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# Some topological applications of a generalized Martin's Axiom

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R. Laver and J. Baumgartner have independently established the consistency of generalizations of variants of Martin's Axiom. We deduce several topological applications of Baumgartner's version. Details will appear elsewhere.
*Definition*. A partial order $P$ is *countably closed* if every countable descending sequence has a lower bound. A subset of $P$ is *linked* if any two members of it are compatible. $P$ is *$\aleph_1$-linked* if $P$ is the union of $\aleph_1$ linked subsets. $P$ is *well-met* if any two compatible elements have an inf. *Baumgartner's Axiom* is the assertion that for each countably closed $\aleph_1$-linked well-met partial order $P$ and each collection $D$ of $<2^{\aleph_1}$ dense subsets of $P$, there is a filter meeting each element of $D$. *Theorem* (Baumgartner). $\mathrm{Con(ZF)} \to \mathrm{Con}(\mathrm{ZFC} + \mathrm{BA} + \mathrm{CH} + 2^{\aleph_1} > \aleph_2)$. *Theorem*. $\mathrm{BA} + \mathrm{CH} + 2^{\aleph_1} = \kappa^+ \to$ there is an L-space such that every uncountable subspace has weight $\kappa$. *Theorem*. $\mathrm{BA} + \mathrm{CH} \to \beta N - N$ is not the union of $<2^{\aleph_1}$ nowhere dense sets. *Theorem*. $\mathrm{BA} + \mathrm{CH} \to$ if $|X| < 2^{\aleph_1}$ and $\pi(X) \le \aleph_1$ and $X$ has caliber $\aleph_1$, then $X$ is separable. *Theorem*. $\mathrm{Con(ZF)} \to \mathrm{Con}(\mathrm{ZFC} + \mathrm{BA} + \mathrm{CH} + 2^{\aleph_1} > \aleph_2 +$ every normal space of character $\le\aleph_1$ is $\aleph_1$-collectionwise Hausdorff, but there is a normal space of character $\aleph_1$ which is not $\aleph_2$-collectionwise Hausdorff.)