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THE WEAK RADON - NIKODYM PROPERTY IN BANACH SPACES

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<u>Definition 1</u>. A Banach space X has the weak Radon-Nikodym property if every X-valued measure ν on a finite complete measure space (S,Σ,μ), which is μ-continuous and of σ-finite variation has a Pettis -integrable derivative $f: S \to X$.

<u>Definition 2</u>. A Banach space X is separably complementable if for every separable space $Y \subset X$ there exists a sparable $Z \subset X$ complemented in X and containing Y.

<u>Theorem 1</u>. The following statements concerning X are equivalent:

(i) X^* has the weak Radon-Nikodym property;

(ii) given any complete finite measure space (S,Σ,μ) and any weak* scalarly integrable function $f: S \to X^*$; then there exists a Pettis integrable function $g: S \to X^*$, which is weak* equivalent to f (i.e. for every $x \in X$ we have, $\langle x,f\rangle = \langle x,g\rangle$ μ-a.e.).

<u>Theorem 2</u>. If X is separably complementable, then the following statements concerning X are equivalent:

(i) X^* has the weak Radon-Nikodym property;

(ii) given any complete finite measure space (S,Σ,μ) and any weak* measurable function $f: S \to X^*$; then f is weak* equivalent to a weakly measurable function;

(iii) given any complete finite measure space (S,Σ,μ) and any weak* scalarly integrable function $f: S \to X^*$; then f is weak* equivalent to a Pettis integrable function;

(iv) X does not contain any isomorphic copy of l_1.
If X is separable then each function f from (ii) is weakly measurable and each f from (iii) is Pettis integrable.

Corollary 1. If X is a subspace of a weakly compactly generated space then X^* has the weak Radon-Nikodym property iff X does not contain any isomorphic copy of l_1.

Corollary 2. If X is separable, X^* is non-separable and X does not contain any isomorphic copy of l_1, then given any finite not purely atomic measure space (S,Σ,μ), there exists a Pettis integrable function $f: S \to X^*$, which is not weak* (and hence also weakly) equivalent to any strongly measurable function $g: S \to X^*$.

Corollary 3. If X is weak* ω_1-sequentially dense in X^{**} (i.e. $X^{**} = \bigcup_{\alpha<\omega_1} X_\alpha$, where $X_0=X$, X_α is the set of all the points from X^{**} which are X^*-limits of sequences from $\bigcup_{\beta<\alpha} X_\beta$ if α is non-limit, and, $X_\alpha = \bigcup_{\beta<\alpha} X_\beta$ if α is a limit ordinal), then X is weak* sequentially dense in X^{**}.

Theorem 3. Let X be such that given any measurable space (S,Σ) and any X^*-valued measure $\nu: \Sigma \to X^*$ of σ-finite variation, the range of ν is a norm separable set. Then, if each norm-separable subspace of X^* is a subspace of a weak* separable subspace of X^* possessing the weak Radon-Nikodym property, then X^* has the weak Radon-Nikodym property.

Definition 3. Let (S,Σ,μ) be a finite measure space and let Σ_0 be a sub-σ-algebra of Σ. If $f: S \to X$ is a Pettis integrable function, then a weakly measurable, with respect to Σ_0, function $E(f|\Sigma_0): S \to X$ is said to be a conditional expectation of f with respect to Σ_0 if and only if for every $F \in \Sigma_0$ the equality

$$\int_F E(f|\Sigma_0)d\mu = \int_F f d\mu$$

holds.

Theorem 4. Let (S,Σ,μ) be complete and let X be a Banach space possessing the weak Radon-Nikodym property. If $f: S \to X$ is a Pettis integrable function and Σ_0 is a sub-σ-algebra of Σ containing all the

μ-null sets, then f has a conditional expectation with respect to Σ_o iff $\nu|\Sigma_o$ is of σ-finite variation, where

$$\nu(E) = \int_E f d\mu \quad , \qquad E \in \Sigma \; .$$

If X has the Radon-Nikodym property then the assumption of the completeness of $\mu|\Sigma_o$ superfluous and, $E(f|\Sigma_o)$ can be taken to be strongly measurable.

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