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EXTREMAL DISCONNECTEDNESS AND DYADICITY

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A topological space X is called extremally disconnected if for each open set U the closure $\overline{U}$ is open. The weight of X is denoted by wX. A system of binary open covers $\Gamma_\alpha = \{H_\alpha^0, H_\alpha^1\}$, $\alpha \in A$, $|A| = \tau$, is called dyadic, if the following properties are satisfied:

1) $H_\alpha^0 \cap H_\alpha^1 = \emptyset$ for every $\alpha \in A$;
2) if $\alpha_1, \ldots, \alpha_s \in A$ are distinct, then

$$H_{\alpha_1}^{i_1} \cap \ldots \cap H_{\alpha_s}^{i_s} = H_{\alpha_1,\ldots,\alpha_s}^{i_1,\ldots,i_s} \neq \emptyset .$$

A similar notion was considered by P. Alexandroff and V. Ponomarev [1]. Such a system is called dense (in the sense of V. Ponomarev [2]) if for every open $U \subset X$ there exists

$$H_{\alpha_1,\ldots,\alpha_s}^{i,\ldots,i_s} \subset U .$$

In the following $\mathrm{p}\mathscr{D}^\tau$ will denote the absolute of the Cantor space $\mathscr{D}^\tau$ of weight τ.

Theorem 1. *The absolute [2] (projective space [3]) $\mathrm{p}\mathscr{D}^\tau$ of the Cantor space $\mathscr{D}^\tau$ coincides with the (unique up to homeomorphism) extremally disconnected bicompact space possessing a dyadic dense system of covers of cardinality τ.*

We say that a cardinal number τ is admissible if $\tau^{\aleph_0} = \tau$.

Theorem 2 (see [4]). *The weight of an arbitrary infinite extremally disconnected bicompact space is an admissible cardinal number.*

This theorem is a "dual" of a theorem of Pierce (see [5]) on cardinality of complete Boolean algebras, however, my proof is topological and shorter.

Let $\chi(x, R)$ be the character of a point x in R, let $\mathrm{d}R$ be the density of R, and let $\log_2 \tau$ be the least cardinal number $\mathfrak{m}$ such that $\exp \mathfrak{m} = 2^{\mathfrak{m}} \geqq \tau$.

Theorem 3. *For an admissible cardinal number τ, we have*

1) $\chi(x, \mathrm{p}\mathscr{D}^\tau) = \tau$ *for each point* $x \in \mathrm{p}\mathscr{D}^\tau$;
2) $w(\mathrm{p}\mathscr{D}^\tau) = \tau$;
3) $\mathrm{d}(\mathrm{p}\mathscr{D}^\tau) = \log_2 \tau$;
4) $\mathrm{card}\,(\mathrm{p}\mathscr{D}^\tau) = \exp \tau$.

It is not known whether $p\mathscr{D}^\tau$ is topologically homogeneous for an admissible cardinal number τ, i.e. whether for each two points $x, y \in p\mathscr{D}^\tau$ there exists a homeomorphism of $p\mathscr{D}^\tau$ onto itself which maps x onto y. However, it is possible to show that $p\mathscr{D}^\tau$ is locally homogeneous, i.e. every two open-closed subsets of $p\mathscr{D}^\tau$ are homeomorphic.

Theorem 4 ([4]). *Every extremally disconnected bicompact space of the weight τ is topologically contained in $p\mathscr{D}^\tau$.*

Theorem 5. *If pR is the absolute of a compact metric space R, then*

$$pR = i_0(p\mathscr{D}^{\aleph_0}) \oplus i_1(\beta N) \oplus i_2 T_k$$

where βN is the Stone-Čech compactification of the space of natural numbers N, T_k is the discrete space of cardinality $\varkappa < \aleph_0$, $\oplus$ denotes the sum (disjoint union), $i_0, i_1, i_2 = 0$ or 1, and $0 . A = \emptyset$, $1 . A = A$. The sequence $i_0, i_1, i_2, \varkappa$ depends on R.

A bicompact space R is called co-absolute (in the sense of V. Ponomarev [2]) with dyadic spaces if there exists a dyadic bicompact space R_0 such that their absolutes coincide, i.e. $pR_0 = pR$.

Theorem 6. *The class of bicompact spaces which are co-absolute with the dyadic spaces contains the least class which is closed under topological products and multivalued irreducible mappings and contains arbitrary compactifications of separable metric spaces. This class is hereditary under formation of canonically closed sets.*

Theorem 7. *Every regular cardinal number is a caliber (in the sense of Šanin [6]) of any bicompact R which is co-absolute with a dyadic space and satisfies the condition $wR \leqq \exp \sup_{x \varepsilon R} \chi(x, R)$.*

References

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