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STABLE PERIODIC OSCILLATIONS IN SYSTEMS WITH MONOTONOUS HYSTERESIS NONLINEARITY

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A wide range of journals and monographic literature is devoted to the study of forced periodic oscillations in systems which are described by different evolutional equations with functional non-linearity. Essentially less facts are known, about forced oscillations in systems with hysteresis nonlinearity. It is explained, in particular, by the difficulty of using standard model of hysteresis while describing all possible behaviours of such systems, since they are adapted, mainly, to define values of hysteresis nonlinearity under the individual inputs of sufficiently simple nature (for example, unimodal periodic).

In [1] an attempt was made to give an explanation of fundamental hysteresis nonlinearities (plays, stops, models of Prager and Ishlinski, continual systems of relay, models of Mizes and Tresca etc.).

In this explanation each hysteresis nonlinearity under consideration is defined to be a transformer with some $\mathbb{P}$ space of p state by a wide class possible inputs u(t), describing evolution lows of states by means of input-state operators

$$(1) \qquad p(t) = W[t_0, p(t_0)]u(t) \qquad (t \geq t_0)$$

and, at last, defining the value of output v(t) under the state p(t) by the state-output operator

$$(2) \qquad v(t) = \Phi[p(t)].$$

Then the hysteresis nonlinearity equations are written in the form of two equations

$$(3) \qquad z' = f(z, v, t) \qquad (z \in \mathbb{R}^N),$$

$$(4) \qquad v(t) = \Phi\{ W[t_0, p(t_0)]g[z(t)] \} \qquad (t \geq t_0)$$

where $u(t) = g[z(t)]$ is the rule (operator) of defining the value of u of the input into hysteresis nonlinearity by vector z.

The individual solution of system (3)-(4) is segregated by the initial conditions

$$(5) \qquad \begin{aligned} z(t_0) &= z_0 \in \mathbb{R}^N, \\ p(t_0) &= p_0 \in \mathbb{P}. \end{aligned}$$

The phase-space for system (3)-(5) is $\mathbb{R}^N \times \mathbb{P}$.

Almost complete information on operators properties (3)-(4) permits us to establish the solvability of problem (3)-(5), to find indicators of the existence of periodic solutions (both for autonomous and nonautonomous equations) to find conditions of the validity differential inequality theorems, to investigate the average principle possibility by Krylov-Bogolubov-Mitropolsky etc. The investigation of solutions stability is connected with overcoming of essential difficulties: unusualness of phase-space $\mathbb{R}^N \times \mathbb{P}$, nonsmoothness of operator (1) on the spaces of continuous functions and with the discontinuety of this operator on the spaces with integral metrices.

2. By the identification theorem [1] each determed and autonomous hysteresis nonlinearity with scalar inputs and outputs is a hysteron if the set of possible outputs forms a segment for each inputs value and if it has properties of controlness and correctness with respect to small amplitude noises. By [1] each hysteron may be considered as a cascade connection of functional factor and the play. Therefore system (3)-(4) with hysteron nonlinearity may be considered as a system with simple hysteresis nonlinearity, i.e.

a generalized play.

3. We consider a system important for control theory

$$(6) \qquad z' = Az + bf(x, v, t) \qquad (z, b \in R^N, x \in R^1),$$

$$(7) \qquad x = (z,c) \qquad (c \in R^N),$$

$$(8) \qquad v(t) = L[t_0, x(t_0)]x(t) \qquad (t \geq t_0),$$

where A – is a Hurwitz matrix, scalar function $f(x, v, t)$ $(x, v, t \in R^1)$ is continuous, satisfies the local Lipshitz conditions by x and v and is T-periodic in t, operator (8) is operator (1), corresponding to the generalized play with disjoint characteristics $\Gamma_r(x)$, $\Gamma_l(x)$ $(-\infty < x < +\infty)$.

Let us denote by $h(t)$ the impulse characteristic

$$(9) \qquad h(t) = (e^{At}b,c)$$

of the linear system

$$(10) \qquad z' = Az + bu(t), \qquad x = (z,c).$$

Theorem 1. Let system (10) be controlled and observed, function (9) be positive for $t \geq t_0$. Let the function $f(x, v, t)$ increase by arguments x and v and the following conditions

$$\overline{\lim_{|x|\to\infty}} \max_{0\leq t\leq T} |x^{-1}f(x, \Gamma_l(x), t)| < (A^{-1}b,c)$$

$$\overline{\lim_{|x|\to\infty}} \max_{0\leq t\leq T} |x^{-1}f(x, \Gamma_r(x), t)| < (A^{-1}b,c)$$

are satisfied. Then problem (6)-(8) has a T-periodic stable (in the sence of Lyapunov) solution.

Different effective conditions of impulse characteristic positivety can be found in the articles by Gyll, Privalsky, the authors and others.

4. The proof of the Theorem 1 is based on a special iteration procedure suggested in [2]. The possibility of using this method is determined by the existence of special cone in the phase-space and by the following property: the shift operator is strictly monotonous with respect to this cone. This statement is of independent interest.

Let us denote by $Q(s, t)$ the shift operator on the trajectories of system for the time s to t.

$$z' = Az + bg[(z,c), t] \qquad (z, b, c \in \mathbb{R}^N) \tag{11}$$

where A – is a Hurwitz matrix, the function $g(x, t)$ $(x, t \in \mathbb{R}^1)$ is continuous and satisfies the local Lipshitz conditions by argument x, and it is T-periodic in t.

Theorem 2. Let system (11) be controlled and observed, function (9) be positive. Let the function $g(x, t)$ increase by the argument x, and the following condition be satisfied

$$\overline{\lim_{|x|\to\infty}} \max_{0\le t\le T} |x^{-1}g(x, t)| < (A^{-1}b,c)$$

Then there exists a solid cone $\mathbb{K}$ such that the difference $Q(s, t)z_2 - Q(s, t)z_1$ is an internal point of the cone $\mathbb{K}$ if $z_2 - z_1 \in \mathbb{K}$, $z_1 \neq z_2$, $t > s$.

References

1. M.A.Krasnosel'skii, A.V.Pokrovskii, Systems with hysteresis. Springer-Verlag, 1989, 1-410.
2. M.A.Krasnosel'skii, A.V.Pokrovskii, Correct solutions of equations with discontinuous nonlinearities, Dokl. AN SSSR, 227 (1976), 547-550.