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PROFESSOR IVO BABUŠKA, FOUNDER OF APPLICATIONS OF MATHEMATICS, PASSED AWAY

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It is with deep sorrow that we inform you that Ivo Babuška passed away at the age of 97 on April 12, 2023. Professor Babuška was a renowned mathematician, a visionary in computational and applied mathematics, and a leading personality connecting rigorous mathematics with engineering applications. Prof. Ing. Dr. Ivo Babuška, DrSc., dr. h. c., was one of the founding fathers of the Mathematical Institute of the Czechoslovak Academy of Sciences and, in 1956, he established the international journal Applications of Mathematics.

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1. Brief curriculum vitae

A worldwide recognized mathematician Ivo Babuška was born on March 22, 1926 in Prague (Praha), Czechoslovakia, in the family of architect Milan Babuška who designed and built many buildings in Prague, including e.g. the National Technical Museum and the National Museum of Agriculture. Milan Babuška and his wife Marie are laid to rest in the family grave at the Vinohrady Cemetery in Prague.

We will outline Ivo Babuška's most significant life periods. After World War II, Ivo studied civil engineering at the Czech Technical University in Prague from where he graduated in 1949 and received his master (Ing.) degree. Next, he received his doctoral (Dr. tech.) degree in Technical Science in 1951. His supervisor was Professor František Faltus, a recognized expert in the design of steel structures. In the period 1949 to 1952, Ivo studied mathematics at the Central Mathematical Institute in Prague as a graduate student of Professor Vladimír Knichal and since 1951 he was a research fellow at that institute. After the reorganization of Czechoslovak science and university education he joined the Mathematical Institute of the Czechoslovak Academy of Sciences (now the Institute of Mathematics of the Czech Academy of Sciences) in Prague.

In 1955, Ivo Babuška received the PhD (CSc.) degree in Mathematics and in 1960 the research professor (DrSc.) degree which was in Czechoslovakia (as well as now in the Czech Republic) awarded only for the highest scientific achievements. From 1955 to 1968 he was the head of the Department of Constructive Methods of Mathematical Analysis of the Mathematical Institute.

Ivo Babuška belonged among the most influential mathematicians in the institute. He spent the first period of his work there as the leader of a numerical mathematics group that analyzed the technology of constructing the 91 meters high gravitational Orlík dam on the Vltava River in south Bohemia. The mathematical problem was to solve a nonlinear heat conduction partial differential equation describing the solidification of concrete. All the computations (about $3 \cdot 10^6$ arithmetic operations) were carried out by a team of people on mechanical desk calculators as no better devices were available in Czechoslovakia that time. An important byproduct was a number of doctoral theses in mathematics defended by Babuška's younger co-workers who formed the next generation of Czech numerical analysts.

In 1956, Ivo Babuška established this journal, Applications of Mathematics (formerly Aplikace matematiky) published in Prague by the Mathematical Institute. He was also one of the founders of the series of the international scientific meetings EQUADIFF. The first EQUADIFF Conference on Differential Equations was held in Prague in 1962 as an exceptional forum, where mathematicians both from the

East and the West could meet. The following conferences take place every four years alternately in Prague, Bratislava and Brno. Later this series of conferences merged with another European series bearing the same name.

Another important application project of Ivo Babuška was the mathematical and numerical modeling of relaxation of residual stresses in the course of thermal treatment of thick-walled steel cylinders in the sixties. Such cylinders were used as the outer shell of the first Czechoslovak nuclear reactor in the nuclear power plant Jaslovské Bohunice.

In 1968, after the August invasion by the Soviet army Ivo Babuška and his family hastened their departure for a planned one-year stay at the University of Maryland in College Park. Because his stay had been successful and fruitful, the university offered him a prolongation of the contract. The Czechoslovak authorities, however, refused to extend his stay abroad. Nevertheless, Ivo declined to return back, which was against the Czechoslovak law at that time. The turn of the political situation in Czechoslovakia and the "normalization" of the political life after 1969 caused that Ivo stayed in the United States ever since.

At the Institute for Physical Science and Technology and the Department of Mathematics of the University of Maryland, his interest in applied and numerical analysis brought Ivo Babuška to the finite element method. He has achieved numerous excellent results in the method itself, in its *hp*-version, in its reliability, a priori and a posteriori estimates, and adaptive approaches, which belong to the fundamentals of the method. In 1970, Ivo Babuška, James H. Bramble, Jim Douglas, Jr., and Bruce Kellogg founded the Finite Element Circus, an informal meeting, which for more than 30 years took place twice a year.

After leaving Prague, Ivo remained in scientific contact with his colleagues in Czechoslovakia. Since 1990 he could resume visiting Prague and he traveled there almost every year. In 1994, he established the Prize for Young Czech Scientists in the field of computational mechanics and mathematics that is funded by his own means and awarded annually.

Ivo Babuška retired at the University of Maryland in 1995 as a Distinguished University Professor Emeritus. The same year, he became a senior research scientist and Robert Trull Professor at the Institute for Computational Engineering and Sciences (now the Oden Institute) at the University of Texas at Austin, where he continued his fruitful scientific work for 23 years and retired only in 2018 at the age of 92.

Along with his other activities, he was involved in supervising several dozens of graduate students, see `www.mathgenealogy.org`. He was a member of editorial boards of numerous mathematical and engineering journals.

Ivo Babuška has received recognition and various awards for his scientific work. Let us name at least some of them: The Czechoslovak State Prize (1968); Alexan-

der von Humboldt Senior U.S. Scientist Award (1976); Gold Medal, Charles University, Prague (1992); George David Birkhoff Prize (1994); John von Neumann Medal, U.S. Association for Computational Mechanics (1995); Distinguished University Professor Emeritus, University of Maryland (1996); Honorary Member, Union of Czech Mathematicians and Physicists (1996); Bolzano Medal for Merit in Mathematical Sciences, Czech Academy of Sciences (1996); Fellow, U.S. Association for Computational Mechanics (1997); Honorary Foreign Member, Learned Society of the Czech Republic (1998); Fellow, International Association for Computational Mechanics (2002); Member, European Academy of Sciences (2003); Honorary editor, Numerische Mathematik (2003); Asteroid 36 060 was named Babuska (2003); Honorary editor, International Journal of Numerical Analysis and Modeling (2003); Honorary editor, International Journal of Computational Methods (2003); Member, National Academy of Engineering, U.S.A. (2005); Member, Academy of Medicine, Engineering and Science of Texas (2005); Medal De Scientia et Humanitate Optime Meritis, Czech Academy of Sciences (2005); Gauss–Newton Medal, International Association for Computational Mechanics (2006); Fellow, Society for Industrial and Applied Mathematics (2009); Leroy P. Steele Prize, American Mathematical Society (2012); and Neuron Fund Prize for Contribution to Science (2014). Ivo Babuška received honorary doctorates from five universities: University of Westminster (1994), Brunel University (1996), Charles University (1997), Helsinki University of Technology (2000), and Czech Technical University (2007). For a more detailed curricula vitae we refer to [11], [12], [13].

It is worth mentioning that three circumstances significantly contributed to Ivo Babuška's scientific achievements. First of all, it was his family background and the constant support from his beloved wife Renata, with whom he had a daughter and a son. His life was very difficult for him when Renata died in 2020. Second, it was his curiosity, which was not limited to mathematical and computational questions, but manifested itself in questions and conversations on all kinds of occasions. He did not forget his native country and asked visitors from the Czech Republic about the situation at home or about former colleagues in academia. However, it were the long and repeated conversations over professional topics that allowed him to sort out his thoughts and find a way to solution. And it was the environment of American universities, with frequent lectures delivered by visitors, that provided him with countless opportunities for debate, which he was happy to take advantage of. This, together with the much better availability of literature than in Czechoslovakia, formed the third supporting pillar of his achievements.

2. A short survey of Babuška's main mathematical results

Professor Babuška substantially influenced the theoretical development of numerical mathematics and computational mechanics. He is also famous for his theoretical results in the theory of partial differential equations. Together with Werner C. Rheinboldt, he founded the field of a posteriori error analysis and adaptive methods and played an essential role in their further development. Together with Jens Markus Melenk, he co-authored the partition of unity finite element method. His work with John E. Osborn on the convergence analysis for eigenvalue problems has become a fundamental and classical reference in the field. His further achievements include works on homogenization techniques and methods for stochastic partial differential equations. Important notions such as the Babuška paradox of a simply supported polygonal plate [1], p. 192, the Ladyzhenskaya-Babuška-Brezzi sufficient stability condition [2], [7], or the Babuška-Aziz theory [4] bear his name. Ideas by Ivo Babuška have influenced mathematics and its applications for more than half a century.

His 1958 paper on the numerical analysis of the construction technology of the Orlík dam formulated the basic principles of reliability of mathematical modeling and numerical calculations that are still valid. The essence of his understanding of the role of mathematical and numerical modeling is represented by the question "Will you sign the blueprint?" that he often asked during his lectures and that perfectly fits in the general concept of validation and verification that gained momentum in scientific and engineering computing after, say, 1990.

In 1966, Ivo Babuška together with Milan Práger and Emil Vitásek in their monograph [6] developed a special finite difference scheme for the equation

$$-(pu')' + qu = f \quad \text{in } (0,1)$$

with mixed boundary conditions and smooth data p, q and f. Using sophisticated numerical quadrature rules, they obtained $\mathcal{O}(h^6)$ accuracy at uniformly distributed nodal points when the discretization parameter h tends to zero. The associated system of linear algebraic equations has only a tridiagonal matrix like for linear finite elements whose accuracy is only $\mathcal{O}(h^2)$.

During the development of the finite element method, it has also been found that the rate of convergence of finite element approximations at some exceptional points in the domain exceeds the optimal global rate if finite element partitions have some regular geometric structure. This phenomenon has come to be known as *superconvergence*. Ivo Babuška with Theofanis Strouboulis and several other mathematicians wrote many papers on this remarkable topic.

Hermann A. Schwarz in Gesammelte Mathematische Abhandlungen (1890) investigated triangulations of smooth curved surfaces. He found that the area of a triangulated surface need not converge to the area of the original surface when no angle conditions are imposed. Later, various angle conditions were proposed. In the seventies of the last century numerical analysts believed that very small angles of triangular elements (i.e., when well-known Zlámal's minimum angle condition is not satisfied) could produce a large discretization error when solving second order elliptic problems by the finite element method. However, Ivo Babuška and Kadir Aziz in SIAM J. Numer. Anal. [4] proved the H^1-convergence under the so-called maximum angle condition. This condition is sufficient for convergence, but more general than Zlámal's condition. Note that up to now no necessary and sufficient condition for convergence is known.

Suitable refinements (adaptivity) of finite element partitions can often increase the accuracy of finite element approximations if they are done near those points, where singularities or oscillations of the solution (and its derivatives) occur. The theory of adaptive procedures in the finite element method for solving differential equations began with Babuška's pioneering paper *The selfadaptive approach in the finite element method* [3]. Currently, various error estimators for many problems of mathematical physics exist. They show where the refinements are actually needed to increase accuracy and minimize computational cost. Such an approach can then be used to construct multilevel sequences of finite element spaces, multilevel preconditioning, adaptive subspace selection, and so on.

Validation and verification (V&V) of problems of mathematical physics and their computer implementation is a very important part of numerical analysis. We always encounter two basic types of errors: modeling errors and numerical errors such as discretization errors, computational errors (iteration errors, algebraic errors, rounding errors) and also undiscovered programming bugs. Validation tries to estimate the modeling error and to answer the question:

Do we solve the correct equations?

On the other hand, verification tries to quantify the numerical errors and to answer the question:

Do we solve the equations correctly?

A joint paper with J. Tinsley Oden [5] on the V&V approach indicates that Ivo Babuška's interpretation of V&V also included an important aspect of modeling in engineering, namely the presence of uncertainty in input parameters. Indeed, a decade earlier Ivo Babuška contributed to the decision of Ivan Hlaváček [8], his former colleague from the Institute of Mathematics, to focus on a rigorous mathematical analysis of the worst-case scenario problems. Ivan Hlaváček's efforts mate-

rialized in a series of scientific papers and in a joint monograph with Ivo Babuška and Jan Chleboun [9]. In this deterministic approach, the goal is to identify, in a set of admissible data, the input data that cause an extreme response of the investigated model.

In practice, however, uncertainty in input data often has a stochastic character. Ivo Babuška addressed this feature in a series of scientific articles starting in 2001. Joint works with Fabio Nobile, Raul Tempone, and Georgios E. Zouraris present a transformation of stochastic boundary value problems to high-dimensional deterministic problems and belong to his eight most cited papers.

The zbMATH Open (formerly known as Zentralblatt für Mathematik) abstracting and reviewing service provides an excellent illustration of Ivo Babuška's visionariness in the field of homogenization, a subject whose goal is to find a limit of differential equations with coefficients oscillating with increased frequency. A search oriented on the homogenization of porous, layered, or periodic materials reveals that a handful of works were published in the early seventies or before. Among them, four papers by Ivo Babuška in 1976. Today, the zbMATH Open delivers more than 4000 items in response to the keyword *homogenization*.

3. Authors' personal stories about Ivo Babuška

Ivo Babuška was the Head of the Department of Constructive Methods of Mathematical Analysis at the Mathematical Institute of the Czechoslovak Academy of Sciences, where all of us worked or still work. As already mentioned above, Ivo could not visit Prague and conversely we and other colleagues from our country could not visit him until the Velvet Revolution in 1989. For this reason, we did not see him until 1990 when he visited Prague after long 22 years. As he always did, he asked us many questions. For example, when I give you USD 100, how fine a finite element mesh are you able to produce for me?

Another question concerned the function which is 0 at all rational points and 1 at all irrational points. Its Lebesgue integral over the interval $(0, 1)$ is equal to 1, whereas any numerical quadrature formula evaluated in a finite computer arithmetic yields the value 0. How is it possible?

To demonstrate that Prof. Babuška was asking questions all the time we would like to add one more story. In 1993, we organized a large international conference "Fifty Years of the Courant Element" at the University of Jyväskylä in Finland. We invited Prof. Babuška to give the first plenary lecture. During the conference Babuška asked everybody to fill out a special questionnaire prepared by him. One of his questions was:

Nobody refused to do this homework, even famous mathematicians such as J. Douglas, R. Durán, R. Glowinski, K. W. Morton, L. A. Oganesijan, R. Rannacher, V. Thomée, and J. R. Whiteman did answer. At the end of the conference Babuška did not forget to carefully evaluate all answers. It was also there that we started to discuss the Babuška Prize for young scientists which was sponsored by him later.

The following story nicely illustrates the very good physical condition of Prof. Ivo Babuška. This is, of course, only a necessary but not sufficient condition to produce really good science. In 1995, his office in the Department of Mathematics of the University of Maryland was on the fifth floor. Prof. Babuška often walked very quickly upstairs to his office and it was quite difficult to follow him. In any case, all his thirty-year old students used the elevator.

In January 2003, we invited Ivo Babuška to the international conference Mathematical and Computer Modeling in Science and Engineering held in Prague. We organized a very special present for him. At the opening of the conference it was announced that by the decision of the International Astronomical Union the asteroid No. 36 060 discovered by Dr. Petr Pravec in the Ondřejov Observatory of the Astronomical Institute of the Czech Academy of Sciences had received the name *Babuska*. Prof. Babuška was so surprised that he jumped up almost three feet high and was very happy. Then the trajectory of this asteroid in the Solar system was projected on the screen. Its Keplerian parameters are $a = 2.591$ au (semimajor axis), $e = 0.207$ (eccentricity), $i = 3.4^\circ$ (inclination), $\Omega = 198^\circ$ (longitude of the ascending node), and $\omega = 29^\circ$ (argument of perihelion). Later Prof. Babuška started to study astronomical literature on minor planets of the Solar system.

We had the privilege to visit Ivo Babuška at the University of Maryland and the University of Texas at Austin. Besides the mathematical training and top professional experience, he cared about our broader education. He insisted that we visit the Smithsonian National Museum of Natural History, the Smithsonian National Air and Space Museum, and also the Bullock Texas State History Museum to learn about the history of Texas including the immigration of Czech people in the 19th century. We still remember the natural, technical, and historical details, for example about the prehistoric creatures, the Apollo missions, or the fascinating history of the independent state of Texas.

4. Conclusions

Professor Ivo Babuška strongly influenced scientific development worldwide, particularly in his fatherland – the Czech Republic. His footsteps in computational mathematics and mechanics are visible not only to experts. He influenced the paths of many young scientists. According to MathSciNet (Mathematical Reviews) and zbMATH Open databases Ivo Babuška produced over 350 publications with more than 150 co-authors. Many of his papers have remarkable impact. His mostly cited paper with J. Melenk [10] has about 800 citations in MathSciNet[1] and was published in 2006 when Ivo was 70 years old. This fact nicely demonstrates his vitality, energy, and exceptional mathematical abilities.

Ivo Babuška deserves to have his memory as an outstanding researcher and university teacher duly preserved. He supervised 40 PhD students and made groundbreaking contributions to applied mathematics, numerical methods, and computational mechanics. We have lost an excellent mathematician, a kind colleague and a good man.

Please, join us in honoring his memory.

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References

[1] *I. Babuška*: Stability of the domain in relation to the main problems of the theory of partial differential equations, especially in connection with the elasticity theory. II. Czech. Math. J. *11* (1961), 165–203. (In Russian.) zbl MR doi

[2] *I. Babuška*: Error bounds for finite element method. Numer. Math. *16* (1971), 322–333. zbl MR doi

[3] *I. Babuška*: The selfadaptive approach in the finite element method. The Mathematics of Finite Elements and Applications II. Academic Press, London, 1976, pp. 125–142. MR

[4] *I. Babuška, K. Aziz*: On the angle condition in the finite element method. SIAM J. Numer. Anal. *13* (1976), 214–226. zbl MR doi

[5] *I. Babuška, J. T. Oden*: Verification and validation in computational engineering and science: Basic concepts. Comput. Methods Appl. Mech. Eng. *193* (2004), 4057–4066. zbl MR doi

[6] *I. Babuška, M. Práger, E. Vitásek*: Numerical Processes in Differential Equations. John Wiley & Sons, New York, 1966. zbl MR

[7] *F. Brezzi*: On the existence, uniqueness and approximation of saddle-point problems arising from Lagrangian multipliers. Rev. Franc. Automat. Inform. Rech. Operat. *8* (1974), 129–151. zbl MR doi

[8] *J. Chleboun, M. Křížek*: Ivan Hlaváček passed away. Appl. Math., Praha *63* (2018), 499–502. zbl MR doi

[9] *I. Hlaváček, J. Chleboun, I. Babuška*: Uncertain Input Data Problems and the Worst Scenario Method. North-Holland Series in Applied Mathematics and Mechanics 46. Elsevier, Amsterdam, 2004. zbl MR doi

[1] Web of Science shows 2378 citations of the paper [10] as of April 2022.

[10] *J. M. Melenk, I. Babuška*: The partition of unity finite element method: Basic theory and applications. Comput. Methods Appl. Mech. Eng. *139* (1996), 289–314. zbl MR doi
[11] *K. Segeth*: Professor Ivo Babuška is seventy. Czech. Math. J. *46* (1996), 351–367. zbl MR doi
[12] *K. Segeth*: Professor Ivo Babuška is eighty. Appl. Math., Praha *51* (2006), 89–92. zbl MR doi
[13] *K. Segeth*: Professor Ivo Babuška. Appl. Math., Praha *60* (2015), 469–472. zbl MR doi

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