Toposym 4-B

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Compactifications by adding a countable number of points

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COMPACTIFICATIONS BY ADDING A COUNTABLE NUMBER OF POINTS

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All spaces are assumed to be completely regular and T_1.

Let αX be a compactification of a space X. Then αX is called a countable-points compactification if the remainder $\alpha X - X$ consists of at most a countable number of points. Concerning this notion K. Morita posed the following problem in [1]: Characterize those spaces which have a countable-points compactification.

As he pointed out there, if a space X has a countable-points compactification then X must be necessarily Čech-complete and semicompact, and in case X is separable metrizable the converse is also true by a theorem of Zippin [4], to which K. Morita gave a proof in [3] based on his results on uniformities [2]. However even if X is Čech-complete semicompact metrizable X does not have a countable-points compactification in general as will be shown by an example below. Thus K. Morita suggested the author to find a necessary and sufficient condition for metrizable spaces to have such a compactification. Namely the purpose of this paper is to give an answer to his suggestion.

Here a space is called semicompact if it has a basis of open sets, each of which has a compact boundary. For any space X let $R(X)$ be the set of all points having no compact neighborhood.

Now our theorems are stated as follows.

<u>Theorem 1</u>. Let X be a Čech-complete semicompact space. If $R(X)$ is separable metrizable then X has a countable-points compactification.

<u>Theorem 2</u>. Let X be collectionwise normal and $R(X)$ paracompact. If X has a countable-points compactification then $R(X)$ is Lindelöf.

<u>Theorem 3</u>. A metrizable space X has a countable-points compactification iff X is Čech-complete semicompact and $R(X)$ is Lindelöf.

Proofs of these theorems need several lemmas and are essentially based on K. Morita's paper [3] above.

<u>Example</u>. Let S be the topological sum of an uncountable number of copies of the space of irrationals. Then S is Čech-complete semicompact metrizable. Since $R(S) = S$ is not Lindelöf, by Theorem 2 S

has no countable-points compactification.

Finally we present a further problem of K. Morita which he communicated to the auther:

Problem. Characterize a semicompact space X whose Freudenthal compactification γX itself is a countable-points compactification of X.

References

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[3] ________, On bicompactifications of semibicompact spaces, Sci. Rep. Tokyo Bunrika Daigaku, Sect. A 4 (1952), 200-207.

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