

Toposym 2

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TYPES OF ULTRAFILTERS ON COUNTABLE SETS¹⁾

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Two ultrafilters x and y on countable sets X and Y are said to be of the same type if there exists a bijective mapping f of X onto Y such that $f[x] = y$ (or equivalently, $f^*x = y$ where x and y are considered as points of the Čech-Stone compactifications of X and Y , and f^* is the Stone-Čech extension of f). Let T be a set and τ be a mapping of the class of all ultrafilters on countable sets onto T such that $\tau x = \tau y$ iff x and y are of the same type.

Let N be the discrete space of the counting numbers. If $x \in \text{cl } X - X$ where X is a discrete countable subset of βN , then the intersections of the neighbourhoods of x with X form an ultrafilter on X which will be denoted by x_X . The type τx_X is called the type of x with respect to X ; τx_N is called the type of x , and the types with respect to subsets of $\beta N - N$ are called the relative types of x .

By [2] the producing relation Φ is defined to be the set of all $\langle t, t' \rangle \in T \times T$ such that $\tau x_N = t'$, $\tau x_X = t$ for some $x \in \beta N - N$ and $X \subset \beta N - N$. If $\langle t, t' \rangle \in \Phi$ then we say that “ t produces t' ” or “ t' is produced by t ”.

Theorem. A. *If $\langle t_1, t_2 \rangle \in \Phi$, $\langle t_2, t_3 \rangle \in \Phi$ then $\langle t_1, t_3 \rangle \in \Phi$.*

B. *No type is produced by itself, i.e. $\langle t, t \rangle \in \Phi$ for no t .*

C. *Any type is produced by at most \aleph_0 types.*

A is simple, B and C are rather profound (B was proved in [3], C in [2]). It should be remarked that B is equivalent to the following theorem on fixed points: No homeomorphism of βN into $\beta N - N$ has a fixed point.

Let us state two applications of C given in [2]: *a proof of nonhomogeneity of $\beta N - N$ without any use of the continuum hypothesis, and an example of a space X such that X^n is countably compact but X^{n+1} is not* (here n is an arbitrary counting number). A very simple proof of nonhomogeneity of $\beta N - N$ without the continuum hypothesis is based on B: if h is a homomorphism of $\beta N - N$ into itself and $hx = y$, then the sets of the relative types of x and y coincide. Thence, according to B, if the type of x is a relative type of y then $hx = y$ for no h .

The properties of $\langle T, \Phi \rangle$ and some related objects are developed in [4].

¹⁾ This is a part of an invited lecture which was cancelled because of a presumed overlap with Gillman's lecture.

References

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