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Operators generating p-stable measures on Banach spaces

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by

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Let E be a real Banach space and let $R_p(E)$, $0<p\leq 2$, be the set of all p-stable symmetric Radon measures on E . Then we investigate operators X from the dual E' into $L_p(\Omega, P)$, $P(\Omega) = 1$, for which the mapping $a \to \exp(-\|Xa\|^p)$ is the characteristic function of a Radon measure μ_X which necessarily belongs to $R_p(E)$. By $\Lambda_p(E', L_p)$ we denote the set of all those operators. This generalizes the concept of so-called γ-Radonifying operators in the case $p=2$.

<u>Theorem 1.</u> For each $\mu \in R_p(E)$ there exists an $X \in \Lambda_p(E', L_p)$ such that

$$\mu_X = \mu .$$

Given $X \in \Lambda_p(E', L_p)$, $0<r<p<2$, we define

$$\lambda_r(X) := \left\{ \int_E \|x\|^r d\mu_X(x) \right\}^{1/r}$$

and

$$l(X) := \lim_{t\to\infty} t\, \mu_X \left\{ \|x\| > t \right\}^{1/p} .$$

<u>Theorem 2.</u> $\Lambda_p(E', L_p)$ is a complete normed $(1 \leq r < p)$ resp. quasi-normed space w.r.t. λ_r . Also l defines a quasi-norm on $\Lambda_p(E', L_p)$.

<u>Corollary.</u> Given r, q with $0<r<q<p$, then there is a constant $c>0$ such that for all Banach spaces E and for all $\mu \in R_p(E)$ the estimation

$$\left\{\int_E \|x\|^q d\mu(x)\right\}^{1/q} \leq c \left\{\int_E \|x\|^r d\mu(x)\right\}^{1/r}$$

holds.

This corollary generalizes a result of Hoffmann-Jørgensen. The next theorem answeres the question in which cases $\Lambda_p(E',L_p)$ becomes complete w.r.t. 1 $(0<p<2)$.

Theorem 3. Let E be a Banach space. Then the following are equivalent:

(1) $\Lambda_p(E',L_p)$ is complete w.r.t. 1 .

(2) E is of stable type p .

(3) $\exists$ $c>0$ s.t. for all $\mu \in R_p(E)$ the following is valid:

$$\sup_{t>0} t^p \mu\{\|x\|>t\} \leq c \lim_{t\to\infty} t^p \mu\{\|x\|>t\} .$$

If we denote by Π_q , $0<q<\infty$, the ideal of q-absolutely summing operators the following holds:

Theorem 4. If $0<p<2$ and $0<q<\infty$ it follows

$$\Lambda_p(E',L_p) \subseteq \Pi_q(E',L_p) .$$

Now, one may ask in which cases there is equality between $\Lambda_p(E',L_p)$ and $\Pi_p(E',L_p)$. S.A. Chobanjan and V.I. Tasieladze proved in 1977 that this happens for $p=2$ if and only if E is of (stable) type 2 . In the case $0<p<2$ we get:

Theorem 5. If $0<p<2$ then the following are equivalent:

(1) $\Lambda_p(E',L_p) = \Pi_p(E',L_p)$.

(2) E is isomorphic to a subspace of some $L_p(\gamma)$ and is of stable type p .

This shows that in contrary to the case $p=2$ for $0<p<2$ the class of Banach spaces with $\Lambda_p(E',L_p) = \Pi_p(E',L_p)$ is far smaller.

Another difficulty arises. In general $X \in \Lambda_p(E', L_p)$ does not imply $AX \in \Lambda_p(E', L_p)$ for each operator A in L_p, $0 < p < 2$.

Thus, there is the following problem:

Characterize Banach spaces E for which

$$AX \in \Lambda_p(E', L_p)$$

whenever $X \in \Lambda_p(E', L_p)$ and A is an operator in L_p.

Remark. It is known that $L_q[0,1]$ has this property for each p if $1 \leq q \leq 2$ and it does not hold for any p if $2 < q \leq \infty$.

Finally we want to give some examples of $\Lambda_p(E', L_p)$.

Theorem 6. a) If $p < q < \infty$ and $1 < q < \infty$, then $X \in \Lambda_p(l_{q'}, L_p)$, $1/q' + 1/q = 1$ if and only if $(\sum_{i=1}^{\infty} |Xe_i|^q)^{1/q} \in$ $\in L_p$ where $\{e_i\}$ denotes the sequence of unit vectors in $l_{q'}$.

b) If $1 < q < p < 2$ then $X \in \Lambda_p(l_{q'}, L_p)$ if and only if $\sum_{i=1}^{\infty} (\int_\Omega |Xe_i|^p dP)^{q/p} < \infty$.

Remark. It is also possible to characterize $\Lambda_p(L_{q'}, L_p)$ in the case a) while the same is not known in the case b). Also the class $\Lambda_p(l_{p'}, L_p)$ is not described.

For further informations about the subject we refer to a forthcomming paper by the author (the same title) and a joint paper of the author with. V. Mandrekar and A. Weron to appear in Lecture Notes of Mathematics.