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SOLVING INTUITIONISTIC FUZZY MULTI-OBJECTIVE LINEAR PROGRAMMING PROBLEM AND ITS APPLICATION IN SUPPLY CHAIN MANAGEMENT

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Abstract. The aim of this paper is solving an intuitionistic fuzzy multi-objective linear programming problem containing intuitionistic fuzzy parameters, intuitionistic fuzzy maximization/minimization, and intuitionistic fuzzy constraints. To do this, a linear ranking function is used to convert the intuitionistic fuzzy parameters to crisp ones first. Then, linear membership and non-membership functions are used to manipulate intuitionistic fuzzy maximization/minimization and intuitionistic fuzzy constraints. Then, a multi-objective optimization problem is formulated containing maximization of membership functions and minimization of non-membership functions. To solve this problem, the minimax and weighted sum methods are used. Then, the described procedure is summarized as an algorithm to solve the problem, and a numerical example is solved by the proposed method. Finally, to investigate the capability and performance of the model, a supplier selection problem, which is one of the important applications in supply chain management, is solved by the proposed algorithm.

Keywords: multi-objective linear programming; intuitionistic fuzzy set; accuracy function; membership function; non-membership function; supplier selection

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1. Introduction

Linear programming is an important topic widely used in many areas of engineering, economy, and management. Since real world problems are very complex, experts and decision makers (DMs) often do not know the values of parameters precisely. Therefore fuzzy linear programming problems (FLPPs) with fuzzy parameters are more effective than the conventional ones in solving real world problems.

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In most cases, judgements and evaluations are made by DMs, who certainly have limitations, such as availability and exactness of data. So DMs hesitate more or less on evaluation activities. When hesitation occures, the classical fuzzy sets (FSs) theory is not able to handle the problem. Therefore, only the membership functions are used in FSs, which represent the degrees of belongingness of the elements to a set. This may not be able to represent the hesitation correctly. So, using both the degree of belongingness and the degree of non-belongingness is more appropriate to represent the hesitation. Therefore, to analyze this situation, we incorporate intuitionistic fuzzy sets (IFSs) introduced by Atanassov [4]. The major advantage of IFSs over FSs is that IFSs separate the degree of acceptance and the degree of non-acceptance of a decision. The IFS theory is a generalization of FS theory. Therefore, any method extended based on IFS theory, is automatically applicable with FS theory as a particular case. So, developing a method for IFS theory is more applicable than that for ordinary FS theory, which is our motivation for writing this paper.

Several researchers have studied IFSs and proposed optimization methods. Angelov [3] broadened the fuzzy optimization into IF optimization. Wan and Li [21], [22] proposed a new Atanassov's intuitionistic fuzzy programming method to solve heterogeneous multiattribute group decision making problem in which the attribute values were given by intuitionistic fuzzy sets, trapezoidal fuzzy numbers, intervals, and real numbers. Recently, Wan and Dong [20] worked on an extension of best-worst method based on IFSs. Deng-Feng Li [10] defined basic arithmetic operations on intuitionistic fuzzy numbers using membership and non-membership values. In another research, he used interval-valued intuitionistic fuzzy (IVIF) sets to capture fuzziness in multiattribute decision making (MADM) problems [11]. Bharati and Singh [5] proposed a computational algorithm to solve a multi-objective linear programming problem in interval-valued intuitionistic fuzzy environment. Garg [8] defined an improved score function by incorporating the idea of weighted average of the degree of hesitation between their membership functions, then used it to solve multi-criteria decision making problem with completely unknown attribute weights. Ye [27] discussed expected value method for intuitionistic trapezoidal fuzzy multicriteria decision making problems. Wan and Dong [19] used possibility degree method for interval-valued intuitionistic fuzzy numbers in decision making. Kabiraj et al. [9] proposed a general tool to model decision making problems under uncertainty, where the degree of rejection was defined simultaneously with the degree of acceptance of a piece of information in such a way that these degrees were not complement to each other. Wei et al. [26] developed an information-based score function of the IVIFSs and applied it to multiattribute decision making. Recently, Wan et al. proposed an intuitionistic fuzzy programming method for group decision making by using interval-valued fuzzy preference relations [23], [25]. Mohan et al. presented a postoptimality analysis for changes in objective functions and constraints with suitable numerical illustrations by dual simplex method using magnitude based ranking of triangular intuitionistic fuzzy numbers [13]. Recently, Malhotra and Bharati [12] have studied intuitionistic fuzzy and its applications in two-stage time minimizing transportation problem.

The problem of supplier selection for shipbuilding enterprises is one of the main problems of shipbuilding supply chain management. In regard to outsourcing supplier selection, Wan et al. [24] proposed a new intuitionistic fuzzy linear programming approach to optimize their proposed two-stage logistic network. Chang [6] proposed a novel supplier selection method based on integrating the intuitionistic fuzzy weighted averaging method and the soft set with imprecise data. Tooranloo and Iranpour [18] developed a supplier selection group decision framework employing interval intuitionistic fuzzy AHP method. Qu et al. [14] proposed an evaluation formula of intuitionistic fuzzy Choquet integral correlation coefficient between an alternative and the ideal alternative. They employed the proposed model to assess the green supply chain choice.

Multi-objective linear programming problem with intuitionistic fuzzy parameters, intuitionistic fuzzy optimization, and intuitionistic fuzzy equality/inequality constraints is a problem which has not been considered in previous researches, to the best knowledge of the authors. Such a problem is considered in this study, and a computational algorithm is developed to solve this problem.

The remainder of the paper is organized as follows: Section 2 represents the preliminary concepts regarding multi-objective optimization and intuitionistic fuzzy sets. Section 3 deals with intuitionistic fuzzy multi-objective linear programming problem (IFMOLPP) and its solution technique. The approach is illustrated through a numerical example and a supplier selection problem in Section 4. Finally, some concluding remarks are reported in Section 5.

2. Preliminaries

In this section, we quote some concept definitions of multi-objective optimization and intuitionistic fuzzy sets which are used in our study.

2.1. Multi-objective linear programming problem. In general, a multi-objective linear programming (MOLP) problem is formulated as follows:

$$\max_{k=1,\dots,p} \{f_k(x) = C_k x\}, \quad \text{s.t. } A_i x \; (\leqslant \text{ or } = \text{ or } \geqslant) \, b_i, \; i = 1, \dots, m, \; x \geqslant 0, \tag{2.1}$$

where x is the n-dimensional vector of decision variables, $C_k = (c_{k1}, \dots, c_{kn})^\top$ is the coefficient vector of the kth objective function, A_i is the ith row of matrix $A = [a_{ij}]_{m\times n}$, and b_i is the ith component of the right-hand side vector $b \in \mathbb{R}^m$.

Definition 2.1 ([15]). Let X be the set of all feasible solutions of problem (2.1). A feasible solution $x^0 \in X$ is called a complete optimal solution for problem (2.1) if for all $x \in X$ we have

$$f_k(x) \leqslant f_k(x^0), \quad k = 1, \ldots, p.$$

Such a complete optimal solution that simultaneously maximizes all of the objective functions does not always exist, because the objective functions sometimes conflict with each other. Thus, instead of a complete optimal solution, a new solution concept, called Pareto optimal or efficient solution, is used as follows.

Definition 2.2 ([15]). A feasible solution $\bar{x} \in X$ is called efficient (Pareto optimal) solution of problem (2.1) if there is no $x \in X$ such that $f_k(x) \geqslant f_k(\bar{x})$ for all $k = 1, \ldots, p$ and $f_j(x) > f_j(\bar{x})$ for at least one $j \in \{1, \ldots, p\}$.

The set of all efficient solutions of problem (2.1) is denoted by X_E and called the efficient set.

Definition 2.3 ([15]). A feasible solution $\bar{x} \in X$ is called weakly efficient (weakly Pareto optimal) solution if there is no $x \in X$ such that $f_k(x) > f_k(\bar{x})$ for all $k = 1, \ldots, p$.

The set of all weakly efficient solutions of problem (2.1) is denoted by X_{wE}. It can be easily seen that $X_E \subset X_{wE}$.

2.2. Scalarization methods. Several computational methods have been proposed for characterizing efficient solutions of multi-objective linear programming (MOLP) problems. Among them, the weighted method and the weighted minimax method are reviewed here.

2.2.1. Weighted sum method. The weighted sum method for obtaining efficient solutions of problem (2.1) is to solve the following single objective problem:

$$\max_{x \in X} \left\{ f(x) = \sum_{k=1}^{p} w_k f_k(x) \right\}. \tag{2.2}$$

The following theorems summarize the relationships between the optimal solutions of problem (2.2) and efficient solutions of problem (2.1).

Theorem 2.1 ([7]). *Let $\bar{x} \in X$ be an optimal solution of* (2.2). *The following statements hold.*

- ▷ *If $\mathbf{w} = (w_1, \ldots, w_p) > 0$ (i.e., all of its components are positive), then $\bar{x} \in X_E$.*
- ▷ *If $\mathbf{w} = (w_1, \ldots, w_p) \geqslant 0$ and $\mathbf{w} \neq 0$ (i.e., all of its components are nonnegative, and at least one of them is non-zero), then $\bar{x} \in X_{wE}$.*

Theorem 2.2 ([7]). *Let X be a convex set and f_k, $k = 1, \ldots, p$, be convex functions. Then the following statements hold.*

▷ *If $\bar{x} \in X_E$, then there is some $\mathbf{w} > 0$ such that $\bar{x}$ is an optimal solution of* (2.2).

▷ *If $\bar{x} \in X_{wE}$, then there is some $\mathbf{w} \geqslant 0$ such that $\bar{x}$ is an optimal solution of* (2.2).

Clearly, the objective functions of MOLP problem are linear, so they are convex functions, and its feasible set is a polyhedral set, which is a convex set. Therefore, all of its efficient solutions can be obtained by the weighted sum method.

2.2.2. The weighted minimax method. The weighted minimax method for obtaining efficient solutions is to solve the following max-min problem:

$$\max_{x\in X} \min_{k=1,\ldots,p} w_k f_k(x), \tag{2.3}$$

where $w_1, \ldots, w_k$ are nonnegative weights. By introducing the auxiliary variable λ, problem (2.3) is equivalently written as follows:

$$\max \lambda, \quad \text{s.t. } \lambda \leqslant w_k f_k(x),\ k = 1, \ldots, p,\ x \in X. \tag{2.4}$$

Theorem 2.3 ([15]). *If $\bar{x} \in X$ is a unique optimal solution of the minimax problem* (2.4) *for some $(w_1, \ldots, w_k) \geqslant 0$, then $\bar{x}$ is an efficient solution of* (2.1).

If the uniqueness of a solution is not guaranteed, only weak Pareto optimality is guaranteed [15].

Theorem 2.4 ([15]). *If $\bar{x} \in X$ is an efficient solution of* (2.1), *then x^* is an optimal solution of the minimax problem for some $(w_1, \ldots, w_k) > 0$.*

According to the above theorem, all efficient solutions of problem (2.1) can be obtained by the weighted minimax method.

2.3. Intuitionistic fuzzy sets. In this section, we review the fundamental notions of IFS theory, used throughout this paper.

Definition 2.4 ([4]). An intuitionistic fuzzy set $\widetilde{A}$ assigns to each element x of the universe X a membership degree $\mu_{\widetilde{A}}(x) \in [0,1]$ and a non-membership degree $\nu_{\widetilde{A}}(x) \in [0,1]$ such that $0 \leqslant \mu_{\widetilde{A}}(x) + \nu_{\widetilde{A}}(x) \leqslant 1$. An IFS $\widetilde{A}$ is mathematically represented as

$$\widetilde{A} = \{\langle x, \mu_{\widetilde{A}}(x), \nu_{\widetilde{A}}(x)\rangle;\ x \in X\}.$$

The membership function $\mu_{\widetilde{A}}(x)$ (non-membership function $\nu_{\widetilde{A}}(x)$) represents the degree of belongingness (non-belongingness) of the element $x \in X$ into the IFS $\widetilde{A}$. The value $\pi_{\widetilde{A}}(x) = 1 - (\mu_{\widetilde{A}}(x) + \nu_{\widetilde{A}}(x))$ represents the degree of hesitation for the element x being in $\widetilde{A}$. A fuzzy set is a special case of IFS in which $\nu(x) = 1 - \mu(x)$ for all $x \in X$, i.e., the hesitation degrees are 0.

Definition 2.5 ([16]). An IFS $\widetilde{A} = \{\langle x, \mu_{\widetilde{A}}(x), \nu_{\widetilde{A}}(x)\rangle;\ x \in X\}$ is called an intuitionistic fuzzy number (IFN) if the following hold:

▷ There exists $m \in \mathbb{R}$ such that $\mu_{\widetilde{A}}(m) = 1$ and $\nu_{\widetilde{A}}(m) = 0$ (m is called the mean value of $\widetilde{A}$),

▷ $\mu_{\widetilde{A}}$ and $\nu_{\widetilde{A}}$ are piecewise continuous functions from $\mathbb{R}$ to the closed interval $[0,1]$ and $0 \leqslant \mu_{\widetilde{A}}(x) + \nu_{\widetilde{A}}(x) \leqslant 1$ for all $x \in \mathbb{R}$, where

$$\mu_{\widetilde{A}}(x) = \begin{cases} g_1(x), & m-\alpha \leqslant x < m, \\ 1, & x = m, \\ h_1(x), & m < x \leqslant m+\beta, \\ 0, & \text{otherwise}, \end{cases} \qquad \nu_{\widetilde{A}}(x) = \begin{cases} g_2(x), & m-\alpha' \leqslant x < m, \\ 0, & x = m, \\ h_2(x), & m < x \leqslant m+\beta', \\ 1, & \text{otherwise}. \end{cases}$$

Here $\alpha \geqslant 0$ and $\beta \geqslant 0$ are the left and right spreads of membership function $\mu_{\widetilde{A}}$, respectively; $\alpha' \geqslant 0$ and $\beta' \geqslant 0$ are the left and right spreads of non-membership function $\nu_{\widetilde{A}}$, respectively; g_1 and h_1 are piecewise continuous, strictly increasing and strictly decreasing functions in $[m-\alpha, m)$ and $(m, m+\beta]$, respectively; and g_2 and h_2 are piecewise continuous, strictly decreasing and strictly increasing functions in $[m-\alpha', m)$ and $(m, m+\beta']$, respectively.

In the above definition, the conditions $\alpha' \geqslant \alpha$ and $\beta' \geqslant \beta$ are necessary for $\mu_{\widetilde{A}}(x) + \nu_{\widetilde{A}}(x) \leqslant 1$ to hold.

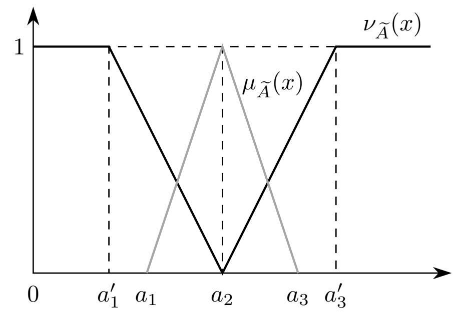

Figure 1. Triangular intuitionistic fuzzy number.

Definition 2.6 ([16], [1]). A triangular intuitionistic fuzzy number (TIFN) $\widetilde{A}$ is an IFN with the membership function $\mu_{\widetilde{A}}$ and non-membership function $\nu_{\widetilde{A}}$ given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \leqslant x < a_2, \\ 1, & x = a_2, \\ \dfrac{a_2-x}{a_3-a_2}, & a_2 < x \leqslant a_3, \\ 0, & \text{otherwise}, \end{cases} \qquad \nu_{\widetilde{A}}(x) = \begin{cases} \dfrac{a_2-x}{a_2-a_1'}, & a_1' \leqslant x < a_2, \\ 0, & x = a_2, \\ \dfrac{x-a_2}{a_3'-a_2}, & a_2 < x \leqslant a_3', \\ 1, & \text{otherwise}, \end{cases}$$

where $a_1' \leqslant a_1 \leqslant a_2 \leqslant a_3 \leqslant a_3'$ (see Fig. 1). This TIFN is denoted by $(m, \alpha, \beta, \alpha', \beta')$ or $(a_1, a_2, a_3; a_1', a_2, a_3')$, where $a_1 = m - \alpha$, $a_2 = m$, $a_3 = m + \beta$, $a_1' = m - \alpha'$ and $a_3' = m + \beta'$. The set of all TIFNs is denoted by $\mathrm{IF}(\mathbb{R})$.

Definition 2.7 ([16]). Let $\widetilde{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $\widetilde{B} = (b_1, b_2, b_3; b_1', b_2, b_3')$ be in $\mathrm{IF}(\mathbb{R})$ and $k \in \mathbb{R}$. Then

- ▷ $\widetilde{A} + \widetilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3; a_1' + b_1', a_2 + b_2, a_3' + b_3')$,
- ▷ $\widetilde{A} - \widetilde{B} = (a_1 - b_3, a_2 - b_2, a_3 - b_1; a_1' - b_3', a_2 - b_2, a_3' - b_1')$,
- ▷ $k\widetilde{A} = \begin{cases} (ka_1, ka_2, ka_3; ka_1', ka_2, ka_3'), & k > 0, \\ (ka_3, ka_2, ka_1; ka_3', ka_2, ka_1'), & k < 0, \end{cases}$
- ▷ $\widetilde{A}\widetilde{B} = (l_1, l_2, l_3; l_1', l_2, l_3')$, where

$$l_1 = \min\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\}, \quad l_3 = \max\{a_1b_1, a_1b_3, a_3b_1, a_3b_3\},$$
$$l_1' = \min\{a_1'b_1', a_1'b_3', a_3'b_1', a_3'b_3'\},$$
$$l_3' = \max\{a_1'b_1', a_1'b_3', a_3'b_1', a_3'b_3'\}, \quad \text{and} \quad l_2 = a_2b_2.$$

Definition 2.8 ([17]). Let $\widetilde{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$ be a TIFN. The score function for the membership function $\mu_{\widetilde{A}}$ is denoted by $\mathcal{S}(\mu_{\widetilde{A}})$ and is defined by

$$\mathcal{S}(\mu_{\widetilde{A}}) = \frac{a_1 + 2a_2 + a_3}{4}.$$

The score function for the non-membership function $\nu_{\widetilde{A}}$ is denoted by $\mathcal{S}(\nu_{\widetilde{A}})$ and is defined by

$$\mathcal{S}(\nu_{\widetilde{A}}) = \frac{a_1' + 2a_2 + a_3'}{4}.$$

The accuracy function of $\widetilde{A}$ is denoted by $\mathcal{R}(\widetilde{A})$ and is defined by

$$\mathcal{R}(\widetilde{A}) = \frac{\mathcal{S}(\mu_{\widetilde{A}}) + \mathcal{S}(\nu_{\widetilde{A}})}{2} = \frac{(a_1 + 2a_2 + a_3) + (a_1' + 2a_2 + a_3')}{8}. \tag{2.5}$$

Theorem 2.5. *The accuracy function $\mathcal{R}$ is a linear function.*

Proof. See the reference [16]. □

Definition 2.9. Let $\widetilde{A} = (a_1, a_2, a_3; a_1', a_2, a_3')$ and $\widetilde{B} = (b_1, b_2, b_3; b_1', b_2, b_3')$ be two TIFNs. Then

- ▷ $\mathcal{R}(\widetilde{A}) \geqslant \mathcal{R}(\widetilde{B}) \Rightarrow \widetilde{A} \geqslant \widetilde{B}$,
- ▷ $\mathcal{R}(\widetilde{A}) \leqslant \mathcal{R}(\widetilde{B}) \Rightarrow \widetilde{A} \leqslant \widetilde{B}$,
- ▷ $\mathcal{R}(\widetilde{A}) = \mathcal{R}(\widetilde{B}) \Rightarrow \widetilde{A} = \widetilde{B}$,
- ▷ $\min(\widetilde{A}, \widetilde{B}) = \widetilde{A}$ if $\widetilde{A} \leqslant \widetilde{B}$ or $\widetilde{B} \geqslant \widetilde{A}$,
- ▷ $\max(\widetilde{A}, \widetilde{B}) = \widetilde{A}$ if $\widetilde{A} \geqslant \widetilde{B}$ or $\widetilde{B} \leqslant \widetilde{A}$.

3. Solving intuitionistic fuzzy multi-objective linear programming problem

Consider the folowing intuitionistic fuzzy multi-objective optimization problem:

$$(3.1)\quad \widetilde{\max_{k=1,\ldots,p}}\{\tilde{f}_k(x)=\widetilde{C}_k x\},\quad \text{s.t. } \tilde{g}_i(x)=\widetilde{A}_i x\,(\widetilde{\leqslant}\text{ or }\widetilde{\cong}\text{ or }\widetilde{\geqslant})\,\tilde{b}_i,\ i=1,\ldots,m,\ x\geqslant 0,$$

where $\widetilde{\leqslant}$, $\widetilde{\geqslant}$ and $\widetilde{\cong}$ are intuitionistic fuzzy inequality and fuzzy equality, respectively.

The method of solving this problem consists of two steps. In the first step, using the accuracy function (2.5) and due to its linearity, we convert the multi-objective linear programming problem (3.1) to the following multi-objective linear programming problem with crisp parameters:

$$(3.2)\quad \widetilde{\max_{k=1,\ldots,p}}\{f_k(x)=C_k x\},\quad \text{s.t. } g_i(x)=A_i x\,(\widetilde{\leqslant}\text{ or }\widetilde{\cong}\text{ or }\widetilde{\geqslant})\,b_i,\ i=1,\ldots,m,\ x\geqslant 0.$$

In problem (3.2), $f_k(x)=\mathcal{R}(\tilde{f}_k(x))$, $g_i(x)=\mathcal{R}(\tilde{g}_i(x))$, $b_i=\mathcal{R}(\tilde{b}_i)$, C_k is an n-dimensional vector with jth element $c_{kj}=\mathcal{R}(\tilde{c}_{kj})$ and A_i is an n-dimensional vector with jth element $a_{ij}=\mathcal{R}(\tilde{a}_{ij})$.

In the second step, we examine intuitionistic fuzzy maximization, fuzzy equalities and fuzzy inequalities.

For any objective function, consider membership function μ_k and non-membership function ν_k as the following linear functions:

$$(3.3)\qquad \mu_k(f_k(x))=\begin{cases}0, & f_k(x)<L_k^{\mu},\\ \dfrac{f_k(x)-L_k^{\mu}}{U_k^{\mu}-L_k^{\mu}}, & L_k^{\mu}\leqslant f_k(x)\leqslant U_k^{\mu},\\ 1, & f_k(x)>U_k^{\mu},\end{cases}$$

and

$$(3.4)\qquad \nu_k(f_k(x))=\begin{cases}1, & f_k(x)<L_k^{\nu},\\ \dfrac{U_k^{\nu}-f_k(x)}{U_k^{\nu}-L_k^{\nu}}, & L_k^{\nu}\leqslant f_k(x)\leqslant U_k^{\nu},\\ 0, & f_k(x)>U_k^{\nu},\end{cases}$$

where $U_k^{\mu}=\max\limits_{1\leqslant r\leqslant p}\{f_k(x^r)\}$, $L_k^{\mu}=\min\limits_{1\leqslant r\leqslant p}\{f_k(x^r)\}$ for membership function, $U_k^{\nu}=U_k^{\mu}-\lambda(U_k^{\mu}-L_k^{\mu})$, $L_k^{\nu}=L_k^{\mu}$ for non-membership function, and $\lambda\in[0,1)$. Here x^r $(r=1,\ldots,p)$ is the individual maximizer of the rth objective function.

In the sequel, for the constraints $g_i(x) \widetilde{\leqslant} b_i$ we consider the membership function μ_i and the non-membership function ν_i as the following linear functions:

$$\mu_i(g_i(x)) = \begin{cases} 1, & g_i(x) < L_i^\mu, \\ \dfrac{U_i^\mu - g_i(x)}{U_i^\mu - L_i^\mu}, & L_i^\mu \leqslant g_i(x) \leqslant U_i^\mu, \\ 0, & g_i(x) > U_i^\mu, \end{cases} \tag{3.5}$$

and

$$\nu_i(g_i(x)) = \begin{cases} 0, & g_i(x) < L_i^\nu, \\ \dfrac{g_i(x) - L_i^\nu}{U_i^\nu - L_i^\nu}, & L_i^\nu \leqslant g_i(x) \leqslant U_i^\nu, \\ 1, & g_i(x) > U_i^\nu, \end{cases} \tag{3.6}$$

where $U_i^\mu = b_i + \Delta_i$, $L_i^\mu = b_i$ for $i = 1, \ldots, m$ and Δ_i is the maximum permissible deviation from b_i to the right, which is determined by the DM. In the non-membership function, $U_i^\nu = U_i^\mu + \lambda(U_i^\mu - L_i^\mu)$, $L_i^\nu = L_i^\mu$, and $\lambda \geqslant 0$.

Similarly, for the constraints $g_i(x) \widetilde{\geqslant} b_i$, we consider μ_i and ν_i as the following linear functions:

$$\mu_i(g_i(x)) = \begin{cases} 0, & g_i(x) < L_i^\mu, \\ \dfrac{g_i(x) - L_i^\mu}{U_i^\mu - L_i^\mu}, & L_i^\mu \leqslant g_i(x) \leqslant U_i^\mu, \\ 1, & g_i(x) > U_i^\mu, \end{cases} \tag{3.7}$$

and

$$\nu_i(g_i(x)) = \begin{cases} 0, & g_i(x) < L_i^\nu, \\ \dfrac{U_i^\nu - g_i(x)}{U_i^\nu - L_i^\nu}, & L_i^\nu \leqslant g_i(x) \leqslant U_i^\nu, \\ 1, & g_i(x) > U_i^\nu, \end{cases} \tag{3.8}$$

where $U_i^\mu = b_i$, $L_i^\mu = b_i - \Delta_i$ for $i = 1, \ldots, m$ and Δ_i is the maximum permissible deviation from b_i to the left, which is determined by the DM. In the non-membership function, $U_i^\nu = U_i^\mu - \lambda(U_i^\mu - L_i^\mu)$, $L_i^\nu = L_i^\mu$, and $\lambda < 1$.

Finally, for the constraints $g_i(x) \widetilde{=} b_i$, we use the triangular membership and non-membership functions as follows:

$$\mu_i(g_i(x)) = \begin{cases} \dfrac{g_i(x) - L_i^\mu}{b_i - L_i^\mu}, & L_i^\mu \leqslant g_i(x) \leqslant b_i, \\ 1, & g_i(x) = b_i \\ \dfrac{U_i^\mu - g_i(x)}{U_i^\mu - b_i}, & b_i \leqslant g_i(x) \leqslant U_i^\mu, \\ 0, & \text{otherwise}, \end{cases} \tag{3.9}$$

and

$$
\nu_i(g_i(x)) = \begin{cases} \dfrac{b_i - g_i(x)}{b_i - L_i^{\nu}}, & L_i^{\nu} \leqslant g_i(x) \leqslant b_i, \\ 0, & g_i(x) = b_i, \\ \dfrac{g_i(x) - b_i}{U_i^{\nu} - b_i}, & b_i \leqslant g_i(x) \leqslant U_i^{\nu}, \\ 1, & \text{otherwise}, \end{cases} \tag{3.10}
$$

where $U_i^{\mu} = b_i + \Delta_i$, $L_i^{\mu} = b_i - \Delta_i$ for $i = 1, \ldots, m$ and Δ_i is the maximum permissible deviation from b_i to the left and right, which is determined by the DM. In the non-membership function, $U_i^{\nu} = U_i^{\mu} + \lambda(U_i^{\mu} - b_i)$, $L_i^{\nu} = L_i^{\mu}$, and $\lambda \geqslant 0$.

Remark 3.1. In constructing non-membership functions (3.4), a parameter $\lambda \in [0, 1)$ was used. The condition $\lambda \in [0, 1)$ is necessary to have an intutionistic fuzzy number. $\lambda = 0$ causes that the function has an ordinary fuzzy number with the degree of hesitation of 0 for the values of objective functions. As λ increases from 0, the non-membership values are decreased and the hesitation is increased. Therefore, by interaction with the DM, the value of λ, which determines the hesitation degree of the DM about the values of objective functions, can be determined. Also, in constructing non-membership functions (3.6), (3.8), and (3.10) a parameter λ was used, which has similar interpretation and can be determined by interaction with the DM.

Remark 3.2. In order to quantify the fuzzy goals of the DM, various membership and non-membership functions such as exponential, hyperbolic, hyperbolic inverse, and piecewise linear functions can also be used in intuitionistic fuzzy optimization. For more details, we refer the reader to [15].

Since the decision maker wants to maximize the acceptance degree and minimize the rejection degree, we are looking for a solution with maximum degree of membership and minimum degree of non-membership. Therefore, according to the fuzzy decision of Belman and Zadeh [15], we have to solve the following multi-objective programming problem:

$$
\begin{aligned}
&\max_{k=1,\ldots p,\, i=1,\ldots m} \{\mu_k(f_k(x)), \mu_i(g_i(x))\}, \qquad \min_{k=1,\ldots p,\, i=1,\ldots m} \{\nu_k(f_k(x)), \nu_i(g_i(x))\}, \\
&\quad \text{s.t. } \nu_k(f_k(x)) \geqslant 0, \ k = 1, \ldots, p, \ \nu_i(g_i(x)) \geqslant 0, \ i = 1, \ldots, m, \\
&\quad \mu_k(f_k(x)) \geqslant \nu_k(f_k(x)), \ k = 1, \ldots, p, \\
&\quad \mu_i(g_i(x)) \geqslant \nu_i(g_i(x)), \ i = 1, \ldots, m, \\
&\quad 0 \leqslant \mu_k(f_k(x)) + \nu_k(f_k(x)) \leqslant 1, \ k = 1, \ldots, p, \\
&\quad 0 \leqslant \mu_i(g_i(x)) + \nu_i(g_i(x)) \leqslant 1, \ i = 1, \ldots, m, \\
&\quad x \geqslant 0.
\end{aligned} \tag{3.11}
$$

The objective functions, membership functions and non-membership functions in problem (3.11) are all linear. This problem consists of $2(m+p)$ objective functions, which can be solved by different methods. Using the minimax method, each set of the objective functions of the above problem can be replaced with an objective function as follows:

$$\max \min_{k=1,\dots,p,\, i=1,\dots,m} \{\mu_k(f_k(x)), \mu_i(g_i(x))\},$$
$$\min \max_{k=1,\dots,p,\, i=1,\dots,m} \{\nu_k(f_k(x)), \nu_i(g_i(x))\}.$$

Using the auxiliary variables α and β, we have the following equivalent bi-objective programming problem:

$$\text{(3.12)} \qquad \max \alpha, \quad \min \beta, \quad \text{s.t. } \begin{aligned}[t] &\mu_k(f_k(x)) \geqslant \alpha,\ k=1,\dots,p,\\ &\mu_i(g_i(x)) \geqslant \alpha,\ i=1,\dots,m,\\ &\nu_k(f_k(x)) \leqslant \beta,\ k=1,\dots,p,\\ &\nu_i(g_i(x)) \leqslant \beta,\ i=1,\dots,m,\\ &\alpha+\beta \leqslant 1, \quad \alpha \geqslant \beta \geqslant 0, \quad x \in \overline{X},\end{aligned}$$

where $\overline{X}$ is the feasible set of problem (3.11). By applying the weighted sum method, problem (3.12) can be transformed into the following single objective linear programming problem:

$$\text{(3.13)} \qquad \max\{w_1\alpha - w_2\beta\}, \quad \text{s.t. } \begin{aligned}[t] &\mu_k(f_k(x)) \geqslant \alpha,\ k=1,\dots,p,\\ &\mu_i(g_i(x)) \geqslant \alpha,\ i=1,\dots,m,\\ &\nu_k(f_k(x)) \leqslant \beta,\ k=1,\dots,p,\\ &\nu_i(g_i(x)) \leqslant \beta,\ i=1,\dots,m,\\ &\alpha+\beta \leqslant 1, \quad \alpha \geqslant \beta \geqslant 0, \quad x \in \overline{X},\end{aligned}$$

where w_1, w_2 are positive weights. The above linear programming problem can be easily solved by the simplex method.

3.1. Computational algorithm. Using the procedure described above, we present the following algorithm to solve problem (3.1), which includes intuitionistic fuzzy parameters, intuitionistic fuzzy maximization, and intuitionistic fuzzy inequations ($\tilde{\leqq}$). With a slight modification, the algorithm can be applied to the situations where the problem involves intuitionistic fuzzy minimization, $\tilde{\eqsim}$ or $\tilde{\geqslant}$ (or a combination of three types of intuitionistic fuzzy constraints and two types of intuitionistic fuzzy optimizations).

Step 1. Convert problem (3.1) to (3.2) using the accuracy function (2.5).

Step 2. Take one objective function out of given p objective functions and solve it as a single objective problem subject to the given constraints. Find an optimal solution x^r and optimal value of its objective function f_r^*.

Step 3. In each point x^r, compute the values of the remaining $k-1$ objectives.

Step 4. Repeat steps 2 and 3 for the remaining $k-1$ objective functions.

Step 5. Tabulate the solutions obtained in steps 2–4 to construct the payoff table as given in Table 1, where $f_r^* = f_r(x^r)$ is the maximum value of rth objective function.

x	f_1	f_2	$\dots$	f_p
x^1	$f_1^*(x^1)$	$f_2(x^1)$	$\dots$	$f_p(x^1)$
x^2	$f_1(x^2)$	$f_2^*(x^2)$	$\dots$	$f_p(x^2)$
$\vdots$	$\vdots$	$\vdots$	$\dots$	$\vdots$
x^p	$f_1(x^p)$	$f_2(x^p)$	$\dots$	$f_p^*(x^p)$

Table 1. Payoff table.

Step 6. Set upper and lower bounds for each objective function and each constraint for the degree of acceptance and the degree of rejection corresponding to the set of solutions obtained in step 5.

For membership functions set

$$U_k^\mu = \max_{1\leqslant r\leqslant p}\{f_k(x^r)\},\quad L_k^\mu = \min_{1\leqslant r\leqslant p}\{f_k(x^r)\},\quad k=1,\dots,p,$$
$$U_i^\mu = b_i + \Delta_i,\quad L_i^\mu = b_i,\quad i=1,\dots,m,$$

where Δ_i is the maximum permissible deviation from b_i to the right, which is determined by the DM. For non-membership functions set

$$U_k^\nu = U_k^\mu - \lambda(U_k^\mu - L_k^\mu),\quad \lambda\in(0,1),\quad L_k^\nu = L_k^\mu,\quad k=1,\dots,p,$$
$$U_i^\nu = U_i^\mu + \lambda(U_i^\mu - L_i^\mu),\quad \lambda\geqslant 0,\quad L_i^\nu = L_i^\mu,\quad i=1,\dots,m.$$

Step 7. Use linear membership functions $\mu_k(f_k(x))$ and $\mu_i(g_i(x))$ for the objective functions and the constraints as Equations (3.3) and (3.5), respectively. Also, consider non-membership functions $\nu_k(f_k(x))$ and $\nu_i(g_i(x))$ for the objective functions and the constraints as Equations (3.4) and (3.6), respectively.

Step 8. Construct problem (3.13), and solve it by the simplex method.

R e m a r k 3.3. Although the proposed algorithm was extended for MOLP problems, it can be used to solve nonlinear programming problems in intuitionistic environment as well. To this end, the single objective problems, which have to be solved in Steps 2, 4 and 8, are not linear, so they cannot be solved by the simplex method and must be solved by an appropriate method or software.

4. Numerical example

In this section, an example for intuitionistic fuzzy multi-objective linear programming problems are used to illustrate the proposed method.

Example 4.1. Let us consider the following intuitionistic fuzzy multi-objective linear programming problem (IFMOLPP):

$$\widetilde{\max}\{f_1(x)=\tilde{5}x_1+\tilde{3}x_2\},\quad \widetilde{\max}\{f_2(x)=\tilde{2}x_1+\tilde{7}x_2\},$$
$$\text{s.t. } \tilde{2}x_1+\tilde{3}x_2\,\tilde{\cong}\,\widetilde{25},\quad \tilde{1}x_1+\tilde{1}x_2\,\tilde{\cong}\,\widetilde{10},\quad \tilde{4}x_1+\tilde{5}x_2\,\tilde{\cong}\,\widetilde{50},\quad x_1,x_2\geqslant 0. \tag{4.1}$$

The parameters estimated by the DM are as follows:

$$\widetilde{25}=(22,25,25;18,25,25),\quad \widetilde{10}=(9,10,10;8,10,10),\quad \widetilde{50}=(50,50,55;50,50,60),$$
$$\tilde{5}=(4,5,6;4,5,7),\quad \tilde{3}=(3,3,4;3,3,4.5),\quad \tilde{2}=(2,2,3;2,2,4),$$
$$\tilde{7}=(7,7,7.5;6,7,8),\quad \tilde{1}=(0.5,1,1;0.2,1,1.5),\quad \tilde{4}=(3,4,4;2,4,4).$$

Using accuracy function (2.5), problem (4.1) is converted to the following problem with crisp parameters:

$$\widetilde{\max}\{f_1(x)=5.125x_1+3.31x_2\},\quad \widetilde{\max}\{f_2(x)=2.37x_1+7.06x_2\},$$
$$\text{s.t. } 2.37x_1+3.62x_2\,\tilde{\cong}\,23.75,\quad 0.9x_1+0.9x_2\,\tilde{\cong}\,9.6,$$
$$3.62x_1+5.125x_2\,\tilde{\cong}\,51.9,\quad x_1,x_2\geqslant 0. \tag{4.2}$$

By considering the first objective function and solving the corresponding single-objective problem by Lingo software, the following optimal solution is obtained:

$$x^1=(x_1,x_2)=(10.0211,0),\quad f_1(x^1)=51.3581.$$

Now we have $f_2(x^1)=23.75$. By repeating this procedure for the second objective function, the payoff table is obtained as Table 2.

$X=(x_1,x_2)$	f_1	f_2
$(10.0211,0)$	51.3581	23.75
$(0,6.560773)$	21.7162	46.3191

Table 2. Payoff table for Example 1.

Now, by considering $\lambda=0.1$ for the objective functions, we have

$$U_1^{\mu}=51.3581,\quad L_1^{\mu}=L_1^{\nu}=21.7162,\quad U_1^{\nu}=48.39391;$$
$$U_2^{\mu}=46.3191,\quad L_2^{\mu}=L_2^{\nu}=23.75,\quad U_2^{\nu}=44.06219.$$

For the constraints, by considering $\Delta_1 = 2$, $\Delta_2 = 1$, and $\Delta_3 = 5$ we have

$$U_1^\mu = 25.75, \quad L_1^\mu = L_1^\nu = 23.75, \quad U_1^\nu = 25.95;$$
$$U_2^\mu = 10.6, \quad L_2^\mu = L_2^\nu = 9.6, \quad U_2^\nu = 10.7;$$
$$U_3^\mu = 56.9, \quad L_3^\mu = L_3^\nu = 51.9, \quad U_3^\nu = 57.4.$$

After constructing problem (3.13) by considering $w_1 = w_2 = 1$, and solving it by Lingo software, the following solution is obtained:

$$x_1 = 5.21, \quad x_2 = 3.35, \quad \alpha = 0.54, \quad \beta = 0.39.$$

5. Application to supplier selection and order allocation problem

Along with advances in technology and the advent of the information age, supply chain competition became the core strategy of enterprises that are in pursuit of a competitive advantage. Supplier selection and evaluation are key issues in the success of a competitive enterprise. Supplier selection for an enterprise is a typical multi-criteria decision-making problem that includes qualitative and quantitative criteria. Moreover, there is a significant amount of fuzzy and intuitionistic fuzzy information in realworld situations, for which the traditional approach in choosing the best supplier becomes no longer applicable. To solve these issues, a numerical example of the supplier selection and order allocation problem in a single-buyer-multi-supplier supply chain is considered, in which appropriate suppliers have to be selected and orders allocated to them. In order to formulate this problem, we first define the following notations:

- ▷ D: demand over period,
- ▷ x_i: the number of units purchased from the ith supplier,
- ▷ P_i: unit net purchase cost from the ith supplier,
- ▷ C_i: capacity of the ith supplier,
- ▷ F_i: percentage of quality level of the ith supplier,
- ▷ S_i: percentage of service level of the ith supplier,
- ▷ n: number of suppliers.

The intuitionistic fuzzy multi-objective linear programming model for purchasing a single item in multiple sourcing network with capacity constraint is

$$\widetilde{\min}\, \widetilde{Z}_1 = \sum_{i=1}^{n} \widetilde{P}_i x_i, \tag{5.1}$$

$$\widetilde{\max}\, \widetilde{Z}_1 = \sum_{i=1}^{n} \widetilde{F}_i x_i, \tag{5.2}$$

$$\widetilde{\max}\, \widetilde{Z}_1 = \sum_{i=1}^{n} \widetilde{S}_i x_i, \tag{5.3}$$

$$\text{s.t.} \quad \sum_{i=1}^{n} x_i \mathrel{\widetilde{\geqslant}} \widetilde{D}, \tag{5.4}$$

$$x_i \mathrel{\widetilde{\leqslant}} \widetilde{C}_i,\ i = 1, \dots, n, \tag{5.5}$$

$$x_i \geqslant 0,\ i = 1, \dots, n. \tag{5.6}$$

The objective function (5.1) minimizes the total monetary cost, the objective function (5.2) maximizes the total quality, and the objective function (5.3) maximizes the service level of purchased items. The constraint (5.4) states that the demand is satisfied by suppliers, the capacity constraints of the suppliers are expressed by the constraint (5.5), and the constraints (5.6) prohibit negative orders.

E x a m p l e 5.1. Suppose three suppliers must be managed to supply a new product. The predicted demand for this product is approximately 1000 units. The purchasing criteria are (i) net price including transportation costs, (ii) quality including defects and manufacturing capabilities and continuous quality improvement and (iii) service including delivery speed and reliability, product development, financial and organizational capabilities [2]. The first one has to be minimized, and the two laters have to be maximized.

It is assumed that the input data from suppliers' performance on these criteria are not accurate and have been reported by intuitionistic fuzzy numbers given in Table 3.

	Supplier 1	Supplier 1
Cost	$\widetilde{3} = (2, 3, 4; 1, 3, 4)$	$\widetilde{2} = (2, 2, 3; 1, 2, 4)$
Quality	$\widetilde{85} = (85, 85, 90; 80, 85, 90)$	$\widetilde{80} = (75, 80, 80; 70, 80, 90)$
Service	$\widetilde{75} = (75, 80, 80; 70, 80, 90)$	$\widetilde{90} = (85, 90, 90; 80, 90, 90)$
Capacity	$\widetilde{500} = (500, 500, 550; 450, 500, 600)$	$\widetilde{600} = (550, 600, 600; 550, 600, 650)$

	Supplier 1
Cost	$\widetilde{4} = (4, 5, 6; 4, 5, 6)$
Quality	$\widetilde{95} = (95, 95, 100; 90, 95, 100)$
Service	$\widetilde{90} = (85, 90, 90; 80, 90, 90)$
Capacity	$\widetilde{550} = (500, 550, 550; 500, 550, 600)$

Table 3. Suppliers' quantitative information.

According to the above information, the intuitionistic fuzzy linear multi-objective programming for this example is as follows:

$$\widetilde{\min}\{\widetilde{Z_1} = \tilde{3}x_1 + \tilde{2}x_2 + \tilde{5}x_3\}, \quad \widetilde{\max}\{\widetilde{Z_2} = \widetilde{0.85}x_1 + \widetilde{0.8}x_2 + \widetilde{0.95}x_3\}, \tag{5.7}$$
$$\widetilde{\max}\{\widetilde{Z_3} = \widetilde{0.75}x_1 + \widetilde{0.9}x_2 + \widetilde{0.85}x_3\},$$
$$\text{s.t. } x_1 + x_2 + x_3 \mathrel{\widetilde{\cong}} \widetilde{1000}, \quad x_1 \mathrel{\widetilde{\cong}} \widetilde{500}, \quad x_2 \mathrel{\widetilde{\cong}} \widetilde{600}, \quad x_3 \mathrel{\widetilde{\cong}} \widetilde{550}, \quad x_i \geqslant 0,\ i = 1, 2, 3.$$

The objective functions Z_1, Z_2 and Z_3 are cost, quality and service, respectively, and x_i is the number of units which have to be purchased from the ith supplier. Demand has been estimated by the DM as the intuitionistic fuzzy number $\widetilde{1000} = (950, 1000, 1050; 950, 1000, 1100)$.

Using the accuracy function (2.5), the above problem is transformed into the following problem with crisp parameters.

$$
\begin{gathered}
\widetilde{\min}\{\widetilde{Z_1} = 2.875x_1 + 2.25x_2 + 5.125x_3\}, \\
\widetilde{\max}\{\widetilde{Z_2} = 0.8575x_1 + 0.79375x_2 + 0.95625x_3\}, \\
\widetilde{\max}\{\widetilde{Z_3} = 0.74375x_1 + 0.88125x_2 + 0.8575x_3\}, \\
\text{s.t. } x_1 + x_2 + x_3 \,\widetilde{\cong}\, 1006.25, \quad x_1 \,\widetilde{\cong}\, 512.5, \quad x_2 \,\widetilde{\cong}\, 593.75, \quad x_3 \,\widetilde{\cong}\, 543.75, \\
x_i \geqslant 0, \ i = 1, 2, 3.
\end{gathered} \tag{5.8}
$$

By considering the first objective function and solving the corresponding single objective problem, and repeating this procedure for the second and third objective functions, the payoff table is obtained as Table 4.

$X = (x_1, x_2, x_3)$	f_1	f_2	f_3
$(412.5, 593.75, 0)$	2521.875	825.0078	830.0391
$(462.5, 0, 543.75)$	4116.406	916.5547	810.25
$(0, 593.75, 412.5)$	3450	865.7422	876.9609

Table 4. Payoff table for Example 2.

By considering $\lambda = 0.1$ for the objective functions, we have

$$
\begin{gathered}
U_1^\mu = 4116.406, \quad U_2^\mu = 916.5547, \quad U_3^\mu = 810.25; \\
L_1^\mu = 2521.875, \quad L_2^\mu = 825.0078, \quad L_3^\mu = 876.9609; \\
U_1^\nu = 3956.953, \quad U_2^\nu = 1008.222, \quad U_3^\nu = 816.9211; \\
L_1^\nu = 2521.875, \quad L_2^\nu = 825.0078, \quad L_3^\nu = 876.9609.
\end{gathered}
$$

For the constraints, by considering $\Delta_1 = 4$, $\Delta_2 = 5$, $\Delta_3 = 6$, and $\Delta_4 = 3$ we have

$$
\begin{gathered}
U_1^\mu = 1006.25, \quad L_1^\mu = L_1^\nu = 1002.25, \quad U_1^\nu = 1005.85; \\
U_2^\mu = 517.5, \quad L_2^\mu = L_2^\nu = 512.5, \quad U_2^\nu = 518; \\
U_3^\mu = 599.75, \quad L_3^\mu = L_3^\nu = 593.75, \quad U_3^\nu = 600.35; \\
U_4^\mu = 546.75, \quad L_4^\mu = L_4^\nu = 543.75, \quad U_4^\nu = 547.05.
\end{gathered}
$$

After constructing problem (3.13) with $w_1 = w_2 = w_3 = 1$ and solving it by Lingo software, the following solution is obtained:

$$x_1 = 413.98, \quad x_2 = 48.52, \quad x_3 = 543.75,$$
$$f_1 = 4086.083, \quad f_2 = 913.462, \quad f_3 = 816.9211, \quad \alpha = 0.9662, \quad \beta = 0.$$

According to the definition of variables α, β, α and β can be called the minimum level of satisfaction and the maximum level of dissatisfaction with the obtained solution, respectively. Therefore, according to the obtained results for the parameters α and β (the value of α close to one and the value of β close to zero), the obtained result is a satisfactory solution.

6. Conclusion

In this paper, a multi-objective linear programming problem in intuitionistic fuzzy environment was considered and an algorithm was proposed to solve it. In this approach, the degree of acceptance and rejection of the objective functions and the constraints were introduced together. The intuitionistic fuzzy multi-objective linear programming problem (IFMOLPP) was transformed to a multi-objective linear programming problem by using the accuracy function, and by applying the scalarization technique it was transformed to a linear programming problem. Since the proposed algorithm in this paper does not change the linearity property of the problem, it is a convenient method to solve IFMOLPP. The proposed computational algorithm is easy and more accurate for dealing with real-life problems with multiple objectives under uncertainty and vagueness. The discussed method was illustrated through an example. Finally, to investigate the capability and performance of the proposed method, a supplier selection problem in intuitionistic fuzzy environment was solved by the proposed algorithm, and observed that the proposed method provides a satisfactory solution of the problem. Although the proposed method was extended for MOLP problems, it can be used to solve nonlinear programming problems in intuitionistic environment as well.

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