## Béla Szökefalvi-Nagy On some properties of the function class $H^{\infty}$ and applications to Hilbert space operators

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Béla Szőkefalvi-Nagy (Szeged):

On some properties of the function class  $H^{\infty}$ and applications to Hilbert space operators

Let  $H^{00}$  denote the Banach algebra of bounded analytic functions in the unit disc. An element u of  $H^{00}$  is called <u>inner</u> if  $|u(e^{it})| = 1$  almost everywhere on the unit circle. For any family of elements  $u_a \in H^{00}$  let  $\bigwedge_a u_a$  denote the largest common inner divisor if not all  $u_a$  equal 0, and let it be 0 if all  $u_a = 0$ .

A useful "arithmetic property" of H<sup>00</sup> is established by the following "Main Lemma":

Let  $u_{ik}$  be an  $n \times m$  matrix over  $H^{\infty}$ , where n and m may be finite or  $\infty$ ; and suppose  $\|u_{ik}\|_{\infty} \in M$ . Let w be an inner function. Then there exists a sequence of complex numbers  $x_1=1, x_2, x_3, \ldots$ , with  $|x_2| + |x_3| + \ldots$  as small as we wish, so that we have, for  $i=1,\ldots,n$ , m m

$$\sum_{k=1}^{m} u_{ik} x_{k} = h_{i} \cdot \bigwedge_{k=1}^{m} u_{ik}, \text{ with } h_{i} \in H^{\infty}, h_{i} \wedge w=1.$$

The proof uses the canonical factorization of functions in  $H^{00}$ , a lemma of M. Sherman (the special case n=1, m=2 of the above statement), function theoretic reasonings (Vitali-Montel), and the Baire category theorem for the space  $\ell^1$ .

The lemma is used to extend a result of E. Nordgren on the "quasi-equivalence" of finite matrices over H<sup>60</sup>, to their diagonalized form, to the case of semi-infinite or (at least partly) to the case of infinite matrices.

When applied to the "characteristic matrix function"  $\mathcal{O}$  of a contraction operator T on a Hilbert space H, such that  $T^{*n} \rightarrow 0$  as  $n \rightarrow \infty$ , and dim  $(I - TT^*)H = m < \infty$ , the results imply sort of similarity relations between T and its "Jordan model".

For detailed exposition, and references, see Béla Sz.-Nagy: Diagonalization of matrices ever H<sup>00</sup>, Acta Sci. Math., <u>38</u>(1976), 223-238.