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ON THE POSSIBILITIES OF COMPUTATIONAL MODELLING OF INTERACTION OF A STRUCTURE WITH SUBSOIL

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Abstract: The following possibilities of reduction of dimension in the computational analysis of strain and stresses transferred to the subsoil massive are available: i) coming from the effective subsoil model by Kolář & Němec (1989), based on the assumptions of the Pasternak's model (1954), where the pair of material parameters of a surface model is evaluated from the energy equivalence, ii) reducing a large sparse matrix of soil massive stiffness to a smaller one, using Schur's complement technique. In both cases i), ii) the steady-state analysis is decisive: inclusion of more complicated combination of loads can be performed without repeated computations.

Keywords: structure-soil interaction, computational modelling, finite element method

MSC: 74M15, 74S05, 74L10

1. Introduction

Evaluation of the soil-rock mass interaction represents a key element in geotechnical engineering, which deals with analysing geological conditions, soil composition, and the physical properties of the subsoil. This information constitutes essential input data for numerical modelling, allowing the simulation of soil-rock mass behaviour under various conditions. When modelling soil-rock masses, it is important to take various factors into account, such as rock types, soil composition, groundwater levels, and other geotechnical parameters. This information enables us to create a realistic representation of the subsoil, and its response to external influences, such as loading from building structures.

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Numerical modelling of the subsoil is essential for the proper design of building structures, especially of their foundations. In the early years of numerical modelling, the subsoil and the structure were treated separately, with no mutual influence, due to the computational complexity and the division between two design teams (geotechnics and structural engineering). This approach worked for simple structures and subsoil conditions (the soil environment under the foundations). For more complex structures, such as dams, high-rise buildings, tunnels, or large underground constructions, a model of the subsoil in interaction with the superstructure is needed, including deformations, tilts and stability considerations, to support the adaptation of foundation design to specific conditions of the given area and the particular structure. However, such advanced modelling techniques require higher computational power and more detailed input data, which must be gathered through geological surveys. Therefore, an essential engineering requirement is to consider the complexity of numerical modelling for various classes of structures reliably and economically.

After this brief motivation (1st section), we shall demonstrate the possibility of modelling the soil-rock mass in interaction with the structure, starting with the physical and mathematical background, including some historical remarks (2nd section). Then the computational design and software implementation (3rd section) is presented, supplied by an illustrative example (4th section) and followed by the sketch of possible generalizations, related to the research priorities for the near future (5th section).

2. Physical and mathematical background

Numerous theories for the modelling of a structure together with its subsoil can be classified by their characterization of subsoil properties and their approach to structure-subsoil interaction. Unlike simple (semi-)analytical historical formulae, such theories can handle viscoelastic and / or viscoplastic behaviour including damage to both a structure and its subsoil due to the class of rather general constitutive models, as presented by [22] and [33] and implemented into the RFEM software package (developed in collaboration with FEM consulting Brno with Dlubal Software Tiefenbach). In this short paper, we shall pay attention to the effective subsoil incorporation into the design of structures. The (quasi-)static approach will be preferred for simplicity; for its modification required by dynamic calculations see [29], for the extensive review of traditional and advanced computational techniques cf. [12].

2.1. Classical theories

The classical analytical models can be derived from the Boussinesq's theory [4], which focuses on the behaviour of subsoil under a single isolated force. A homogeneous isotropic subsoil which is defined by two key parameters: Young's modulus of elasticity E [Pa] and Poisson's ratio ν [–]. Consequently a full 3-dimensional model in the Cartesian coordinate system (x, y, z) can be formulated, using displacements (non-zero in general, related to the initial configuration) $u(x,y,z)$, $v(x,y,z)$, $w(x,y,z)$. Its later modification, well-known as the Westergaard's theory [36], focuses on the non-uniform distribution of pressure on the foundation surface, adding corrections for wider foundations; the zero-valued u and v are considered, for more details in the modern formulation see [8]. Another significant contribution is the Mindlin's theory [21] where a closed-form solution for the displacement field caused by horizontal and vertical forces acting at any point in an elastic half-space can be found.

Biot's theory [3] describes the interactions in a porous elastic medium filled with an incompressible fluid; so-called Terzaghi-Wegmann's model [32] can be seen as an application of this theory in engineering practice, namely for the analysis of the influence of foundation geometry on subsoil stresses. Skempton's model [31] is frequently employed for analyzing subsoil deformations induced by shrinkage. Seed-Idriss's theory [30] contributes to the analysis of the behaviour of a cohesive subsoil under dynamic loading, namely in seismically active areas. Vesić's theory [35] describes the behaviour of soft subsoil under a foundation: the pore pressure within the subsoil is regarded as the combined effect of foundation-induced stress and hydrostatic pressure. Janbu-Meyerhof's approach [14] addresses the analysis of slope stability, accounting for the influence of the foundation on a plastic subsoil.

2.2. Winkler's and Pasternak's models

Simple (but still frequently used) Winkler's subsoil model [37] needs only one parameter C_1 [N/m^3], the vertical modulus of compressibility (coefficient of support). Since the displacements u and v are supposed to be negligible in comparison to w, we can take $w(x,y,z) = \widetilde{w}(x,y)\psi(z)$, ψ is a prescribed function. The stress p under a foundation structure (and also the subsoil reaction) can be expressed as $p(x,y) = C_1\widetilde{w}(x,y)$. The disadvantage of this model is the omission of shear stresses, which can lead to a sudden change in deformation immediately at the edge of the foundation structure where the deformation is zero, see Fig. 1, part A).

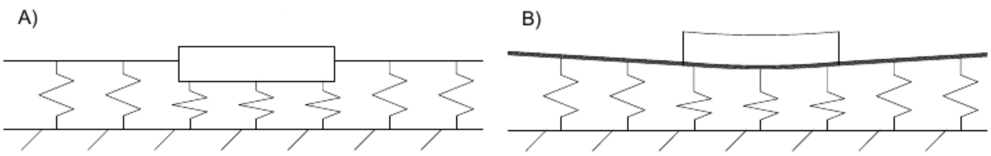

Figure 1: Subsoil models: A) 1 parameter by Winkler, B) 2 parameters by Pasternak.

Later studies try to suppress such a disadvantage, cf. [11]. In Pasternak's model [24], Winkler's model is extended by the parameter C_2 [N/m], which takes the effect of both normal and shear stresses into account, i. e. (under the assumption of isotropic behaviour of a subsoil, for simplicity here) $p(x,y) = C_1\widetilde{w}(x,y) - C_2\Delta\widetilde{w}(x,y)$, utilizing the Laplace operator $\Delta = \partial^2/\partial x^2 + \partial^2/\partial y^2$; cf. Fig. 1, part B). A more general class of foundation models of this type, involving additional parameters typically is introduced in [16]; for their detailed classification and numerous historical remarks cf. [19], [10] and [34].

2.3. Full 3-dimensional models

The 3-dimensional modelling requires the computational analysis of boundary value problems for partial differential equations, for a building structure and its subsoil separately, rarely reducible to those well-known from linear elasticity, together with a detailed analysis of all corresponding interfaces, thus rare (semi-)analytical solutions are available and an appropriate numerical approach is needed, based on the finite element techniques by [2] typically, as [22], [23] and [15]; another approach [1] relies on the boundary element method and certain integrals transforms.

Such approaches enable us to perform various specialized studies, such as stability analysis, permeability analysis of the subsoil, slope stability analysis, and seismic behaviour and response analysis of the subsoil. The proper analysis of complex shapes of subsoil layers and intricate structures, as well as their mutual interactions, can be seen as the principal advantage of this approach. Nevertheless, its evident disadvantages must be mentioned, too: at least i) rather high hardware and software requirements, ii) a tricky choice of the appropriate size of subsoil area, iii strong dependency of the reliability of all results on the correct choice of parameters and model validation, which must be provided by the user everywhere for ii), cf. [20]. In particular, in [17] the subsoil model is extended to a distance of approximately 115 m from the structure.

From the point of view of ii), the linear elastic model can be appreciated as simple and effective, allowing easy simulation of soil deformations under low stresses. Beyond this simplification, higher stresses lead to irreversible plastic (or viscous, etc.) deformations, as evaluable from historical Mohr-Coulomb's model [6], upgraded by [25], from later Drucker-Prager's [9] or Hoek-Brown's [13] ones, or from those developed especially from the soil analysis, referenced as Cam-Clay [26], working with a relation between stress, strain, and porosity, and Hardening Soil [27] for cyclic loading.

3. Computational approach

The subsoil can be modelled using the computational approaches mentioned in the 2nd section. The implementation into RFEM software makes it possible to perform a wide range of mechanical analyses, simplifying the analysis of the subsoil and its interaction with the structure. This is especially important in such tasks where only some specific loading cases influence the subsoil significantly.

3.1. Stress in the subsoil

In the analysis of stress within the subsoil, various types of stress arising from loading and site conditions can play a crucial role. The vertical (normal) stress is determined using the following formula $\sigma_z = \gamma h$ where h is the depth of the layer in the soil and γ is the unit weight of the soil. The horizontal (lateral) stress can be expressed as $\sigma_x = K_b \sigma_z$ where the lateral dimensionless pressure coefficient K_b depends on the type of soil: i) for cohesive soils it is considered as $K_b = \nu/(1-\nu)$

(cf. Subsection 2.1), whereas ii) for granular (cohesion-free) soils the relation $K_b = 1 - \sin\phi$ is used, ϕ is the angle of internal friction. In the case that the groundwater level occurs, the total stress is usually expressed as $\sigma_{\mathrm{tot}} = \sigma_{\mathrm{eff}} + u$ where the pore pressure $u = \gamma_u$h depends on one additional constant γ_u and the effective stress $\sigma_{\mathrm{eff}} = \gamma_{\mathrm{su}} h$, γ_{su} is the unit weight of dry soil.

Namely the stress at a depth z in an elastic, homogeneous, and isotropic soil caused by a single point load P with $\nu = 0$ can be determined by Boussinesq as

$$\sigma_z = \frac{3P}{2\pi z^2 \left(1 + (r/z)^2\right)^{5/2}}\,.$$

By Westergaard, infinitely thin soil layers are assumed, together with $0 \leq \nu < 1$, which results

$$\sigma_z = \frac{P(1-2\nu)(2-2\nu)}{2\pi z^2 \left((1-2\nu)/(2-\nu) + (r/z)^2\right)^{3/2}}\,.$$

The depth of the deformation zone is defined according to the technical standards CSN EN 1997-1 (731000) and Eurocode 7: Design of Geotechnical Structures – Part 1: General Rules, obligatory in the Czech Republic. These methods determine the depth below the foundation where the substantial increase in vertical stress occurs. The first method is the primary stress limitation method, which is expressed by the formula $\sigma_z = p\sigma_{\mathrm{or}}$ where σ_{or} represents the original geostatic stress and p is its considered percentage. The second method refers to the structural strength theory with the (formally similar) result $\sigma_z = m\sigma_{\mathrm{or}}$, m is the structural strength coefficient. The stress distributions and the deformation depths for both methods are illustrated by Fig. 2.

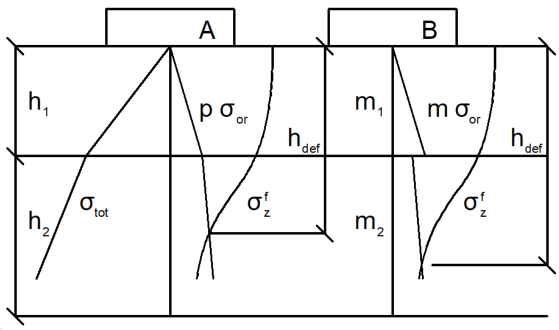

Figure 2: Evaluation of the deformation depth.

3.2. Effective subsoil approach

The approach working with dimension reduction comes from Pasternak's model, allows us to develop a relatively accurate model of the structure-subsoil interaction using only the foundation plane and its boundaries. However, an iterative evaluation of the deformation depth (zone) is necessary. All needed relations between the parameters of the surface and the spacial model for individual layers are compatible with [19]. We receive (as introduced in Subsection 2.2)

$$C_1 = \int_0^h E_z \left(\frac{\partial f(z)}{\partial z}\right)^2 \mathrm{d}z, \qquad C_{2x} = \int_0^h G_{xz} f^2(z)\,\mathrm{d}z, \qquad C_{2y} = \int_0^h G_{yz} f^2(z)\,\mathrm{d}z$$

for certain function $f(z)$; E_z is the deformation modulus, G_{xz} and G_{yz} are the shear moduli in the respective axes. In a non-isotropic environment, the parameters C_{2x} and C_{2y} can be different; in this presentation, we shall assume a homogeneous and isotropic environment for simplicity, which allows us to work with one constant value C_2, similarly to the case of C_1.

On the boundary line, the spring constants k_w and k_φ for the displacement w and the rotation φ can be derived in the form $k_w = \sqrt{C_1 C_2}$, $k_\varphi = \frac{1}{2} C_2 \sqrt{C_2/C_1}$. For corner nodes or changes in the curvature of the line (e. g. on a polygonal boundary) a nodal spring constant $\mathcal{K} = \frac{1}{2} C_2 \alpha \mathfrak{K}(\alpha)$ must be added where α denotes the angle measured between normals of adjacent curves and $\mathfrak{K}(\alpha)$ is a certain additional function taking further geometric properties into account. In particular, for $\alpha = \pi/2$ the spring constant $\mathcal{K} \approx \frac{1}{2} C_2$ can be considered; for the justification see [18], p. 60. For multiple subsoil layers, it is possible to calculate C_1 and C_2 from n parameters C_{1i} with $i \in \{1, \ldots, n\}$ in the form

$$C_{1i} = \frac{E_i\,(1-\nu_i)}{h_i\,(1+\nu_i)\,(1-2\nu_i)}, \qquad C_1 = 1 \Big/ \sum_{i=1}^n (1/C_{1i}),$$

$$C_2 = \frac{1}{6} C_1^2 \sum_{i=1}^n \left(\frac{E_i h_i}{1+\nu_i} \left[\left(\sum_{j=i}^n \frac{1}{C_{1j}} \right)^2 + \left(\sum_{j=i}^n \frac{1}{C_{1j}} \right) \left(\sum_{j=i+1}^n \frac{1}{C_{1j}} \right) + \left(\sum_{j=i+1}^n \frac{1}{C_{1j}} \right)^2 \right] \right).$$

3.3. Stiffness matrix reduction

As discussed in Subsection 2.3, the finite element (or similar) techniques are needed for the full 3-dimensional modelling, with the result of the solution of large systems of linear algebraic equations (frequently iterative, handling various nonlinearities, as mentioned at the beginning of the 2nd section), with sparse or banded stiffness matrices as system ones. From the physical point of view, in certain cases where the effects and behaviour of the soil mass have been precisely calculated for the critical loading conditions, these stiffness values can be used for subsequent states that are less significant for the behaviour of the soil mass. This complexity can be reduced by using Schur's complements by [28] and [7]; for their effective applications in numerical analysis see [5]. Schur's complement technique involves partitioning the large square stiffness matrix into particular blocks. Namely the stiffness matrix

$$K = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

contains 4 matrices A, B, C, D: A corresponds to internal degrees of freedom (DOFs), D to boundary DOFs, B, C are cross-reference terms connecting internal and boundary DOFs. The reduction of a matrix K can be then introduced (if D is invertible) as $K/D = A - BD^{-1}C$, $K/A = C - AD^{-1}B$ which implies

$$K^{-1} = \begin{bmatrix} (K/D)^{-1} & -(K/D)^{-1}BD^{-1} \\ -D^{-1}C(K/D)^{-1} & D^{-1} + D^{-1}C(K/D)^{-1}BD^{-1} \end{bmatrix}.$$

Some well-known properties of Schur's complements help to simplify our practical calculations, namely $\mathrm{rank}(K) = \mathrm{rank}(D) + \mathrm{rank}(K/D)$ (rank additivity formula), $A/B = (A/C)/(B/C)$ (quotient identity), etc. Even in the case that A or D is singular, the generalized inverse (pseudoinverse) instead of the standard one on K/A and K/D yields generalized Schur's complement.

4. Illustrative example

A simple example was prepared to verify the implemented formulas for the parameters C_1, C_2. The problem involves a plate of size $10 \times 10 \times 0.3$ m, subjected to the uniform perpendicular load 40 kPa. The material characteristics are $E = 25$ MPa, $\nu = 0.28$, $\gamma = 17$ kPa, the total height is $h = 8$ m.

Three variants of computational modelling cover: i) one single layer with $h = 8$ m, ii) three layers with $h_1 = 5$ m, $h_2 = 2$ m, $h_3 = 1$ m, iii) stress-based calculations (cf. Subsection 3.2). The parameters C_1 and C_2 based on the effective subsoil model were identical in variants i) and ii), with $C_1 = 2.604167$ MPa/m and $C_2 = 3.995028$ MN/m. For the approach iii) using the deformation zone, all results were computed for each finite element at the centroid and the stress distribution below this point was obtained. It was found that the formulae for calculating the stress σ_z are not quite suitable because they approach infinity near the surface (ground level). Therefore it is better to use modified formulas for distributed loads where this effect is eliminated. Implementing such additional formulas shortly is feasible; it requires adjusting the calculation of σ_z to obtain a more accurate distribution only, as shown at Fig. 3 (with non-constant values of C_1) and Fig. 4.

For the variant of reducing the stiffness matrix using Schur's complements, the results are still too large to be displayed for this example. The original stiffness matrix had 7 497 nodes with 6 degrees of freedom with a total number of columns of 44 982, and the total number of non-zero elements was 3 304 413, indicating that a significantly large and sparse system is being solved. For the reduced system, from the original 7 497 nodes to a surface with 441 nodes, the number of columns decreased to 2 646, and the total number of non-zero elements was 1 750 329. We can see that the total size of the matrix decreased 17 times, but the number of non-zero elements decreased approximately 1.9 times. This is therefore advantageous for us, but it is necessary to continue the development and implementation, as well as the use of this reduction in subsequent analyses.

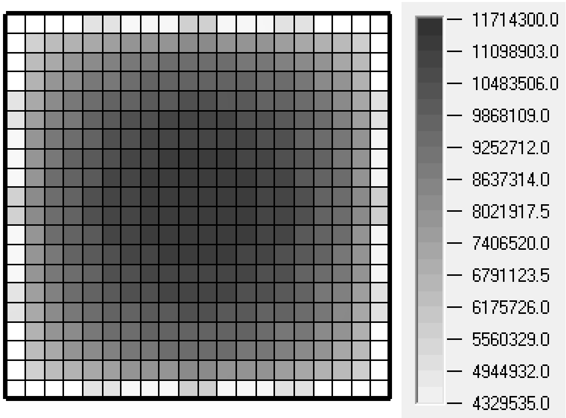

Figure 3: Distribution of the parameter C_1 for variant iii).

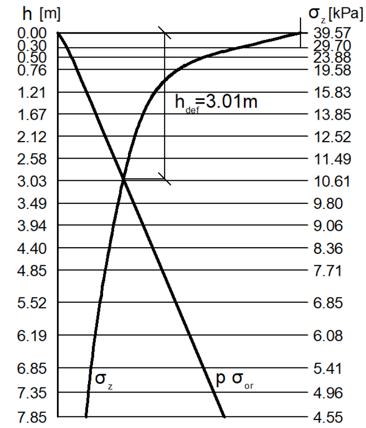

Figure 4: Stress distribution under the finite element.

5. Conclusions

This paper aims to demonstrate the possibility of modelling of the soil-rock mass and its reduction. For surface models, the implementation of an effective subsoil model was presented, including the calculation of parameters C_1 and C_2 for multiple layers of subsoil. The stress calculation σ_z in the subsoil and the determination of the deformation depth were also included. This solution is already fully implemented and can be used in RFEM software in collaboration with FEM consulting.

For advanced modelling using 3D objects, a reduction method via stiffness matrix condensation was proposed. Schur's complement technique was applied to reduce the stiffness matrix, resulting in a smaller system of equations. This solution is not yet fully implemented in the program and cannot be used routinely due to the fact that it is not a fully general solution and can only be applied with the correct setup of the calculated analyses on the computational core side. Therefore, the software user cannot control this part, this can be seen as a major challenge for us in generalizing this approach and releasing it in the relevant software as soon as possible. This reduction is especially significant for subsequent calculations where it is no longer necessary to analyze the soil-rock mass in detail, but rather the superstructure, for example in dynamic problems.

Based on the above findings, our plans focus on further development and improvement of methods for modelling the soil-rock massif and its reduction. Our priority is to complete the implementation into the RFEM software and enable the creation of more test cases more easily. Above all, to determine the appropriateness of using individual variants and identify their limitations. This will require a large number of test and benchmark examples to validate and compare these approaches. In addition, the reduction of the stiffness matrix using Schur's complement can be used in other analyses, not only within the soil-rock massif, which increases the importance of this implementation. So our drive remains unchanged to create the most sophisticated tools possible to support structural engineers in solving increasingly complex problems, both in terms of computational speed and accuracy.

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