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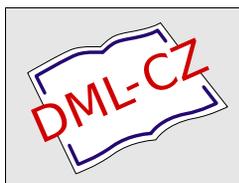
NUMERICAL ASPECTS OF THE IDENTIFICATION OF THERMAL CHARACTERISTICS USING THE HOT-WIRE METHOD

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Abstract

The hot-wire method, based on the recording of the temperature development in time in a testing sample, supplied by a probe with its own thermal source, is useful to evaluate the thermal conductivity of materials under extremal loads, in particular in refractory brickworks. The formulae in the technical standards come from the analytical solution of the non-stationary equation of heat conduction in cylindric (finally only polar) coordinates for a simplified formulation of boundary conditions, neglecting everything except the first terms of the decomposition of related exponential integrals to infinite series, and least-squares based data fitting; such approach reduces the validity of results and obstructs the simultaneous evaluation of heat capacity.

This paper demonstrates that substantial improvements can be obtained without any requirements to additional measurements, both i) under the assumption of a wire of zero-thickness and an infinite sample (following the valid Czech technical standard) with proper exponential integrals and ii) for a more realistic geometrical configuration and physical simplification (taking into account the thermal characteristics of the wire), based on the properties of Bessel functions. The suggested algorithms have been implemented in the MATLAB environment.

1. Introduction

Reliable evaluation of thermal characteristics of materials used in mechanical, civil, etc. engineering, including their dependence on temperature, moisture, strain and other fields, even for advanced materials, structures and technologies where no reasonable values from practical experience are available, determines the range of applications of computational modelling of all multi-physical processes. In particular, identification of thermal properties of refractory brickworks (discussed later in more details), of hardening cement pastes and concrete structures [14], as well as of foods stored in freezing and cooling plants [9], requires some simple methodology, applicable under hard conditions, with negligible disturbing effect of other physical processes.

For simplicity, let us restrict to the identification of two basic characteristics of heat conduction in engineering materials: the thermal conductivity λ [W/(m·K)]

(as a crucial thermal insulation characteristic) and the volumetric heat capacity κ [J/(m²·K)] (important for thermal accumulation); the thermal diffusivity α [m²/s] can be then introduced as $\alpha := \lambda/\kappa$. For the evaluation of λ, European technical standards offer the i) hot-plate, ii) hot-wire and iii) hot-ball approaches. The physical background of all these approaches is very similar: temperature (or temperature difference) is recorded in some (sufficiently small) range, whose development is forced by the carefully controlled generation of heat fluxes, during a (rather short) time interval. The principal difference consists in the geometrical configuration: in the case i) we have one or more parallel heating (or also additional non-heating) thin plates [11], in the case ii) a thin heating wire (see [1] and the following section) and in the case iii) a small heating ball (see [8]); the heat fluxes generated into the measurement system is controlled by direct voltage in all cases. The arrangement should be as simple as possible, with the aim to reduce the dimensions of corresponding heat transfer problems as much as possible; consequently (most frequently) working i) with Cartesian coordinates, ii) with cylindrical and iii) with spherical ones.

Our more detailed analysis will be devoted to the case ii). The relevant European standard [4] contains a (seemingly strange) explicit logarithmic formula for the evaluation of λ, supplied (for uncertain measurements) by the least-square (linear regression) approach to data fitting. However, as shown in [1], this formula can be identified with the fundamental solution of a heat conduction equation, satisfying the realistic boundary conditions in certain limit sense, well-known from [2], where in the additive decomposition of an exponential all terms except the first two are removed; this can be justified by the location of temperature sensors close to the heating wire. Such approach enables us to calculate (approximately) λ without the a priori knowledge of α; unfortunately, no information referring to κ is then available (because it was hidden in the removed terms containing α). We shall demonstrate that the proper analysis of the above sketched problems offers a possibility to identify both λ and κ from the same data set. Moreover, we shall show how some unpleasant physical and geometrical assumptions can be modified to be more realistic, using the properties of Bessel functions by [3] instead of the classical analytical results from [2].

2. Improved computations with exponential integrals

Following [4], let us assume that some constant heat Q [W/m], starting from the zero initial time, is generated per unit length of a very long and thin wire, located in the axis of the circular cylinder with a very large radius, occupied by the material specimen. Let $T(r,t)$ be the temperature field defined for any positive radius r (distance from the axis of rotation) and any positive time t (in practice for some measurement time interval) and T_0 the constant temperature of the surrounding environment. Then, by [1] (referring to [2]), using the notation $\beta_0 := Q/(4\pi\lambda)$, $\beta := 1/(4\alpha)$, we have

$$T = \beta_0 \operatorname{Ei}(\beta r^2/t) + T_0 \qquad \text{with} \ \operatorname{Ei}(.) := \int_.^\infty \frac{\exp(-u)}{u}\,\mathrm{d}u\,. \qquad (1)$$

Indeed, using dot symbols for partial derivatives with respect to t and prime symbols for those with respect to r, it is easy to verify that T from (1) satisfies the classical Fourier equation of heat conduction (without internal heat sources) with constant characteristics λ and κ in polar coordinates

$$\kappa \dot{T} + \frac{\lambda}{r}(rT')' = 0 \tag{2}$$

together with the obvious initial condition $T(.,0) = T_0$ and the with the couple of boundary conditions

$$\lim_{r \to \infty} T(r,.) = 0\,, \qquad \lim_{r \to 0+} \frac{-\lambda T'(r,.)}{Q/(2\pi r)} = 1 \tag{3}$$

where the first limit guarantees the absence of heat fluxes from external environment and both the numerator and the denominator in the second limit represent the heat flux [W/m^2] on the surface of cylinder with a fixed small radius (this is just the announced way how to avoid the realistic finite radius and material characteristics of a wire). Clearly the data for $t = 0$ (and also $t \to 0$ in practice), thanks to the discontinuity of heat generated into the system (forcing the application of Dirac measures and Heaviside functions in [2]), are then not employable in any credible identification procedure for λ and κ, in particular for λ and α from (1); for the special case of the simplified evaluation of λ this observation is reflected by [4], too.

Let us assume that all sensors recording the temperature are located at $r = \delta$ where distance δ must be very small positive number by [4] (the measurement could be performed as close as possible to the wire surface), but is allowed to be finite in our considerations. Let m be a number of measurement time steps; the initial time $t = 0$ is not included. Using the notation $t_1, \ldots, t_m$ $(0 < t_1 < \ldots < t_m)$ for discrete measurement times and T_s for corresponding temperature values at $r = \delta$. All differences $T_s - T_{s-1}$ with $s \in \{2, \ldots, m\}$ should correspond to the experimental temperature differences τ_s; for simplicity, only one recorded temperature value is considered in every discrete time; the generalization over all available data is obvious. Thus, using the notation $\beta_1 := \beta \delta^2$, we have to minimize a function

$$\Phi = (1/2) \sum_{s=2}^{m} (\tau_s - (T_s - T_{s-1}))^2 \tag{4}$$

of two positive variables β_0 and β_1 (transformed from λ and α easily) where, for simplicity, only one recorded temperature value is considered in every discrete time; the generalization over all available data is obvious.

Let $\Phi_{,i}$ and $\Phi_{,ij}$ denote the derivatives $\partial \Phi / \partial \beta_i$ and $\partial^2 \Phi / \partial \beta_i \partial \beta_j$ with $i, j \in \{0, 1\}$. For $\beta_{1s} := \mathrm{Ei}(\beta_1/t_s)$, $\widetilde{\beta}_{1s} = \exp(-\beta_1/t_s) - \exp(-\beta_1/t_{s-1})$ and $\varepsilon_s := \beta_0 \beta_{1s} - \tau_s$ with $s \in \{2, \ldots, m\}$ we receive explicit formulae (the MAPLE support is welcome)

$$\Phi = (1/2) \sum_{s=2}^{m} \varepsilon_s^2\,, \qquad \Phi_{,0} = \sum_{s=2}^{m} \varepsilon_s \beta_{1s}\,, \qquad \Phi_{,1} = -(\beta_0/\beta_1) \sum_{s=2}^{m} \varepsilon_s \widetilde{\beta}_{1s}\,,$$

$$\Phi_{,00} = \sum_{s=2}^{m} \beta_{1s}^2 \,, \qquad \Phi_{,01} = -(1/\beta_1) \sum_{s=2}^{m} (2\beta_0\beta_{1s} - \tau_s)\widetilde{\beta}_{1s} \,, \qquad \Phi_{,11} = (\beta_0/\beta_1)^2 \sum_{s=2}^{m} \widetilde{\beta}_{1s}^2$$
$$+ (\beta_0/\beta_1) \sum_{s=2}^{m} \varepsilon_s \left(\mathrm{Ei}\,(\beta_1/t_s)/t_s - \mathrm{Ei}\,(\beta_1/t_{s-1})/t_{s-1} \right) + (\beta_0/\beta_1^2) \sum_{s=2}^{m} \varepsilon_s \widetilde{\beta}_{1s} \,.$$

Clearly we need $\Phi_{,0} = \Phi_{,1} = 0$. Taking (for sufficiently small δ) $\beta_1 \approx 0$ together with $\mathrm{Ei}\,(.) \approx -C_e - \ln(.)$ (the Euler-Mascheroni constant C_e is not needed in numerical calculations), for $\gamma_s := \ln(t_s/t_{s-1})$ with $s \in \{2, \ldots, m\}$ we receive the very simple formula

$$\beta_0 \approx \sum_{s=2}^{m} \gamma_s \tau_s / \sum_{s=2}^{m} \gamma_s^2 \,, \tag{5}$$

which is identical with that for the identification of λ from [4]. More generally, we are allowed to choose β_0 from (5) as the first estimate together with

$$\beta_1 \approx \sum_{s=2}^{m} (1/t_s - 1/t_{s-1})(\tau_s/\beta_0 - \gamma_s) / \sum_{s=2}^{m} (1/t_s - 1/t_{s-1})^2$$

and apply the Newton iteration algorithm

$$\begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} \leftarrow \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} - \begin{bmatrix} \Phi_{,00} & \Phi_{,01} \\ \Phi_{,01} & \Phi_{,11} \end{bmatrix}^{-1} \begin{bmatrix} \Phi_{,0} \\ \Phi_{,1} \end{bmatrix} \,;$$

this enables us to determine (more exactly) both β_0 and β_1, consequently also λ and κ (even without evaluations of inverse matrices in the computational practice).

3. A generalized approach applying Bessel functions

The generalization of the above sketched approach, removing mathematical and physical simplifications, can be done in more directions. However, being motivated from the results of MATLAB supported practical calculations with data coming from experiments with fire-clay bricks at high temperatures, we shall try to replace rather artificial boundary conditions (3) by more realistic ones. Let a be the outer radius of a specimen and $\delta < a$ a wire radius. Following [5], let us introduce the brief notation for scalar products in the special Lebesgue weighted spaces

$$(\phi, \widetilde{\phi})_r = \int_0^a \phi(.) r \widetilde{\phi}(.) \,\mathrm{d}r \qquad \text{for all} \quad \phi, \widetilde{\phi} \in L_r^2(0, a) \,,$$

$$(\phi, \widetilde{\phi})_{r0} = \int_0^\delta \phi(.) r \widetilde{\phi}(.) \,\mathrm{d}r \qquad \text{for all} \quad \phi, \widetilde{\phi} \in L_r^2(0, \delta) \,,$$

$$(\phi, \widetilde{\phi})_{r1} = \int_\delta^a \phi(.) r \widetilde{\phi}(.) \,\mathrm{d}r \qquad \text{for all} \quad \phi, \widetilde{\phi} \in L_r^2(\delta, a) \,.$$

Material characteristics λ, κ, α will be taken as simple functions of r, with values equal to a priori known constants $\lambda_0, \kappa_0, \alpha_0$ for $0 \le r \le \delta$ and unknown ones $\lambda_1, \kappa_1, \alpha_1$ for $\delta \le r \le a$ (although their rather good estimates may be available by the previous section); moreover we shall need $\lambda_* := \lambda_1/\lambda_0$, $\kappa_* := \kappa_1/\kappa_0$ and $\alpha_* := \alpha_1/\alpha_0$.

Let $\mathcal{V}$ be the space of admissible test functions, i.e., applying the notation of special Sobolev weighted spaces from [5] again, the space of all $v \in W_r^{1,2}(0,a)$ such that $v(r) = v_0(r)$ for $0 \le r \le \delta$ and some $v_0 \in W_r^{1,2}(0,\delta)$, as well as $v(r) = v_1(r)$ for $\delta \le r \le a$ and some $v_1 \in W_r^{1,2}(\delta,a)$ satisfying $v_1(a) = 0$. Let $\mathcal{H}$ be the space introduced in the same way as $\mathcal{V}$ except L_r^2 inserted instead of $W_r^{1,2}$ everywhere. Using such notation, we are able to convert (2) into the form

$$(v, \kappa \dot{T})_r = (v, \lambda(rT')'/r)_r + (v, g)_r \qquad (6)$$

where $g := Q/(\pi\delta^2)$ for $0 \le r \le \delta$ (any better information on the distribution of g in a wire is usually missing), zero otherwise. For positive times t we have to find $T(.,t) - T_0$ from $\mathcal{V}$ with $\dot{T}(.,t)$ from $\mathcal{H}$.

Let us consider the decomposition $T(r,t) = T_\sigma(r) + \theta(r,t)$ where

$$T(r,t) = T_\sigma(r) + \theta(r,t) \qquad \text{with} \ \ \theta(r,t) = \sum_{i=1}^{\infty} \varphi_i(r)\psi_i(t) \, ; \qquad (7)$$

the corresponding initial conditions are $T(.,0) = T_0$ and $\theta(.,0) = T_0 - T_\sigma(.)$ and the boundary (including the internal interface) ones are

$$\begin{aligned}
T'(0,.) &= 0 \, , & \lambda_0 T'(\delta_-,.) &= \lambda_1 T'(\delta_+,.) \, , & T(a,.) &= 0 \, , \\
\theta'(0,.) &= 0 \, , & \lambda_0 \theta'(\delta_-,0) &= \lambda_1 \theta'(\delta_+,0) \, , & \theta(a,.) &= 0 \, , \\
& & \lambda_0 T_\sigma'(\delta_-) &= \lambda_1 T_\sigma'(\delta_+) \, , & T_\sigma(a) &= T_0 \, .
\end{aligned} \qquad (8)$$

(δ_+ and δ_- refer to left and right limits for r tending to δ). Here T_σ can be derived as an analytical solution for the stationary case (with zero κ formally)

$$T_\sigma(r) = \begin{cases} Q/(2\pi\lambda_1)\ln(a/\delta) + Q/(4\pi\lambda_0)(1 - (r/\delta)^2) & \text{for } 0 \le r \le \delta \, , \\ Q/(2\pi\lambda_1)\ln(a/r) & \text{for } \delta \le r \le a \, . \end{cases} \qquad (9)$$

Utilizing the properties of Bessel functions

$$J_n(r) = \frac{1}{\pi} \int_0^\pi \cos(r\sin\xi - n\xi) \, \mathrm{d}\xi \qquad \text{with} \ \ n \in \{0,1,2,\dots\} \, ,$$

namely $J_0'(r) = -J_1(r)$, $J_1'(r) = J_0(r) - J_1(r)/r$, etc., by [3], we can see that

$$r^{-1}(rJ_0'(\omega r))' + \omega^2 J_0(\omega r) = 0 \qquad (10)$$

for any real ω, it is natural to find the zero points of Bessel functions, i.e. to solve $J_0(\omega_i a/\sqrt{\alpha_*}) = 0$ for unknown parameters ω_i with $i \in \{1,2,\dots\}$, and to choose

$$\varphi_i(r) = \begin{cases} \beta_i J_0(\gamma_i \omega_i r) & \text{for } 0 < r < \delta \, , \\ J_0(\omega_i r/\sqrt{\alpha_*}) & \text{for } \delta < r < a \, , \end{cases} \qquad (11)$$

to satisfy boundary conditions $\varphi_i'(0) = 0$, $\varphi_i(a) = 0$ automatically and

$$\varphi_i(\delta_-) = \varphi_i(\delta_+) \, , \qquad \lambda(\delta_-)\varphi_i'(\delta_-) = \lambda(\delta_+)\varphi_i'(\delta_+) \qquad (12)$$

for a priori unknown values of β_i and γ_i, coming from the auxiliary systems of two nonlinear equations

$$\beta_i J_0(\gamma_i \omega_i \delta) = J_0(\omega_i \delta / \sqrt{\alpha_*}) \,, \qquad \beta_i \gamma_i J_1(\gamma_i \omega_i \delta) = (\lambda_* / \sqrt{\alpha_*}) J_1(\omega_i \delta / \sqrt{\alpha_*}) \,. \qquad (13)$$

It is easy to see that β_i can be evaluated from (13) as a function of γ_i directly. Consequently (13) degenerates to just one nonlinear equation for the evaluation of γ_i; all technical details for the Newton iterative algorithm can be found in [13].

Inserting (11) and (7) into (6), for any $v \in \mathcal{V}$ we receive

$$[(v, \varphi_i)_{r0} + \kappa_*(v, \varphi_i)_{r1}]\dot{\psi}_i - \alpha_0[(v, (r\varphi_i')'/r)_{r0} + \lambda_*(v, (r\varphi_i')'/r)_{r1}]\psi_i = 0 \,. \qquad (14)$$

Taking (10) into account, (14) gets tho form

$$[(v, \varphi_i)_{r0} + \kappa_*(v, \varphi_i)_{r1}]\dot{\psi}_i + \alpha_0 \omega_i^2 [\gamma_i^2(v, \varphi_i)_{r0} + \kappa_*(v, \varphi_i)_{r1}]\psi_i = 0 \,. \qquad (15)$$

Simultaneously, applying the Green-Ostrogradskiĭ theorem, (14) yields

$$\begin{aligned}[(v, \varphi_i)_{r0} + \kappa_*(v, r\varphi_i)_{r1}]\dot{\psi}_i + \alpha_0[(v', \varphi_i')_{r0} + \lambda_*(v', \varphi_i')_{r1}]\psi_i \\ = \alpha_0[(v(\delta_-)\varphi_i'(\delta_-) - \lambda_* v(\delta_+)\varphi_i'(\delta_+)] \,.\end{aligned} \qquad (16)$$

In particular for $v = \varphi_j$ with arbitrary $j \in \{1, 2, \ldots\}$, comparing (15) and (16), we have

$$(\varphi_j', \varphi_i')_{r0} + \lambda_*(\varphi_j', \varphi_i')_{r1} = \omega_i^2[\gamma_i^2(\varphi_j, \varphi_i)_{r0} + \kappa_*(\varphi_j, \varphi_i)_{r1}] \,.$$

The mutual exchange of indices i and j then results certain quasi-orthogonality condition

$$(\omega_i^2 - \omega_j^2)\kappa_*(\varphi_i, \varphi_j)_{r0} + (\omega_i^2\gamma_i^2 - \omega_j^2\gamma_j^2)(\varphi_i, \varphi_j)_{r1} = 0 \,;$$

in practice $\gamma_i^2 \approx \gamma_j^2 \approx \kappa_*$ can be considered.

To find all ψ_i contained in (7), we must solve an eigenproblem $M_{ji}\dot{\psi}_i + K_{ji}\psi_i = 0$ for $M_{ji} := (\varphi_j, \varphi_i)_{r0} + \kappa(\varphi_j, \varphi_i)_{r1}$, $K_{ji} := \alpha_0\omega_i^2[(\varphi_j, \varphi_i)_{r0} + \kappa(\varphi_j, \varphi_i)_{r1})]$ and for the decomposition $\psi_i = V_{ip}\exp(-\Lambda_p t)C_p$, using the Einstein summation rule for all indices $i, j, p \in \{1, 2, \ldots\}$; Λ_p here are eigenvalues, $V_{i1}, V_{i2}, \ldots$ eigenvectors (in the matrix form we could write $MV\Lambda = KV$ only) and C_p unknown parameters, needed to be set due to our initial condition. The resulting formulae (assuming $i \neq j$) for effective numerical evaluation (obtained with the support of MAPLE) for the effective evaluation of D_{ji}, M_{ji} and K_{ji}, separately for diagonal and non-diagonal terms, can be found in [13]. The evaluation of constants C_p then comes from the equation

$$(v, T_0 - T_\sigma)_{r0} + \kappa_*(v, T_0 - T_\sigma)_{r1} = [(v, \varphi_i)_{r0} + \kappa_*(v, \varphi_i)_{r1}]V_{ip}C_p \,,$$

i. e. $F = MVC$, consequently $C = (MV)^{-1}F$, where most parts of integrals F_j with $j \in \{1, 2, \ldots\}$, coming from (9), as presented in all details in [13], can be evaluated analytically, thanks to the properties of Bessel functions J_0 J_1, J_2 and J_3.

Our final aim is, exploiting the same data as in the preceding section, to minimize a function Φ from (4) of two positive variables λ_* and κ_* (transformed from λ_1 and α_1). Clearly a (sufficiently large) finite number of Bessel functions is considered in (7) in numerical calculations, thus all matrices M and K, vectors F, etc. are finite. However, it is not so easy to perform the minimization procedure because no simple explicit formulae employable in the Newton iterations are available, thus numerical evaluations of approximate first and second derivatives of Φ are necessary. Fortunately, this can be done e.g. with the support of selected functions from the MATLAB optimization toolbox.

4. Applications, conclusions and generalizations

In addition to the i) simplified approach recommended by [4], both algorithms presented in ii) Section 2 and iii) Section 3 of this paper have been implemented in MATLAB environment as the support of measurement tools in the Laboratory of Building Physics at Brno University of Technology. The limited extent of this paper does not allow to present results of practical calculations; the reader can find corresponding figures and graphs, together with more detailed description (and photo) of the original hot-wire measurement equipment in [10], devoted to the material design for the high-temperature thermal accumulator, as one part of the large Swedish-Czech research project of the efficient exploitation of solar energy using optical fibers.

Up to now, the computational results under hard conditions (far from room temperatures) demonstrate that i) gives only the rough estimate of λ, but no reasonable value of κ at all, whereas ii) is able to improve this estimate substantially. The system error of ii), coming from the neglected size and heat capacity of a hot wire, can be removed by iii) effectively, but making use of much more numerical computations. Nevertheless, other disturbing effects, coming from thermal convection and radiation, namely from the heat transfer at the wire / specimen interface, as well as those connected with the more complicated real geometrical conditions, cannot be handled in this way. More general formulations of heat transfer (together with other physical, chemical, etc. processes) need extensive applications of finite element, volume or difference methods, accompanied by the proper uncertainty analysis, as that based on Sobol sensitivity indices and Monte Carlo stochastic simulations like [7], or that substituting the Lebesgue measure by some probabilistic one, directed to stochastic finite element, etc. approaches, like [15]. Consequently Φ the optimization problem of the type (4) is not a function of two (or finite, for the best low) number of positive parameters, but a rather general functional in some space of abstract functions; some results and open questions of such analysis, containing direct, sensitivity and adjoint problems, have been presented in [12].

Acknowledgements

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