ON $\mathfrak{m}$-ADIC SPACES

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S. Mrówka [4] generalizing the notion of dyadicity, introduced the class of $\mathfrak{m}$-adic spaces. Denoting by $A_{\mathfrak{m}}$ the one-point compactification of a discrete space of cardinality $\mathfrak{m}$, a T_2-space X is said to be $\mathfrak{m}$-adic if it is a continuous image of a suitable topological power of $A_{\mathfrak{m}}$. It is not difficult to prove that a space is dyadic iff it is $\aleph_0$-adic.

S. Mrówka also proposed the following generalization of $\mathfrak{m}$-adicity: let us denote by $W(\xi + 1)$ the order-topological space of the ordinal numbers $<\xi + 1$ for an ordinal ξ. A Hausdorff space which is a continuous image of some topological power of $W(\xi + 1)$ will be called a ξ-adic space. This class of spaces is wider than that of $\mathfrak{m}$-adic spaces; indeed $A_{\mathfrak{m}}$ is a continuous image of $W(\xi + 1)$, where ξ is any ordinal of cardinality $\mathfrak{m}$.

S. Mrówka raised the following question as an open problem: is it true that an $\mathfrak{m}$-adic space with character $\leqq \mathfrak{n}$ ($\mathfrak{n} \leqq \mathfrak{m}$) is necessarily $\mathfrak{n}$-adic? Our aim is to give an affirmative answer to this question; indeed, the following more general theorem holds:

Theorem 1. *The weight and the character of a ξ-adic space are equal.*

The method of the proof is very similar to a method of N. A. Shanin (the "calibers" [5]).

Definition. Let $\mathfrak{n}$ denote an infinite cardinality. A topological space X is said to have the *property $B(\mathfrak{n})$* if for any family $\{G_\alpha; \alpha \in A\}$, $|A| = \mathfrak{n}$, of non-empty open subsets of X a set $B \subset A$, $|B| = \mathfrak{n}$, and a point $p \in X$ can be selected such that each neighbourhood of p meets almost all sets G_β in the sense that

$$|\{\beta \in B; V \cap G_\beta = \emptyset\}| < \mathfrak{n}$$

for each neighbourhood V of p.

Our main tool for the investigation of ξ-adic spaces is the following theorem:

Theorem 2. *An arbitrary product of spaces with property $B(\mathfrak{n})$ has this property as well.*

The continuous image of a space with property $B(\mathfrak{n})$ also has this property and the spaces $W(\xi + 1)$ obviously have the property $B(\mathfrak{n})$, hence we have

Corollary. *If the space X is ξ-adic then X has the property $B(\mathfrak{n})$ for each infinite cardinality $\mathfrak{n}$.*

Using this Corollary and some other theorems of R. Engelking [4] and R. Marty [3] Theorem 1 can be proved.

Our Corollary implies also some related theorems for ξ-adic spaces. The following results are direct generalizations of two theorems of R. Engelking and A. Pelczynski [2].

Theorem 3. *If the Stone-Čech compactification of a Tychonoff space T is ξ-adic for an ordinal ξ, then T is pseudocompact.*

Theorem 4. *There is no infinite extremally disconnected ξ-adic Hausdorff space.*

To prove these two theorems it is enough to apply our Corollary to the case $\mathfrak{n} = \aleph_0$.

Using a different method, applying an argument due to Efimov [1] for a more general situation, we obtain

Theorem 5. *Let X be a ξ-adic space. If X has a dense set each point of which has a character $\leqq \mathfrak{n}$ and $|\xi| \leqq \mathfrak{n}$, then the weight of $X \leqq \mathfrak{n}$.*

Corollary. *If the Tychonoff space X has a ξ-adic compactification αX for some ordinal ξ, then the weight of αX does not exceed the weight of X.*

Problem. Has each metrizable space M an $\mathfrak{m}$-adic compactification? (By Theorem 5, if such an $\mathfrak{m}$ exists, then it can be chosen as the weight of the space M.)

A detailed paper with proofs will appear in Periodica Math. Hungarica.

References

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