ON MONOTONE DECOMPOSITIONS OF SMOOTH CONTINUA

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The notion of smoothness of fans, dendroids, and hereditarily unicoherent continua has been discussed in [1], [4], and [6], respectively. We shall define a class of continua, called smooth, which contains the class of smooth hereditarily unicoherent continua, and we shall discuss some of the basic properties of such continua.

A *continuum* is a compact, connected, metric space. A continuum is said to be *hereditarily unicoherent at the point p* provided that the intersection of any two subcontinua, each of which contains p, is connected. Clearly a continuum M is hereditarily unicoherent at p if and only if given any point x in M there exists a unique subcontinuum which is irreducible between p and x. If the continuum M is hereditarily unicoherent at p, and q is a point of M, then pq will denote the unique subcontinuum which is irreducible between p and q.

A continuum M is said to be *smooth at the point p* if M is hereditarily unicoherent at p, and for each convergent sequence of points a_n of M the condition $\lim a_n = a$ implies that the sequence of continua pa_n is convergent and $\mathrm{Lim}\, pa_n = pa$. The set of points at which a continuum M is smooth is called the *initial set* of M and is denoted by $I(M)$. If $I(M) \neq \emptyset$, then M is said to be *smooth*.

Theorem 1. *If M is a smooth continuum then* (i) *M is locally connected at each point of $I(M)$,* (ii) *M is a dendrite if and only if $I(M) = M$,* (iii) *M is unicoherent, and* (iv) *every indecomposable subcontinuum of M has void interior.*

Theorem 2. *If M is a smooth continuum then there exists a decomposition D of M* (*called the canonical decomposition*) *such that* (i) *D is upper semicontinuous,* (ii) *the elements of D are continua,* (iii) *the decomposition space of D is arcwise connected, and* (iv) *if E is a decomposition satisfying* (i), (ii), *and* (iii) *then D refines E. Moreover, the decomposition space of D is a smooth dendroid and each element of D has void interior.*

The decomposition of Theorem 2 is similar to the decomposition obtained for λ-dendroids in [2]; however, the canonical decomposition of a λ-dendroid may consist of a single element [3] while the canonical decomposition of a smooth continuum is never degenerate.

For a detailed discussion of these results including generalizations to compact Hausdorff continua, see [5].

References

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[4] *J. J. Charatonik and C. Eberhart:* On smooth dendroids. Fund. Math. *67* (1970), 297—322.

[5] *G. R. Gordh, Jr.:* On decompositions of smooth continua. Fund. Math. *75* (1972), 51—60.

[6] *L. Mohler:* A characterization of smoothness in dendroids. Fund. Math. *67* (1970), 369—376.

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