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CONCERNING THE TOPOLOGICAL PRODUCT OT TWO FRÉCHET SPACES

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Let (X,u) be a Hausdorff space. Denote u^* the following operator: $x \in u^*A$ if there are points $x_k \in A$ such that each neighborhood of x contains x_k for all but a finite number of k, i.e. if $\lim x_k = x$. Let $\{S_m\}$ be a <u>twofold sequence</u>, i.e. a sequence of sequences S_m of points of X. If S'_{m_i} is a subsequence of S_{m_i}, then we have <u>twofold subsequence</u> $\{S'_{m_i}\}$ of $\{S_m\}$. We define: $\{S_m\}$ converges to x_0 provided that $x_0 \in u \bigcup \dot{S}'_{m_i}$ for each subsequence $\{S'_{m_i}\}$ of $\{S_m\}$. Here $\dot{S}'_{m_i}$ denotes the set of all points of the sequence S'_{m_i}. A sequence $\{x_k\}$ is a <u>crosssequence</u> in $\{S_m\}$ provided that there is a subsequence $\{m_k\}$ of $\{m\}$ such that $x_k \in S_{m_k}$.

Classify all points in a Hausdorff space into three (not necessarily disjoint) classes. We define the point $x_0 \in X$ to be a <u>π point</u> provided that the following condition is fulfilled: if a twofold sequence $\{P_m\}$ converges to x_0, then there is a subsequence of $\{P_m\}$ each crosssequence in which converges to x_0. A point x_0 is called a <u>ρ point</u> if there is a twofold sequence $\{R_m\}$ converging to x_0 no crosssequence in which converges to x_0. A point x_0 is a <u>σ point</u> if there is a twofold sequence $\{S_m\}$ converging to x_0 in each subsequence of which there is a crosssequence converging to x_0 and another one containing no subsequence converging to x_0; moreover, if $\lim S_m = x_0$ for each m, then x_0 is called a <u>σ_1 point</u> and if $\lim S_m = x_m$ and $\lim x_m = x_0$ where x_m is one-to-one, then we have a <u>σ_2 point</u>.

Let a twofold sequence $\{S_m\}$ converge to x_0 in (X,u) and $\{T_m\}$ converge to y_0 in (Y,v). The points x_0 and y_0 are said to be <u>coupled</u> if the following statement holds: If a crosssequence in $\{S_m\}$

converges to x_0 , then the corresponding crosssequence in $\{T_m\}$ does not converge to y_0 and vice versa: If a crosssequence in $\{T_m\}$ converges to y_0 , then the corresponding crosssequence in $\{S_m\}$ does not converge to x_0 .

Theorem. Let (X,u) and (Y,v) be Hausdorff Fréchet non isolated spaces and let $(X \times Y,w)$ be their topological product. Then $(X \times Y,w^*)$ is a Fréchet space iff there is no ρ point either in X or in Y and there are neither $\sigma_1\sigma_2$ nor $\sigma_2\sigma_2$ coupled points.

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