

Toposym 4-B

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Some topological applications of a generalized Martin's Axiom

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R. Laver and J. Baumgartner have independently established the consistency of generalizations of variants of Martin's Axiom. We deduce several topological applications of Baumgartner's version. Details will appear elsewhere.

Definition. A partial order P is countably closed if every countable descending sequence has a lower bound. A subset of P is linked if any two members of it are compatible. P is \aleph_1 -linked if P is the union of \aleph_1 linked subsets. P is well-met if any two compatible elements have an inf. Baumgartner's Axiom is the assertion that for each countably closed \aleph_1 -linked well-met partial order P and each collection D of $< 2^{\aleph_1}$ dense subsets of P , there is a filter meeting each element of D . Theorem (Baumgartner). $\text{Con}(\text{ZF}) \rightarrow \text{Con}(\text{ZFC} + \text{BA} + \text{CH} + 2^{\aleph_1} > \aleph_2)$. Theorem. $\text{BA} + \text{CH} + 2^{\aleph_1} = \kappa^+ \rightarrow$ there is an L-space such that every uncountable subspace has weight κ . Theorem. $\text{BA} + \text{CH} \rightarrow \aleph\text{N-N}$ is not the union of $< 2^{\aleph_1}$ nowhere dense sets. Theorem. $\text{BA} + \text{CH} \rightarrow$ if $|X| < 2^{\aleph_1}$ and $\pi(X) \leq \aleph_1$ and X has caliber \aleph_1 , then X is separable. Theorem. $\text{Con}(\text{ZF}) \rightarrow \text{Con}(\text{ZFC} + \text{BA} + \text{CH} \cdot 2^{\aleph_1} > \aleph_2 +$ every normal space of character $\leq \aleph_1$ is \aleph_1 -collectionwise Hausdorff, but there is a normal space of character \aleph_1 which is not \aleph_2 -collectionwise Hausdorff.)