C x_3 + \dot{\alpha}_2 + x_1 z_2$.

For simulations, x_{1_d}, the changes, and other characteristics are same as for Case 1. From $t = 0s$ to $t = 4.8s$, the network runs without any connection ($c_{ij} = 0$). From $t = 4.8s$, the coupling strengths are set constant, randomly chosen with $c_{ij} \geq 30$. Then, at $t = 5s$, the proposed control law is applied; the state x_1 of the whole network is stabilized at a constant reference, which is selected as, $\mathbf{x}_s = 7.9373$. Afterwards, at $t = 10s$, chaotic Lorenz system (32) ($\mathbf{x}_s = x_{s_1}(t)$) is selected as reference; furthermore, at $t = 15s$, the value of coupling strengths c_{ij} changes for 25% of nodes of the entire network.

For all the described events c_{ij} always fulfill equation (13). Figure 8 displays the time response for the output $y = x_1$ (only one state of the whole network). As can be seen, tracking is achieved for both a constant reference and a chaotic one. Figure 9 shows the control input signal u_3 applied to the pinned node; due that the whole network (50 nodes) track a desired trajectory controlling only one node, the required energy is large and depends on the number and location of the pinned nodes. Moreover, Figure 10 shows simulation results for the state evolutions. Before $t = 5s$, trajectory evolutes freely without control; when the proposed control law is applied, the state x_1 tracks the desired trajectory, note that x_2 and x_3 track the trajectory given by Chen oscillator one. Furthermore, at $t = 15s$, the perturbations are incepted that in case 1, validating robustness properties of the proposed controller with 0.6921% MSE. The effects of the interconnected nodes and the coupling strengths variations can be seen as unmodeled dynamics and external disturbances.

Differing from previously published results, our controller fulfills output tracking using pinning control strategy ensuring ultimately bounded tracking errors even in presence of unmodeled dynamics and bounded external disturbances.

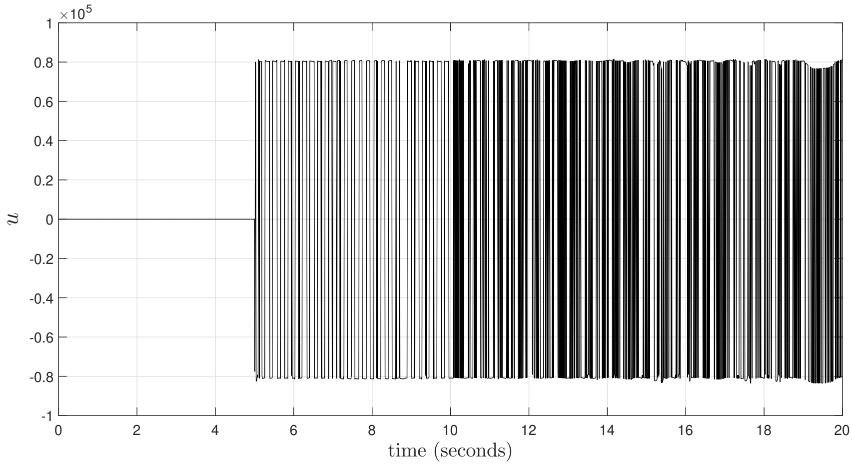

Fig. 9. Control signal u_3 for the pinned node.

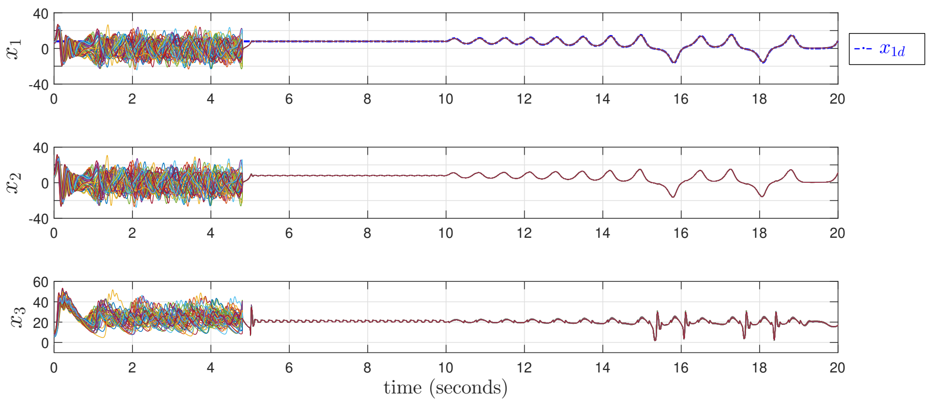

Fig. 10. Network state evolutions.

5. CONCLUSIONS

This paper presents a new control strategy for trajectory tracking on complex networks, based on pinning control and the sliding-mode technique. Simulation results illustrate that trajectory tracking for the scale-free network of chaotic nodes can be effectively achieved by using the proposed control scheme. Two cases are presented; the first case presents sliding-mode pinning control for the whole network to follow a reference for each one of the states, this controller is applied on the pinned node to control 50 nodes(150 states). It is easy to see that trajectory tracking is achieved. The second case considers the control for only one state, applying backstepping technique; desired output tracking is also achieved.

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